Cordial Labeling Of One Point Union Of Graphs Related To **triple -Antena** Of C_7 and invariance.

Mukund V.Bapat[1]

1. Abstract: We discuss graphs of type $G^{(k)}$ i.e. one point union of k-copies of G for cordial labeling. We take G as triple-antena graph. A triple-antena graph also called as triple tail graph is obtained by attaching a path P_m to any three vertices which forms a path p_3 in given graph C_7. It is denoted by triple- tail(G,P_m) where G is given graph and all the three tails may or may not be identical to p_m. We take G as C_7 and restrict our attention to m = 2.We have taken care that the sum of pendent edges on path on all the three vertices is same , in this case upto 2. Further we consider all possible structures of $G^{(k)}$ by changing the common point and obtain non-isomorphic structures. We show all these structures as cordial graphs. This is called as invariance of different structures of $G^{(k)}$ under cordial labeling.

Key words: cordial, one point union, triple-tail graph, cycle, vertex.,C_7, labeling
Subject Classification: 05C78

2. Introduction

The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Holton [4] , Graph Theory by Harary [5], A dynamic survey of graph labeling by J.Gallian [6] and Douglas West.[7].I.Cahit introduced the concept of cordial labeling [3]. f:V(G)→{0,1} be a function. From this label of any edge (uv) is given by |f(u)-f(v)|. Further number of vertices labeled with 0 i.e $v_f(0)$ and the number of vertices labeled with 1 i.e.$v_f(1)$ differ at most by one .Similarly number of edges labeled with 0 i.e.$e_f(0)$ and number of edges labeled with 1 i.e.$e_f(1)$ differ by at most one. Then the function f is called as cordial labeling. Cahit has shown that : every tree is cordial; Kn is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; the friendship graph $C_3^{(t)}$ (i.e., the one-point union of t copies of C_3) is cordial if and only if t is not congruent to 2 (mod 4); all fans are cordial; the wheel W_n is cordial if and only if n is not congruent to 3 (mod 4).A lot of work has been done in this type of labeling. One may refer dynamic survey by J. Gallian [6].

So far only one type of one point unions were discussed at varies stages. But for a given graph there are different one point unions (upto isomorphism) structures possible in $G^{(k)}$. It depends on which point on G is used to fuse to obtain one point union. We discuss all such structures for cordial labeling. We show that all these graphs are cordial graphs.It is called as invariance under cordial labeling. We use the convention that $v_f(0,1)$ = (a,b) to indicate the number of vertices labeled with 0 are a in number and that number of vertices labeled with 1 are b in number. Further $e_f(0,1)$ = (x,y) we mean the number of edges labeled with o are x and number of edges labeled with 1 are y. The graph whose cordial labeling is available is called as cordial graph. In this paper we define triple -tail graph and obtain one point union graphs on it. For this we consider C_7 and t-pendent edges attached to each of any three vertices forming a path on C_7.($t \leq 3$).In this paper we discuss the graphs obtained from C_7 by fusing an edge each or fusing two edges at each consecutive three vertices.

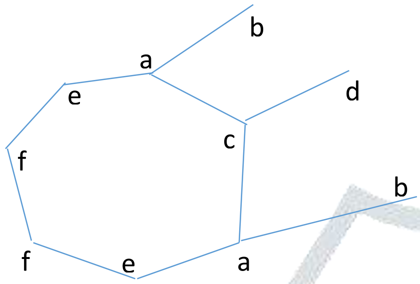
3. Preliminaries

3.1 Tail Graph: A (p,q) graph G to which a path P_m is fused at some vertex. This also can be explained as take a copy of graph G and at any vertex of it fuse a path P_m with it's one of the pendent vertex. It's number of vertices are P+m-1 and edges are by q + m-1. It is denoted by tail(G, P_m). 3.2 double-tail graph of G is denoted by double-tail(G,Pm).It is obtained by attaching (fusing) path P_m to a pair of adjacent vertices of G .It has q+2m-2 edges and p + 2m-2 vertices.($m \geq 2$) 3.3 Fusion of vertices. Let $u \neq v$ be any two vertices of G. We replace these two vertices by a single vertex say x and all edges incident to u and v are now incident to x. If loop is formed then it is deleted.[4] 3.4 $G^{(K)}$ it is One point union of k copies of G is obtained by taking k copies of G and fusing a fixed vertex of each copy with same fixed vertex of other copies to create a single vertex common to all copies. If G is a (p, q) graph then $|V(G_{(k)}| = k(p-1)+1$and $|E(G)|= k.q$

3.4 triple-tail graph of G is denoted by triple-tail(G,P_m).It is obtained by attaching (fusing) path Pm to each of three vertices of G that forms a path P_3 .It has q+3m-3 edges and p + 3m-3 vertices.($m \geq 2$)

4. Results Proved:

Theorem 4.1 All non- isomorphic one point union on k-copies of graph obtained on G = triple –tail(C_7,p_2) given by $G^{(k)}$ are cordial graphs. Proof: From fig.4.1 it follows that there are six non-isomorphic structures of one point union possible at vertices a, b, c ,d, e, f Define f:V(G)→{0,1} that gives us labeled copies of G as given below..We extend the same f : V($G^{(k)}$):→{0,1} to obtain cordial labeling of $G^{(k)}$. When the one point union is taken at a, d , b or e then type A and type B label are fused alternately at vertex desired vertex of these vertices.. The first copy being type A .



