AESTHETICS OF MATHEMATICS

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Abstract: Mathematics is the most prestigious subject in the curriculum at any level. It is one subject where the gap between the intended and the implemented objectives is wide. As a consequence, it is considered as the most difficult subject and abhorred by a majority of the educated people. This attitude of parents and educated people in the society is creating a negative impact upon the younger generation. But there is a world of mathematics that most people live and die without ever getting to see. The aim of this paper is to present this other world before all to introduce them the extreme beauty of mathematics.

Keywords: aesthetics, golden ratio, birkhoff, Fibonacci sequence.

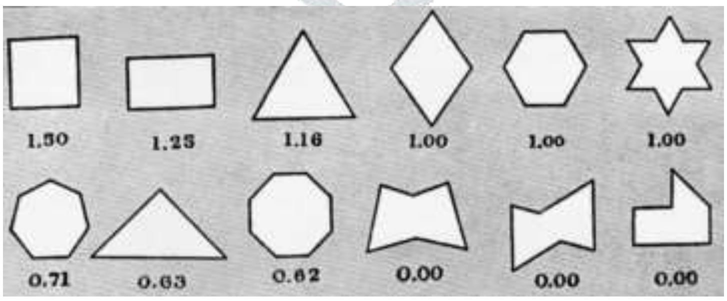
WHAT DOES IT MEAN FOR MATHS TO BE BEAUTIFUL?

Its not about the appearance of the symbols on the page. That, at best, is secondary. Maths becomes beautiful through the power and elegance of its arguments and formulae through the bridges it builds between previously unconnected worlds. For those who learn the language, maths has the same capacity for beauty as heart, music, a full blanket of stars on the darkest night .

AESTHETICS BY BIRKHOFF

Mathematician George David Birkhoff (1884-1944) is best known for his work on differential equations and dynamics. His Ergodic theorem gave the kinetic theory of gases a rigorous basis. He solved important problems in celestial mechanics and made contributions to the mathematical foundations of relativity theory and quantum mechanics. Birkhoff also had a keen interest in aesthetics; the qualities that make a painting, sculpture, musical compositions, or poem pleasing to the eye, ear, or mind. He sought a formula, a mathematical measure, that would capture an object's beauty. As an undergraduate at Harvard, he was intrigued by the structure of western music and pondered the riddle of what makes something melodious. In the early 1930s, Birkhoff spent a year traveling around the world studying art, music and poetry in various countries. He came up with a formula that encapsulated his insights into aesthetic value and described his theory in a 1933 book, Aesthetic Measure, published by Harvard University Press. Professor Birkhoff presents the following mathematical formulation of the fundamental problem: "Within each class of aesthetic objects, to define the order and the complexity so that their ratio M=O/C yields the aesthetic measure of any object in the class." Where M is aesthetic measure, O is aesthetic order, and C is complexity. In other words, Birkhoff put a high aesthetic value on orderliness and a low one on complexity. In his view, beauty increases as complexity decreases. Here's how Birkhoff's formula for aesthetic measure could be applied to isolated polygonal figures. In this case, $O = V + E + R + HV – F$, where V is vertical symmetry, E is equilibrium, R is rotational symmetry, HV is the relation of the polygon to a horizontal-vertical network, and F is a general negative factor (unsatisfactory form). C is the number of distinct straight lines containing at least one side of the polygon.

AESTHETIC SCALE AND POLYGON:



Applied to polygons, birkhoff's formula for aesthetic value gives a square the highest rating.. A five – pointed star similar to the one that appears on the flag of united states has a rating of 0.90. his thoery applies to polygonal forms, ornaments and tilings and vases.

AESTHETIC SCALE AND PAINTING

Birkoff used white lines, following the composition's principal lines, to show how antonio correggio's 1531 painting "DANAE" consists of comprehensible geometric forms.The 'complexity' of paintings is usually so considerable that they are analogous to ornamental patterns whose constituent ornaments must be appreciated one by one. However, it is decidedly interesting to remark in this connection how a fine composition is always arranged so as to be easily comprehensible. In any work of art, imaginary lines can be drawn across from point to point, following the principal lines of the composition. These lines define geometric areas that generally have a comprehensible form. Similarly, light and dark areas have a certain order or pattern. There should be a natural primary center of interest in the painting and also suitable secondary centers. Such a primary center of interest is often taken in the central vertical line of the painting or at least near to it. The elements of order are of course taken to be the same as in the three-dimensional object represented. Finally there are the connotative elements which play a decisive part; a good painting requires a suitable subject just as much as a poem requires a poetical idea.

