MUTUALLY EXCLUSIVE OF COMPLEX SPACETIME

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INTRODUCTION

The phase-space coordinates of a particle are in one-to-one correspondence with the initial conditions which determine its classical motion, i.e. its world-line. Hence the phase space can be identified with either the set of all initial conditions or the set of worldliness of the classical particle. the physical case $s = 3$, i.e. three- dimensional space. Our approach will be to leave space-time intact and, instead, consider its group of symmetries, the *Poincare group*, which is defined as follows: Let u be a four vector. We denote its time component (with respect to an arbitrary reference frame) by u^0 and its space components by $\boldsymbol{u}= (u^1, u^2, u^3)$.

The component connected to the identify (whose elements reverse neither the orientations of time nor of space) is called the restricted Lorentz group and denoted by L_0. If we let $u^4 \equiv cu^o$, so that $uv = -u \cdot v + u^4 v^4$, we can identify L_0 with $SO(3,1)_{+}$. The *Poincare group P* is defined as the set of all Lorentz transformations combined with spacetime.

The classical phase space resulted from the Weyl-Heisenberg group *W*, which, unlike P_o was not a symmetry group of the theory but merely a Lie group generated by the fundamental dynamical observables of position and momentum at a fixed time. We will now see that *W* is related to the non-relativistic limit of P_o in two distinct ways: as a normal subgroup, and as a homogeneous space. This insight will play a key role in generalizing the canonical coherent states to the relativistic case.

Keywords : worldliness, dimensional space, homogeneous space, relativistic case

Galilean Frames

The general case where the configuration space is IR^s instead of IR^3. For simplicity, we restrict ourselves to spinless particles. It is not difficult to include spin, as will be shown later. The states of such particles are described by complex-valued wave functions $f(x.t)$ of position x and time t which are square-integrable with respect to x at any time t. Their evolution in time is given by the Schrödinger equation

$$i\frac{\partial f}{\partial t} = Hf \qquad \ldots(1)$$

where

$$H = -\frac{1}{2m}\Delta \qquad \ldots(2)$$

is the Hamiltonian operator, and Δ is the Laplacian acting on L^2 (IR^s). Since H is self-adjoint, though unbounded, the solutions are given through the unitary one-parameter group

$$U(t) = \exp(-itH):$$

$$f(x,t) = (U(t)f)(x)$$

$$= (2\pi)^{-s}\int_{IR^s} d^s p \exp\left[-itp^2/2m + ip\cdot x\right]\hat{f}(p) \qquad \ldots(3)$$

where it is assumed that $f(x,0)$, hence also its Fourier transform $\hat{f}(p)$, is in $L^2(IR^s)$.

Relativistic Frames

The (unique) relativistically covariant statement of this condition gives rise to a canonical complexification of spacetime which embodies in its geometry the structure of quantum mechanics as well as that of Special Relativity. The complex spacetime also has the structure of a classical phase space underlying the quantum system under consideration. The Klein-Gordon equation, which describes a simple relativistic particle in the same way that the Schrödinger equation describes a non-relativistic particle. The spectral condition will enable us to analytically continue the solutions of this equation to complex spacetime, and the evaluation maps on the space of these analytic solutions will be bounded linear functionals, giving rise to a reproducing kernel.

Our formalism extends them to $z = x - iy$, with the new parameters y playing the role of a control vector for the energy-momentum observables. Thus, in place of a set of pairs of canonically conjugate observables X_k, P_k. we have a set of observables P□ and a dual set of complex parameters z^{μ}. The symmetry between position- and momentum operators in the non-relativistic theory was based on the Weyl-Heisenberg group, and we have seen that this symmetry is "accidental," being broken in the transition to relativity.

The reproducing kernel by itself is of limited use. Although it makes it possible to establish the interpretation of τ_+ as an extended classical phase space, it does not provide us with a direct physical interpretation of the function values $f(x)$. The inner product in k is borrowed from $L_+^2(d\tilde{p})$ hence a probability interpretation exists, so far, only in momentum space. In the standard formulation of Klein-Gordon theory, it is possible to define the inner product in configuration space, but the corresponding density turns out not to be positive, thus precluding a probabilistic interpretation. This is one of the well-known difficulties with the first quantized Klein-Gordon theory, and is one of the reasons cited for the necessity to go to quantum field theory (second quantization. We will see that the phase-space approach does admit a covariant probabilistic picture of relativistic quantum mechanics, thus making the theory more complete even before second quantization. These topics will be discussed further in the next section and the next chapter. At this point we wish only to define an "autonomous" inner product on K as an integral over a "phase space" lying in τ_+. This will provide us with a normal frame of evaluation.

Geometry and Probability

It is therefore reasonable to expect that σ and $d\sigma$ merely represent one choice out of many. Our purpose here is to construct a large natural class of such phase spaces and associated measures to which our previous results can be extended. This class will include σ and will be invariant under P_o. In this way our formalism is freed from its dependence on σ and becomes manifestly covariant. As a byproduct, we find that positive energy solutions of the Klein-Gordon equation give rise to a conserved probability current, so the probabilistic interpretation becomes entirely compatible with the spacetime geometry.

The relativistic quantum mechanics (RQH) is any Poincare' Covariant formulation of quantum mechanics. It is valid for massive particle for all velocities upto velocity of light and accommodated to massless particles. This theory has very wider applications in particle physics, acceleration physics high energy physics, condensed matter physics, atomic physics and chemistry.

Before RQM, for getting prediction of antimatter, spin magnetic moments of elementary spin ½ fermions, quantum dynamics of charged particles in electromagnetic fields and fine structure, terms have to be introduced artificially into the Hamiltonian operator to achieve agreement with the experimental observations facts.

Space and time

In non-relativistic QM one has for a many particle system $\Psi\,(r_1, r_2, \ldots., t, \sigma, \sigma_2 \ldots.)$ while in relativistic mechanics, the spatial coordinates(r) and temporal coordinate (t) are not absolute. The position and time coordinates combine naturally into a four dimensional space-time position X(ct, r) corresponding to events, and the energy and 3 momentum combine naturally into the four momentum $P\left({E}/{c}, P\right)$ of a dynamic particle, as measured in some reference frame, change according to a Lorentz transformation as one measures in a different frame boosted and/or rotated relative the original frame is consideration.

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