Fir 4.1 triple tail (C_7,p_2) : The one point union for non-isomorphic structures the common point may be any of a, b, c, d, e,f

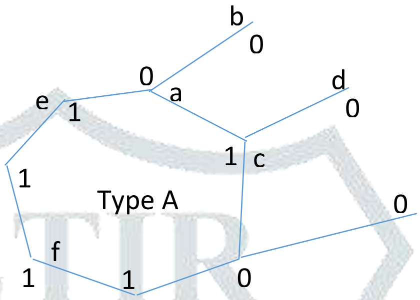


Fig 4.2 $v_f(0,1)$ = (5,5) $e_f(0,1)$ = (5,5)

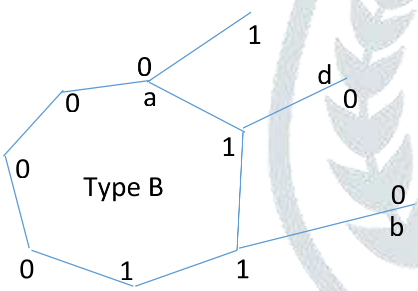


Fig 4.3 $v_f(0,1)$ = (6,4) $e_f(0,1)$ = (5,5)

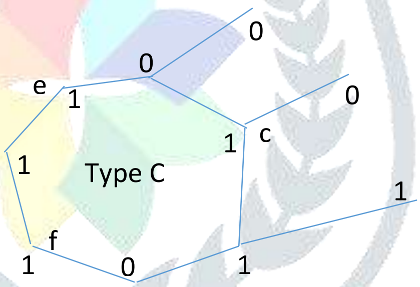


Fig 4.4 $v_f(0,1)$ = (4,6) $e_f(0,1)$ = (5,5)

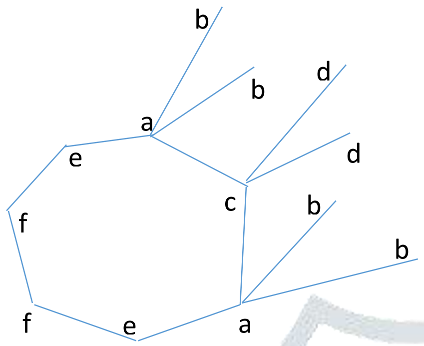
To obtain one point union of k copies of G at any of the vertices a, b,,c, d, e when k =1 we use type A label.

For k>1 For one point union at the vertices a, b, d and for k = 2x copies we fuse x copies of type A and type B label. When k = 2x+1 there will be x+1 copies of type A label and x copies of type B label. The label number distribution is $v_f(0,1)$ =(5+9x,5+9x), $e_f(0,1)$= (5k,5k).when k = 2x+1, x= 0,1, 2, .. The label number distribution is $v_f(0,1)$= (10+9(x-1), 9+(x-1)) when k = 2x, x= 1, 2, .. In this case the common vertex is with label 0

When one point union is taken at point c or e or f and k= 1 Type A label is used. For k>1 , for k = 2x there will be x copies of type A and type C each. When k = 2x+1 there will be x+1 copies of type A label and x copies of type C label are used. The label number distribution is $v_f(0,1)$ =(5+9x,5+9x),.when k = 2x+1, x= 0,1, 2, .. The label number distribution is $v_f(0,1)$= (9+9(x-1),10+9(x-1))).when k = 2x, x= 1, 2, ..and $e_f(0,1)$= (5k,5k) for all k In this case the common vertex is with label 1.

Thus we observe that All non- isomorphic one point union on k-copies of graph obtained on G = triple –tail(C_7,p_2) given by $G^{(k)}$ are cordial graphs. #

Theorem 4.2 All non- isomorphic one point union on k-copies of graph obtained on G =triple-tail(C_7,$2P_2$) given by $G^{(k)}$ are cordial graphs. Proof: From fig 4.5 it follows that there are 6 non-isomorphic structure at points a, b, c, d, e and f possible. At these points one can obtain on one point union of k copies of graph.