AESTHETIC SCALE AND POETRY

Birkhoff's formula for poetry, $O = aa + 2r + 2m - 2ae - 2ce$, where *aa* stands for alliteration and assonance, *r* for rhyme, *m* for musical sounds, *ae* for alliterative excess, and *ce* for excess of consonant sounds.Here's a poem by Alfred Lord Tennyson (1809–1892) that ranks near the top on Birkhoff's aesthetic scale (0.77):

Come into the garden, Maud,

For the black bat, Night, has flown,

Come into the garden, Maud,

I am here at the gate alone;

And the woodbine spices are wafted abroad,

And the musk of the roses blown

Birkhoff himself attempted to apply the principles embedded in his formula to the composition of a poem. This is what he came up with:

Wind and wind the wisps of fire,

Bits of knowledge, heart's desire;

Soon within the central ball

Fiery vision will enthrall.

Wind too long or strip the sphere,

See the vision disappear!

This short poem earns an aesthetic rating of 0.62.

Birkhoff suggested that his formula has practical value. He indicates very briefly how application of this theory may be made, in decorative design, painting, sculpture, architecture, and music, to include the qualitative aspects of form. A glimpse of the quixotic nature of Birkhoff's quest to quantify beauty, however, was revealed in his "practical" advice on what shape a movie screen should have. In the days of silent movies, the proportion of the length to the height of the screen was 4 to 3. When sound films were introduced, a portion of the width of the film was used for the soundtrack, leaving a narrower picture that was almost square. Even though the square is the most ideal of all polygons, but that doesn't necessarily mean that the square is the best shape for a movie screen. Any obvious numerical ratio of dimensions such as 1 to 1 or 2 to 1 is to be avoided in a picture frame, because it is often desirable that the rectangle be a purely neutral accessory, not producing irrelevant associations. Birkhoff himself conceded that an intuitive appreciation is better than any attempt to analyze the source of one's delight in beautiful object. Nevertheless, he argued, that pleasure is due to an unconscious appreciation of the mathematical proportions of the object. The 2 to 1 proportion has the additional disadvantage that it isn't well adapted to fill the circular field of effective vision. And a rectangle that is nearly a square is disagreeable because of the resulting ambiguity.

MATHEMATICS AND ART

Mathematics and art are related in a variety of ways. Mathematics has itself been described as an art motivated by beauty. Mathematics can be discerned in arts such as music, dance, painting, architecture, sculpture, and textiles. Mathematics and art have a long historical relationship. Artists have used mathematics since the 4th century BC when the Greek sculptor Polykleitos wrote his *Canon*, prescribing proportions based on the ratio 1:√2 for the ideal male nude. Persistent popular claims have been made for the use of the golden ratio in ancient art and architecture, without reliable evidence. In the Italian Renaissance, Luca Pacioli wrote the influential treatise *De Divina Proportione* (1509), illustrated with woodcuts by Leonardo da Vinci, on the use of the golden ratio in art. Another Italian painter, Piero Della Francesca, developed Euclid's ideas on perspective in treatises such as *De Prospectiva Pingendi*, and in his paintings. The engraver Albrecht Dürer made many references to mathematics in his work *Melencolia I*. In modern times, the graphic artistM. C. Escher made intensive use of tessellation and hyperbolic geometry, with the help of the mathematician H. S. M. Coxeter, while the De Stijl movement led by Theo van Doesberg and Piet Mondrian explicitly embraced geometrical forms.

MUSIC AND MATHEMATICS

Music theory has no axiomatic foundation in modern mathematics, yet the basis of musical sound can be described mathematically (in acoustics) and exhibits "a remarkable array of number properties". Elements of music such as its form, rhythm and metre, the pitches of its notes and the tempo of its pulse can be related to the measurement of time and frequency, offering ready analogies in geometry. The attempt to structure and communicate new ways of composing and hearing music has led to musical applications of set theory, abstract algebra and number theory. Some composers have incorporated the golden ratio and Fibonacci numbers into their work. Western music has its roots in the harmonics discovered by Pythagoras – himself a keen musician – over 2000 years ago. Pythagoras noticed that certain string ratios would produce sounds that were in harmony with each other.

SYMMETRY, GEOMETRY AND DANCE

Dance comes in many different forms. All types of dancing, from break-dancing to ballet, use ideas found in maths. Dancers must understand symmetry and geometry, as well as being able to count in time to the music. Choreographers also use these ideas to design new dances.A dancer while spinning around ,to stay in control and not get dizzy, use a technique called ‘spotting’. As they turn their body, they keep their head fixed for as long as possible, and then quickly rotate their neck to catch up with their body. They try to keep their head looking in the same direction after each rotation, because this helps them to balance and prevent dizziness.

