REVIEW OF ORIGIN AND DISCOVERY OF VERSATILE DIGIT ZERO

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ABSTRACT: *In this paper, we study about the invention of the most important number zero. There are versatile applications of zero in different scientific fields. On surveying the literature available, we have pointed out the ideas--about the origin of the number zero; how it was symbolized and used as a mathematical number; how it becomes popular throughout the world and finally the Indians credits for coming up on their own. It has been established that the great fundamental contribution of ancient India to the progress of civilization is the invention of the number zero including the decimal number system of numeration. Although today, the division of zero by zero (0/0) is a concept in philosophy, mathematics and physics without a definite solution.*

Keywords**:** Zero, Infinity, Indeterminate, Unique zero, History of zero, Invention of zero, etc.

I. INTRODUCTION

Historically, the word “zero” came via the French word zéro, and cipher came from the Arabic word safira which means “it was empty.” Also sifr, meaning “zero” or “nothing,” was the translation for the Sanskrit word sunya, which means void or empty. The story of zero is actually a story of two zeroes: zero as a symbol to represent nothing and zero as a number that can be used in calculations and has its own mathematical properties. It is the last digit to come into use. In the English language, zero may also be called nil when a number, when a numeral, and nought/naught in either context. In the English language, 0 may be called - zero, oh, null, nought/naught or nil. The first indubitable appearance of a symbol for zero appears in 876 in India on a stone tablet in Gwalior. Documents on copper plates, with the same small o in them, dated back as far as the sixth century C.E. abound. 0 (zero) was the last numerical digit to come into use. 0 (zero) is both a number and a numerical digit used to represent that number in numerals. As a number, zero means *nothing*—an absence of other values. It plays a central role in mathematics as the identity element of the integers), real numbers, and many other algebraic structures. As a digit, zero is used as a placeholder in place value systems.

In around 1200, Leonardo Fibonacci wrote *Liber Abaci* where he described the nine Indian symbols together with the sign ‘0’. However, the concept of *zero* took some time for acceptance. It is only around 1600 that *zero* began to come into widespread use after encountering a lot of supports and criticisms from mathematicians of the world. That is how *shunyam* given by our forefathers was recognised in the world and made its place permanently as *zero*. Interestingly, the word *zero* probably came from Sanskrit word for *shunyam* or the Hindi equivalent of *shunya*. The word *shunyam* was translated to Arabic as *al-sifer*. Fibonacci mentioned it as *cifra* from which we have obtained our present *cipher*, meaning empty space. From this original Italian word or from alteration of Medieval Latin *zephirum*, the present word *zero* might have originated. Mathematicians in ancient India are credited with being the pioneers in both these inventions. The whole world routinely uses the decimal number system with the ten digits 0,1,2,3,4,5,6,7,8,9 serving to build arbitrarily large numbers with the power of their place value. This is the so-called Hindu Arabic system, developed in India and propagated through the Arabic Mathematical sources across Europe.

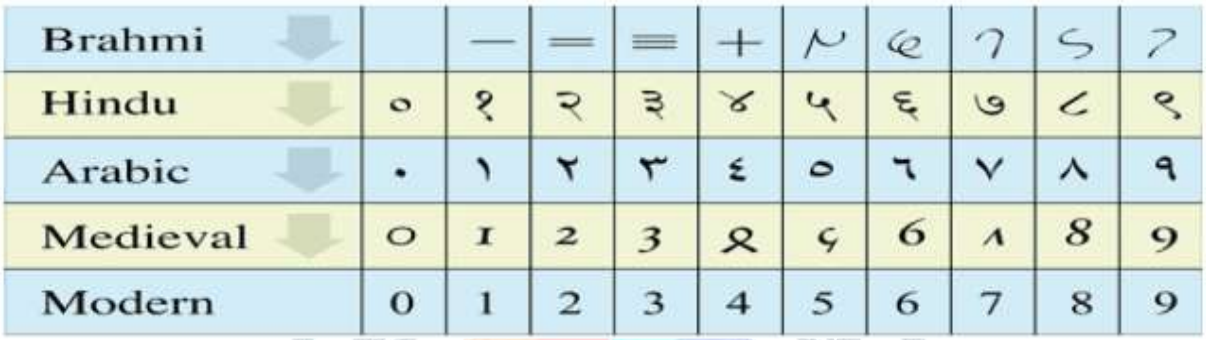
II. ETYMOLOGY

The word, *zero* comes through the Arabic literal translation of the Sanskrit *śūnya* meaning *void* or *empty*, into *ṣifr* meaning *empty* or *vacant.* Through transliteration this became *zephyr* or *zephyrus* in Latin. The word *zephyrus* already meant "west wind" in Latin; the proper noun Zephyrus was the Roman god of the west wind (after the Greek god Zephyros). With its new use for the concept of zero, zephyr came to mean a light breeze—"an *almost*

