Similarity Solution of Cylindrical shock wave with radiation energy & material pressure in Magnetohydrodynamic

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Abstract : The interaction of a perfect gas behind a cylindrical shock wave with radiation energy and material pressure is studied numerically .The proposed physical model is the self similar solution which are solved by similarity method .This physical model and numerical simulation used in the present work can be employed in solving shock wave interactions with other problems.

Keywords : *Radiation Energy, Material Pressure, Magnetohydrodynamic, Radiation Flux*

Introduction

The theory of shock wave and related flows are considerable physical interest Shock waves conceivably driven by solar flares are observed to propagate into interplanetary medium. Rogers [1], Deb Ray [2] have obtained an exact analytic solution for the shock wave problem with an atmosphere of varying density. They have considered the problem taking effect of radiation energy and material pressure shock propagation at very high temperature in which the radiation effects might play a very important role through the coupling of radiation and magnetogasdynamics fields on account of the high temperature, gases are ionized over the entire region of interest in the shock and the medium behaves like a medium of very high electrical conductivity.

The propagation of shock waves has been studied by Greifigner and Cole[3] ,Green span [4], Christer [5] and Ranga Rao [6] without taking into account the radiation effects. Elliot [7], Wang [8] and Helliwell [9], have considered the effect of thermal radiation in their studies of gas dynamic using similarity method of Sedov [10]. Singh [11] and Cheny [12] have studied the problem in ordinary gas dynamics. Theoretical and experimental studied of radiative shock have been treated by Michaut & et al. [13].In recent years several studies have been performed concerning the problems of shock waves in radiative gasdynamics with variable density, in particular Singh et.al[14], Nath[15] ,Srivastava et.al.[16].In this paper we consider the problem of cylindrical shock wave in magnetogasdynamics when the atmosphere is non-uniform and conducting taking counter pressure and radiation flux into account. The radiation pressure and radiation energy have been considered. The gas in the undisturbed field is assumed to be at rest and it is grey and opaque where the explosion along a line in a gas cloud has been discussed

We suppose that the magnetic field H_θ and density ρ_1 of distributions ahead of shock vary as an inverse power of radial distance from centre of symmetry i.e.

$H_{\theta_1} = \frac{A}{r}$ and $\rho_1 = \frac{B}{r^w} \left(-2 \le w \le 2\right)$,where A, B and w are constant.

Self Similar Formulation

The cylindrical polar coordinate, where r is the radial distance from axis of symmetry is used here. The equation of conservation of mass, momentum, magnetic flux and energy in the infinite conduction region behind the wave are,

$$\frac{d\rho}{dt} + \frac{\rho}{r}\frac{\partial}{\partial r}(ru) = 0, \qquad (01)$$

$$\rho\frac{\partial u}{\partial t} + \frac{\partial P}{\partial r} + \frac{\mu H_\theta}{r}\frac{\partial}{\partial r}(rH_\theta) = 0, \qquad (02)$$

Where $P = p + P_r$

$$\frac{dH_\theta}{dt} + H_\theta\frac{\partial u}{\partial r} = 0, \qquad (03)$$

$$\frac{d}{dt}(e + e_r) + (p + p_r)\frac{d}{dt}\left(\frac{1}{\rho}\right) + \frac{1}{\rho r}\frac{\partial}{\partial r}(rq) = 0, \qquad (04)$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r},$$

And ρ is the density, p the pressure, u the radial velocity, H_θ the azimuthal magnetic field; q the heat flux, t the time, e is the material energy, e_r the radiation energy and p_r is the radiation pressure. The magnetic permeability μ is taken to be unity. For an ideal gas we have

$$e = \frac{P\rho}{(\gamma - 1)}, \qquad P = \Gamma\rho T \qquad (05)$$

where γ is the adiabatic gas index, T the temperature and γ the gas constant. Assuming local thermodynamic equilibrium and taking Roseland diffusion approximation we have

$$q = -\frac{cv}{3}\frac{\partial}{\partial r}\left(\sigma T^4\right) \qquad (06)$$

where $\frac{\sigma c}{4}$ is Stefan Boltzmann constant; C the velocity of light; and v, the mean free path of radiation a function of density and temperature. Following Wang [13], we take

$$v = v_0\, \rho^\alpha\, T^\beta \qquad (07)$$

where v_0, α and β are constant

Following Singh & Vishwakarma [14] the jump conditions at an isothermal shock front

are taken to be

$$\rho_2 = N\rho_1, \qquad (08)$$

$$P_2 = \frac{N\rho_1 v^2}{\gamma M^2}, \tag{09}$$

$$H_{\theta_2} = NH_{\theta_1}, \tag{10}$$

$$u_2 = \left(1 - \frac{1}{N}\right) V, \tag{11}$$

$$p_{r2} = \frac{N\rho_1 V^2}{\gamma M^2}, \tag{12}$$

$$q_2 = (N-1)\left[\frac{1}{M_A^2} - \frac{N-1}{2N^2}\right] \rho_1 \; v^3 ; \tag{13}$$