The power of symmetry

Symmetry is important, but so is geometry. Dancers form shapes with their bodies, and choreographers think about how to use lines and angles to make their dances more interesting. A choreographer called Rudolf Laban, has even created a system of notation for dance that can be manipulated like a mathematical equation.

Dancers use symmetry and geometry to improve their performances and make them visually appealing. We enjoy looking at symmetrical things because our brains like to hunt for patterns. We like regular geometrical shapes for the same reason, and these often form the basis for dance. So when a dancer lifts their arms together, or traces out a square as they move, they’re making use of maths to create a better dance.

THE MATH BEHIND THE BEAUTY

Mathematics has got a lot to do with beauty. Physical attraction depends on **ratio**. Our attraction to another person's body increases if that body is symmetrical and in proportion. Likewise, if a face is in proportion, we are more likely to notice it and find it beautiful. Scientists believe that we perceive proportional bodies to be more healthy.

Leonardo da Vinci's drawings of the human body emphasised its proportion. The ratio of the following distances in the Vitruvian Man image is approximately the Golden Ratio:
(foot to navel) : (navel to head)
Similarly, buildings are more attractive if the proportions used follow the Golden Ratio.

Golden Ratio

The Golden Ratio (or "Golden Section") is based on **Fibonacci Numbers**, where every number in the sequence (after the second) is the sum of the previous 2 numbers:
1, 1, 2, 3, 5, 8, 13, 21, ...
we will see (below) how the Fibonnaci Numbers lead to the **Golden Ratio:** Φ = 1.618 033 ...

The Fibonacci Sequence

Let's look at the ratio of each number in the Fibonacci sequence to the one before it:
Let’s create a new sequence of numbers by dividing each number in the Fibonacci sequence by the previous number in the sequence….
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...
So, dividing each number by the previous number gives: 1 / 1 = 1, 2 / 1 = 2, 3 / 2 = 1.5, and so on up to 144 / 89 = 1.6179…. The resulting sequence is:
1, 2, 1.5, 1.666..., 1.6, 1.625, 1.615…, 1.619…, 1.6176…, 1.6181…, 1.6179…
We notice that they keep oscillating around and getting tantalizingly closer and closer to 1.618—the value of phi: the golden ratio. Indeed, completely unbeknownst to Fibonacci, his solution to the rabbit population growth problem has a deep underlying connection to the golden ratio that artists and architects have used for thousands of years.This ratio was used by architects and artists throughout history to produce objects of great beauty (like Michelangelo's "David" and the Greek temples.)

Phi (Φ) is like pi (π) in the sense that it is an irrational number. There is no equivalent fraction for Φ and its decimal keeps going and never stops.
The Golden Ratio also occurs in nature, in the patterns we see in sunflowers, pine cones and so on. This is largely because one of the best ways to efficiently pack things tightly together is using the Fibonacci sequence.

SOME EXAMPLES IN NATURE WHERE WE SEE MATH BEHIND BEAUTY:

- Bright, bold and beloved by bees, sunflowers boast radial symmetry and a type of numerical symmetry known as the Fibonacci sequence, which is a sequence where each number is determined by adding together the two numbers that preceded it. For example: 1, 2, 3, 5, 8, 13, 21, 24, 55, and so forth.
- This is not uncommon; many plants produce leaves, petals and seeds in the Fibonacci sequence. It's actually the reason it's so hard to find four-leaf clovers.
- Sunflowers can pack in the maximum number of seeds if each seed is separated by an irrational-numbered angle.
- The nautilus, showcase Fibonacci numbers. A nautilus shell is grown in a Fibonacci spiral. The spiral occurs as the shell grows outwards and tries to maintain its proportional shape
- Not every nautilus shell makes a Fibonacci spiral, though they all follow some type of logarithmic spiral.
- Pinecones have seed pods that arrange in a spiral pattern. They consist of a pair of spirals, each one twisting upwards in opposing directions.
- The number of steps will almost always match a pair of consecutive Fibonacci numbers. For example, a three–to–five cone meets at the back after three steps along the left spiral and five steps along the right
- Hexagonal figures in honeycombs.
- Honeycombs are an example of wallpaper symmetry. This is where a pattern is repeated until it covers a plane.
- Mathematicians believe bees build these hexagonal constructions because it is the shape most efficient for storing the largest possible amount of honey while using the least amount of wax. Shapes like circles would leave gaps between the cells because they don't fit perfectly together.
- Spiders create near-perfect circular webs that have near-equal-distanced radial supports coming out of the middle and a spiral that is woven to catch prey.
- Some scientists theorize that orb webs are built for strength, with radial symmetry helping to evenly distribute the force of impact when a spider's prey makes contact with the web. This would mean there'd be less rips in the thread.
- The shape of a perfect room was defined by the architects of the Renaissance to be a rectangular-shaped room that has a certain ratio among its walls—they called it the "golden section." A rectangular room with the golden-section ratio also has the property that the ratio between the sum of the lengths of its two walls (the longer one and the shorter one) to the length of its longer wall is also the golden section, 1 plus the square root of 5 over 2. Architects today still believe that the most harmonious rooms have a golden-section ratio. This number appears in many mathematical phenomena and constructions (e.g., the limit of the Fibonacci sequence). Leonardo da Vinci observed the golden section in well-proportioned human bodies and faces—
in Western culture and in some other civilizations the golden-section ratio of a well-proportioned human body resides between the upper part (above the navel) and the lower part (below the navel).
- Mosaic is an art form where solid pieces (wood, stone, glass, etc.) are assembled on a flat surface with no overlaps and no gaps. In its sophisticated form, the mosaic has recognizable patterns, which are repeated in two different directions, where no center, no boundary, no preferred direction, and no focus, is identified. The pattern of a mosaic work gives a sense of infinity.
In mathematical terms, mosaics are referred to as tiling. To form a tiling means to fully cover a two-dimensional plane with geometric forms (polygons or shapes bounded by curves) with no overlaps. A tiling is called symmetric if one can virtually rotate it or reflect it without causing the tiling picture to change. There are different angles of rotation (half twist, quarter twist, etc.) and different axes of mirror reflection (horizontal, vertical, diagonal, etc.). A rich mosaic represents a variety of tiling symmetries, which can be understood via the resulting geometry.
The most impressive mosaics in history were done by artists working in the Islamic World in the Middle Ages, and, in particular, those who created the beautiful, sophisticated mosaic of the Alhambra Palace in Spain. The Alhambra was built by the Moors in the beginning of the thirteen century, on a red-soil hill overlooking the old city of Granada. It is a showcase of Muslim architecture and design with an immense wealth of patterns, ornaments, calligraphy, and stone carvings. It includes virtual night-sky ceilings crafted from thousands of pieces of wood, as well as the most sophisticated, gorgeous, colorful mosaics.
The Dutch artist M. C. Escher paid two visits to the Alhambra to sketch and catalogue the ornate patterns that are found in the tilings throughout the palace and surrounding courtyards. Escher's tilings are not necessarily periodic, meaning that the tiles appear or occur at regular intervals. In the late nineteenth century it was proven that, from a mathematical point of view, there are only seventeen possible symmetries. In the early twentieth century, it was discovered that the tilings in the Alhambra represent all possible seventeen symmetries! Hundreds of years of skilled construction, tiling, a deep respect for symmetry (as a harmonious force), and the study and knowledge of geometry (for religion as well as for commerce) resulted in all seventeen possible symmetry groups being represented on the Alhambra walls! In 1944, the assertion that all seventeen symmetries could be found there was challenged, but lately, once flexibilities of colors and interlacing were introduced, it was verified again.
- Crystals in nature (e.g., snowflakes, minerals, precious stones) are atomically built with order and with respect to symmetry rules. They contain two, three, four, and six-fold structural symmetries, and they are periodic. Aperiodic tiling, i.e., tiling with

no periodicity, was proven mathematically possible in the 1960s, but it was then believed that there were no solid structures in nature with order but with no periodicity. In 1982, Dan Shectman, a Professor at the Technion in Israel, predicted the existence of nature-made aperiodic crystals, later known as quasicrystals (left). A quasicrystal should have a polygon with five edges in its pattern. The first such nature-made stones were found in a Russian mountainous region. In 2009, this discovery was announced scientifically by Paul Steinhardt, a Professor at Princeton. In 2011, Shectman received the Nobel Prize in Chemistry for his prediction.

CONCLUSION:

Mathematics is the Queen of the Sciences. It forms the basis of Science, Engineering and Technology. Thus to teach mathematics at all levels of the educational system is an extremely important and honorable part of mathematical activity that is difficult to overstate. It may happen that in thinking and talking about Mathematics, some people emphasize its merely educational component but it is special type in comparison with all other sciences which is based on logic and has beauty and harmony as its strongest criteria. The most powerful results in Mathematics are surely the most beautiful, and the aesthetic component is a driving force of mathematics.

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