nothing." This became *zefiro* in Italian, which was contracted to *zero* in Venetian, giving the modern English word. In different languages it known by different name with different meaning. it is given below:
1. Czech/Slovak: *cifra*, digit; *šifra,* cipher, 2. Danish: *ciffer*, digit, 3. Dutch: *cijfer*, digit, 4. French: *zéro*, zero, 5. German: *Ziffer*, digit, figure, numeral, cipher 6. Hindi: *shunya*, 7.Hungarian: *nulla*, 8.Italian: *cifra*, digit, numeral, cypher; *zero*, zero, 9. Kannada: *sonne*, 10. Norwegian: *siffer*, digit, numeral, cypher; *null*, zero, 11. Persian: *Sefr*, 12. Polish: *cyfra*, digit; *szyfrować,* to encrypt; *zero*, zero Portuguese: *cifra*, figure, numeral, cypher, code; *zero*, zero, 13. Russian: *цифра (tsifra),* digit, numeral; *шифр (shifr)* cypher, code, 14. Slovenian: *cifra*, digit, 15. Spanish: *cifra*, figure, numeral, cypher, code; *cero*, zero, 16. Swedish: *siffra*, numeral, sum, digit; *chiffer*, cipher, 17. Serbian: *цифра (tsifra),* digit, numeral; *шифра (shifra)* cypher, code; *нула (nula),* zero, 18. Turkish: *Sıfır, 19.* Urdu: *Sifer*, Anda, Zero.

III. THE DECIMAL NUMBER SYSTEM

The Indian numerals are elements of Sanskrit and existed in several variants well before their ormal publication during the late Gupta Period (c. 320-540 CE). In contrast to all earlier number systems, the Indian numerals did not relate to fingers, pebbles, sticks or other physical objects.



IV. DIFFERENCE BETWEEN OVAL AND CIRCULAR-SHAPED ZERO

The number and the letter O are both round, but there is great difference in modern science. The difference is important on a computer. For one thing, a computer will not do arithmetic with the letter O, because it does not know that it should have been a zero. The oval-shaped zero and circular letter O came into use together on modern character displays. I t is shown in fig. [1]: (a) and (b) as:

Oval-shaped zero Circular-shaped zero

Fig. [1]: (a) and (b)

The zero with a dot in the centre seems to have begun as a choice on IBM-3270 controllers (this has the problem that it looks like the Greek letter theta). The slashed zero, looking like the letter O with a diagonal line drawn inside it, is used in old-style ASCII graphic sets that came from the default type-wheel on the well-known ASR-33 teletype. This format causes problems because it looks like the symbol, representing the empty set, as well as for certain Scandinavian languages which use Ø as a letter.

V. SIMPLE MEANING OF ZERO

Zero is a special number. If there are zero things, there are no things at all. There are none. For example, if Vinay has zero hats, that means he does not have a hat at all. Some peculiar properties of zero are:

- The zero is not a number but it is one of the digit.
- The zero is a whole number.
- The zero is not a positive number.
- The zero is not a negative number.

- The zero is neither negative nor positive.
- The zero is a neutral number.
- Any number divided by itself equals one, except if that number is zero. In operations: 0 ÷ 0 = not a number, but it is the indeterminate form.

VI. DISCOVERY OF ZERO

Initially, the zero as a number was not available. There was the idea of empty space, which may be thought conceptually similar to zero. Babylonians around 700 BC uses three hooks to denote an empty place in the positional notation.

Around 650 AD, the use of zero as a number came into Indian mathematics. The Indian used a place-value system and zero was used to denote an empty place. In fact there is evidence of an empty placeholder in positional numbers from as early as 200AD in India. Around 500AD Aryabhata devised a number system, which had no zero, as a positional system, but used to denote empty space. There is evidence that a dot had been used in earlier Indian manuscripts to denote an empty place in positional notation. India's first satellite Aryabhata, was named after him. The lunar crater Aryabhata is named in his honor.

In 628 AD, **Brahmagupta** wrote ***Brahmasphutasiddhanta*** (The Opening of the Universe), and attempted to give the rules for arithmetic involving zero and negative numbers. He explained that given a number then if you subtract it from itself you obtain zero. He gave the following rules for addition, which involve zero: The sum of zero and a negative number is negative, the sum of a positive number and zero is positive; the sum of zero and zero is zero. Similarly, he gave the correct rules for subtraction also.

Brahmagupta then said that any number when multiplied by zero is zero but when it comes to division by zero, he gave some rules that were not correct. However, it was an excellent attempt to visualize number system in the light of negative numbers, zero and positive numbers.

In 830, another Indian mathematician **Mahavira** wrote *Ganita Sara Samgraha* (Collections of Mathematics Briefings), which was designed as an update of Brahmagupta's book. He correctly stated the multiplication rules for zero but again gave incorrect rule for division by zero.

After 500 years of Brahmagupta, mathematician **Bhaskara** tried to solve the problem of division by stating that any number divided by zero as infinity. Well, conceptually though it is still incorrect, however, Bhaskara did correctly state other property of zero, such as square of zero is zero and square root of zero is also zero.

It is therefore clear that Indian mathematicians developed the concept of zero and stated different mathematical operations involved with zero. But this the concept spread to all over the world.