Where suffices 2 and 1 are for the region just behind and just ahead of the shock surface, respectively and V denote the shock velocity. Also

$$N = \left(\alpha + \frac{1}{2}\right) + \left[\left(\alpha + \frac{1}{2}\right)^2 + 2\gamma \; M^2 a\right]^{1/2},$$

$$\alpha = \frac{1}{\gamma}\left(\frac{M_h^2}{M}\right)$$

In which M and M_h denotes the Mach number and Alfven mach number respectively given by

$$M^2 = \frac{\rho_1 V^2}{\gamma P_1} \quad and \quad M_h^2 = \frac{\rho_1 V^2}{\mu H_{\theta_1}^2} \tag{14}$$

Similarity Solution

By the standard dimensional analysis of Sedov [10], the non dimensional variables η is defined by

$$\eta = \frac{m}{n} \; rt^{-\delta}, \text{where} \;\; \delta = \frac{2}{(4-w)}, \;\; (w < 2) \tag{15}$$

$\eta = \left(A^2 / B\right)^{b/2}$ And the dimensionless constant m is defined such that η assumes the value one on the shock front. This choice enables us to write $\eta = \frac{r}{R}$ and $V = \frac{dR}{dt} = \frac{\delta R}{t}$, where R is shock radius.

Then the field variables of the flow variables of the flow pattern in terms of dimensionless function of η are

$$u = \frac{r}{t}\ U\ (\eta)\ , \quad (16)$$

$$P = \frac{A^2}{r^2}\ P\ (\eta)\ , \quad (17)$$

$$H_\theta\ = \frac{A}{r}\ H_\theta\ (\eta)\ , \quad (18)$$

$$\rho = \frac{A^2 t^2}{r^4}\ G\ (\eta)\ , \quad (19)$$

$$q = \frac{A^2}{rt}\ F\ (\eta)\ , \quad (20)$$

$$p_r\ = \frac{A^2}{r^2}\ p_r\ (\eta) \quad (21)$$

Solutions of Equation of Motion The equation (1) – (4) and (6) are now transformed into the forms

$$\frac{dG}{d\eta} = \frac{G}{\eta(U-\delta)}\left[2(U-1)-\eta\ \frac{dU}{d\eta}\right] \quad (22)$$

$$\frac{dP}{d\eta} = \frac{G}{(U-\delta)}\left[\frac{H^2}{G} - (U-\delta)^2\right]\frac{dU}{d\eta} + \frac{G}{\eta}\left[\frac{2P}{G} - U\ (U-1)\right] = 0 \quad (23)$$

$$\frac{dH_\theta}{d\eta} = \frac{-H_\theta}{(U-\delta)}\ \frac{dU}{d\eta} \quad (24)$$

$$\frac{dF}{d\eta} = \frac{G}{(\gamma-1)}\left\{(U-\delta)^2 - \frac{H_\theta^2}{G} + \frac{\gamma\left(P+P_r\right)}{G}\right\}$$

$$-\ \frac{1}{\eta}\ \frac{G\ (U-\delta)}{(\gamma-1)}\ \frac{2\,(P+\Pr)}{G}\ - U\ (U-1) + \left(2\left(P+P_r\right)U + F\right) \quad (25)$$

$$\frac{du}{d\eta} = \frac{2\left(P+P_r\right)}{G}\left(U-2\delta+1\right) - U\ (U-1)\,(U-\delta) + \frac{F}{K}\ \frac{G^{\beta-\alpha+3}}{\left(P^{\beta+3}+P^{1\beta+3}\right)}\ (U-\delta) \quad (26)$$

$$\left[(U-\delta)^2 - \frac{H_\theta^2}{G} + \frac{\left(P+P_r\right)}{G}\right]$$

where $k = \frac{4c\sigma v_0}{3\Gamma A}$, a non dimensional radiation parameter. $F\ = L G^{\alpha'-\beta'+4} p^{\beta'+4}\left[\frac{1}{P}\ \frac{dP}{d\eta} - \frac{1}{G}\ \frac{dG}{d\eta}\right]$

(27)

Where

$$L=\frac{4\sigma c v_0\,\rho_c^{\alpha^1-1}}{3\gamma^{(\beta'+4)}} \qquad \alpha^1=\frac{1}{2}\text{ and }\beta^1=-3$$