Fgr 4.5 triple tail (C_7,p_2) : The one point union for non-isomorphic structures the common point may be any of a, b, c, d, e,f

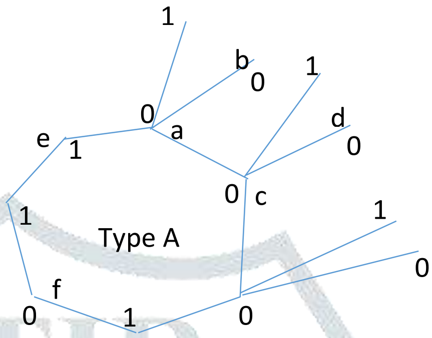


Fig 4.6$v_f(0,1)$ = (7,6) $e_f(0,1)$ = (6,7)

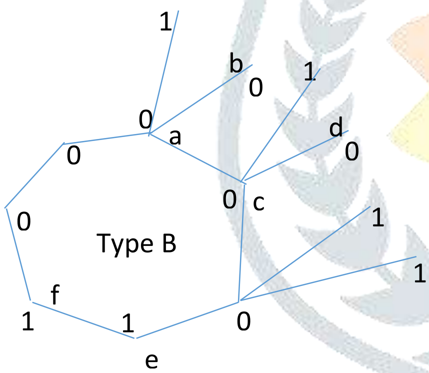


Fig 4.7 $v_f(0,1)$ = (7,6) $e_f(0,1)$ = (7,6)

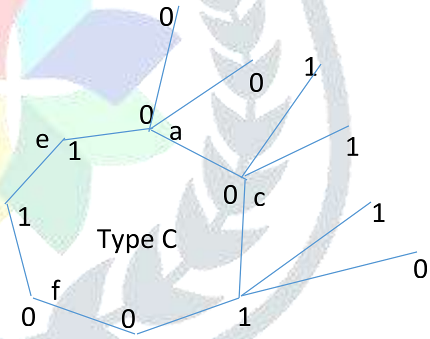


Fig 4.8 $v_f(0,1)$ = (7,6) $e_f(0,1)$ = (6,7)

Define f:V(G)→{0,1} that gives us labeled copies of G as above. We extend the same f : V($G^{(k)}$):→{0,1} to obtain cordial labeling of $G^{(k)}$. To obtain one point union at points a or b or c or d or f we fuse type A label with type B label at one of these required points. When k= 1 we use type A label .When k = 2x type A and type B are used x times each. When k = 2x+1 then type A label is used x+1 times and type B label for x times to obtain $G^{(K)}$.The label distribution is $v_f(0,1)$ =(6k+1,6k) for all k and $e_f(0,1)$= (6+13x,7+13x) .when k = 2x+1, x= 0,1, 2, .. The label number distribution is $e_f(0,1)$= (13+13(x-1),13+13(x-1)).when k = 2x, x= 1, 2, … . The common vertex label is 0.

The one point union at point e is taken then type B and Thpe C label is used.When k = 1 only type B label is used. When k = 2x type B and type C are used x times each. When k = 2x+1 then type B label is used x+1 times and type C label for x times to obtain $G^{(K)}$.The label distribution is $v_f(0,1)$ =(6k+1, 6k+ 1) for all k and $e_f(0,1)$= (7+13x,6+13x) .when k = 2x+1, x= 0,1, 2, .. The label number distribution is .when k = 2x, x= 1, 2, for edges ; $e_f(0,1)$= (13+13(x-1),13+13(x-1)). The common vertex label is 1. Thus even if we change point common to all copies in $G^{(k)}$ the cordiality is preserved.

Conclusions: In this paper we define some new families obtained from C_7.We take a copy of C_7 and to any three of it's adjacent vertices fuse t pendent edges each. We call this as triple-tail (G,tP_2) graph.. We show that

1) All non- isomorphic one

point union on k-copies of graph obtained on G = triple-tail(C_7,P_2) given by $G^{(k)}$ are cordial graphs.

2) All non- isomorphic one point union on k-copies of graph obtained on G = triple-tail(C_7,2P_2) given by $G^{(k)}$ are cordial graphs.

It is necessary to investigate the cordiality and invariance for one point union graph for the general case when t pendent edges are attached at each three vertices of C_7

References:

[1] Bapat Mukund, Ph.D. thesis submitted to university of Mumbai. India 2004.

[2] Bapat Mukund V. Some Path Unions Invariance Under Cordial labeling, IJSAM feb.2018 issue.

[3] I.Cahit, Cordial graphs: a weaker version of graceful and harmonious graphs, Ars Combin., 23 (1987) 201-207.

[4] J. Clark and D. A. Holton, A first look at graph theory; world scientific.

[5] Harary, Graph Theory, Narosa publishing ,New Delhi

[6] J.Gallian, Dynamic survey of graph labeling, E.J.C 2017

[7] D. WEST, Introduction to Graph Theory, Pearson Education Asia.

[1]Mukund V. Bapat, Hindale, Tal: Devgad, Sindhudurg
Maharashtra, India 416630
mukundbapat@yahoo.com