VII. APPROXIMATION OF VALUE OF (π)

Aryabhata worked on the approximation for pi (π), and may have come to the conclusion that π is irrational. In the second part of the *Aryabhatiyam* (gaṇitapāda 10), he writes: *caturadhikam śatamaṣṭaguṇam dvāṣaṣṭistathā sahasrāṇām ayutadvayaviṣkambhasyāsanno vṛttapariṇāhaḥ*. "Add four to 100, multiply by eight, and then add 62,000. By this rule the circumference of a circle with a diameter of

20,000 can be approached." This implies that the ratio of the circumference to the diameter is ((4 + 100) × 8 + 62000)/20000 = 62832/20000 = 3.1416, which is accurate to five significant figures. It is speculated that Aryabhata used the word *Samna* (approaching), to mean that not only is this an approximation but

that the value is incommensurable (or irrational). If this is correct, it is quite a sophisticated insight, because the irrationality of pi was proved in Europe only in 1761 by Lambert. After Aryabhattya was translated into Arabic (820 CE) this approximation was mentioned in Al-Khwarizmi's book on algebra.

VIII. RULES OF BRAHMAGUPTA

Brahmagupta lived in Ujjain, India from 598 to 668 AD and made notable contributions to mathematics and astronomy. In 628 AD he is reported to have written a text called the *Brahmasphutasiddhanta, The Opening of the Universe* in English, within which is found the earliest known attempt of defining division by zero.

The rules governing the use of zero appeared for the first time in the book *Brahmasputha Siddhanta* written in 628 by **Brahmagupta** (598-670). Here, Brahmagupta considers not only zero but also negative numbers, and the

algebraic rules for the elementary operations of arithmetic with such numbers. In some instances, his rules differ from themodern standard. Brahamagupta's rules are given below:

- The sum of two positive quantities is positive.
- The sum of two negative quantities is negative.
- The sum of zero and a negative number is negative.
- The sum of a positive number and zero is positive.
- The sum of zero and zero is zero.
- The sum of a positive and a negative is their difference; or, if they are equal, zero.
- In subtraction, the less is to be taken from the greater, positive from positive.
- In subtraction, the less is to be taken from the greater, negative from negative.
- When the greater however, is subtracted from the less, the difference is reversed.
- When positive is to be subtracted from negative, and negative from positive, they must be added together.
- The product of a negative quantity and a positive quantity is negative.
- The product of a negative quantity and a negative quantity is positive.
- The product of two positive, is positive.
- Positive divided by positive or negative by negative is positive.
- Positive divided by negative is negative. Negative divided by positive is negative.
- A positive or negative number when divided by zero is a fraction with the zero as denominator.
- Zero divided by a negative or positive number is either zero or is expressed as a fraction with zero as numerator and the finite quantity as denominator.
- Zero divided by zero is zero.

In saying "zero divided by zero is zero," Brahmagupta differs from the modern position. Mathematicians normally do not assign a value, whereas computers and calculators will sometimes assign NaN, which means "not a number." Moreover, non-zero positive or negative numbers when divided by zero are either assigned no value, or a value of unsigned infinity, positive infinity, or negative infinity. Once again, these assignments are not numbers, and areassociated more with computer science than pure mathematics, where in most contexts no assignment is made.

Table 1: A tabular layout with examples with a condensed, and generalized form are given in the table below, in which x represents any number.

Operation	Rule	Example
Addition	$x + 0 = x$	$3 + 0 = 3$
Subtraction	$x - 0 = x$	$3 - 0 = 3$
Multiplication	$x \times 0 = 0$	$5 \times 0 = 0$
Division	$0 \div x = 0$, when $x \neq 0$	$0 \div 5 = 0$
	$x \div 0$ is undefined	$5 \div 0$ is undefined
Exponentiation	$0^x = 0$	$0^5 = 0$
	$x^0 = 1$	$5^0 = 1$
Root	$\sqrt{0} = 0$	
Logarithm	$\log_b(0)$ is undefined	
Factorial	$0! = 1$	
Sine	$\sin 0^\circ = 0$	
Cosine	$\cos 0^\circ = 1$	
Tangent	$\tan 0^\circ = 0$	
Derivative	$0' = 0$	
Integral	$\int 0 \, dx = 0 + C$	

IX. CONCLUSIONS

When and who did the invention of zero is in dark till date, but it is established across the world that zero was invented in India. There are stories telling that zero was first invented in Babylon and second time it was invented by Mayans but both the inventions could not influenced the numeral system. Hindus invented third time zero in India around the middle of the fifth century. The oldest extant mathematical document produced on the Indian subcontinent is the birch bark *Bakhshali Manuscript*, discovered in 1881, in the village of Bakhshali, near Peshawar in Pakistan. The exact date of the documentation of it is uncertain till now, but it is believed by some scholars that it is to pre-date Aryabhatta in which zero was symbolized and used as a number. If the early dating is accepted, it would be also pre-date Brahamagupta.

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