The transformed shock conditions (8) – (13) are

$$U(1)=\left(1-\frac{1}{N}\right)\delta \tag{28}$$

$$H_\theta\,(1)=N \tag{29}$$

$$G\,(1)=\frac{N\,M_h^2}{\delta^2} \tag{30}$$

$$P\,(1)=\frac{N\,M_h^2}{\gamma M^2} \tag{31}$$

$$p_r\,(1)=\frac{NM_h^2}{M^2} \tag{32}$$

$$F(1)=L\,M_h^2\,\delta \tag{33}$$

where $L=(N-1)\left[\dfrac{1}{M_h^2}-\dfrac{N-1}{2N^2}\right]$,

Result and Discussion

In order to exhibit the numerical solution it is convenient to write the field variables in the non dimensional form as

$$\frac{u}{u_2}=\eta\frac{U(\eta)}{\left(1-\frac{1}{N}\right)\delta}\quad , \tag{34}$$

$$\frac{\rho}{\rho_2}=\frac{\delta^2}{\eta^4\;N\;M_h^2}\quad G\;(\eta)\;, \tag{35}$$

$$\frac{P}{P_2}=\frac{1}{\eta^2}\;\frac{\gamma M^2}{N\,M_h^2}\;P\;(\eta)\quad , \tag{36}$$

$$\frac{Pr}{Pr_2}=\frac{1}{\eta^2}\;\frac{\gamma M^2}{N\,M_h^2}\;Pr\;(\eta)\quad , \tag{37}$$

$$\frac{H_\theta}{H_{\theta_2}}=\frac{1}{\eta}\;\frac{1}{N}\;H\;(\eta)\quad , \tag{38}$$

$$\frac{q}{q_2}=\frac{1}{\eta\;M_h^2\;\delta}\,\eta^2\;\frac{P(\eta)}{G(\eta)}\quad , \tag{39}$$

$$\frac{T}{T_2} = \gamma \left(\frac{M}{\delta}\right)^2 \eta^2 \frac{P(\eta)}{G(\eta)}$$

The numerical results for certain choice of parameter are reproduced in tabular form. We have calculated our results for $\gamma = 1.4, M_h^2 = 20, \; M^2 = 10$, $\delta = 5, 2$ and $k = 10$ The nature of field variables is illustrated through table 01 and 02.

We can easily see that the radiation parameters has an important effect on the flow variables. Magnetoradiative effects are prominent on the field variables when we compare our results with the results of ordinary gas dynamics.

Table-01

$K = 10, \; \delta = 2$

η	u/u_2	ρ/ρ_2	p/p_2	H/H_2	q/q_2	T/T_2
1.00	1.200000	1.200000	1.200000	1.200000	1.200000	1.200000
0.99	1.189713	1.211770	1.233358	1.259207	2.205242	1.201495
0.98	1.179343	1.237695	1.252275	1.288322	3.225350	1.104061
0.97	1.168883	1.258038	1.277232	1.317315	4.252065	1.207675
0.96	1.158326	1.283072	1.298743	1.346149	6.297158	1.212310
0.95	1.147663	1.303076	1.317355	1.374785	7.312421	1.217943
0.94	1.136886	1.328336	1.343649	1.413179	8.339657	1.224552
0.93	1.125985	1.349134	1.388237	1.431281	10.350662	1.232112
0.92	1.114949	1.355754	1.391761	1.459036	11.377211	1.240601
0.91	1.103765	1.378466	1.424887	1.496383	13.491037	1.249999
0.90	1.192420	1.397525	1.454297	1.523254	15.513795	1.260283
0.89	1.180898	1.423162	2.512686	1.549573	17.537041	1.271435
0.88	1.169183	1.435570	2.538742	1.575256	18.55182	1.283434
0.87	1.157254	1.464890	2.557137	1.590211	20.570432	1.296262
0.86	1.145091	1.491199	2.578501	1.614334	22.612750	1.209902

Table-02

$K = 10, \; \delta = 5$

η	u/u_2	ρ/ρ_2	p/p_2	H/H_2	q/q_2	T/T_2
1.00	1.200000	1.200000	1.200000	1.200000	1.200000	1.200000
0.99	1.180721	1.227090	1.308584	1.402352	1.557591	1.551374
0.98	1.171376	1.252716	1.356953	1.424803	2.575334	1.593582
0.97	1.161957	1.277757	1.456264	1.457320	4.597607	1.616606
0.96	1.152454	1.303175	1.487805	1.479862	5.619113	1.650427
0.95	1.132855	1.350017	1.493009	1.502382	6.664906	1.705027
0.94	1.093147	1.459420	1.534625	1.534821	7.690399	1.720391
0.93	1.083317	1.492608	1.550913	1.637111	9.701375	1.766503
0.92	1.073348	1.530890	2.597283	1.699173	10.733997	1.793348
0.91	1.053221	2.585648	2.614659	1.710912	12.754799	1.820913
0.90	1.042917	2.628327	2.655302	1.752220	13.790675	1.859184
0.89	1.032411	2.666048	2.701621	1.782970	15.808854	1.878150
0.88	1.021676	2.693373	3.736155	1.793013	17.826841	1.897801
0.87	1.010680	3.718656	3.751529	1.822180	19.852347	1.908128
0.86	1.009388	3.737572	3.790382	1.880276	21.893165	1.929125

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