

# VELOCITY, MASS AND TEMPERATURE ANALYSIS OF FREE CONVECTIVE MHD SECOND GRADE FLUID FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE IN THE PRESENCE OF THERMO-DIFFUSION AND HEAT GENERATION / ABSORPTION THROUGH POROUS MEDIUM

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**Abstract :** *This paper is concerned with the study of flow, heat and mass transfer characteristics on the free convection MHD flow of second grade fluid past an exponentially accelerated vertical plate. It is assumed that the bounding plate has a ramped temperature and isothermal temperature in a porous medium. Analytic solution of momentum, energy and concentration equations are obtained in closed form by Laplace transform technique. From velocity, temperature and concentration fields, the exact expression of skin friction, Nusselt number and Sherwood number are derived. The results obtained show the impact of Prandtl number  $Pr$ , Grashof number  $Gr$ ,  $Gm$ , Schmidt number  $Sc$ , magnetic parameter  $M$ , Soret number  $Sr$ , heat generation / absorption parameter  $H$  and time  $t$  on velocity, heat and mass transfer and clarified with the help of graphical illustrations.*

**Keywords:** *Magneto hydrodynamics; Second grade fluid, isothermal temperature; Ramped temperature; Skin friction; Nusselt number; Sherwood number.*

## I. INTRODUCTION

The non-Newtonian fluids are considered as more appropriate models of fluids in industrial and technological applications than Newtonian fluids. Non-Newtonian fluid flow analysis is much more complicated and subtle in comparison with Newtonian fluids. The governing equations are very complex and the solutions of the resulting equations are more difficult to obtain. Due to the complexity of fluids, several models of non-Newtonian fluids have been proposed in the literature. Second grade fluid is one of the non-Newtonian fluids. M. Sheikholeslami [1-2] has introduced influence of EFD viscosity on nanofluid forced than M. Sheikholeslami and M. M. Bhatti [3] have done research work on nanofluid heat transfer enhancement by means of EHD.

S.A. Shehzad and M. Sheikholeslami [4] studied thermal radiation of Lorentz forces considering viscosity. T. Hayat et al. [5] studied numerical simulation of nanofluid forced convection heat transfer. M. Shamlooei and M. Sheikholeslami [6] discussed natural convection in presence of thermal radiation. M. Sheikholeslami [7] and T. Hayat et al. [8] obtained numerical solution of MHD nanofluid free convective heat transfer. On the other hand, flow in porous media has practical applications in heat removal from debris, storage of food stuffs, paper production, nuclear fuel, oil exploration, underground disposal of radioactive waste material etc. Some of related research studies are due to Kataria and Mittal [9-10]. M.M. Rashidi et al. [11-16] discussed unsteady MHD free convective flow with numerical technique. M Hatami et al. [17-21] studied hydrodynamic heat transfer. D D Ganji et al. [22-26] studied micropolar fluid flow and heat transfer. Katariya and Patel [27-31] obtained analytic solution of MHD fluid flow with ramped wall temperature through porous medium. H R Seyf et al. [32] discussed heat transfer through a porous using two-equation energy model. S M Rassoulinejad-Mousavi and S Abbasbandy [33] did analysis in a circular tube using Homotopy analysis method. S M Rassoulinejad-Mousavi et al. [34-37] discussed fluid flow in moving wall. Due to increasing significant, Application of unsteady second grade MHD flow with heat and mass transfer is important in engineering and technology. Many researchers like, Cortell [38], Hayat et al. [39-42]. Bataineh et al. [43] obtained the solution using bernstein method for the MHD flow and heat transfer of a second grade fluid in a channel with porous wall.

In present investigation, we have studied Analytic expression of MHD second grade fluid flow past an exponentially accelerated in the presence of thermal radiation and concentration with ramped wall temperature and ramped surface concentration through porous medium is obtained using Laplace transform technique.

### Nomenclature:

$B_0$  Uniform magnetic field  
 $u'$  Fluid velocity  
 $t'$  Time  
 $T'$  Fluid temperature

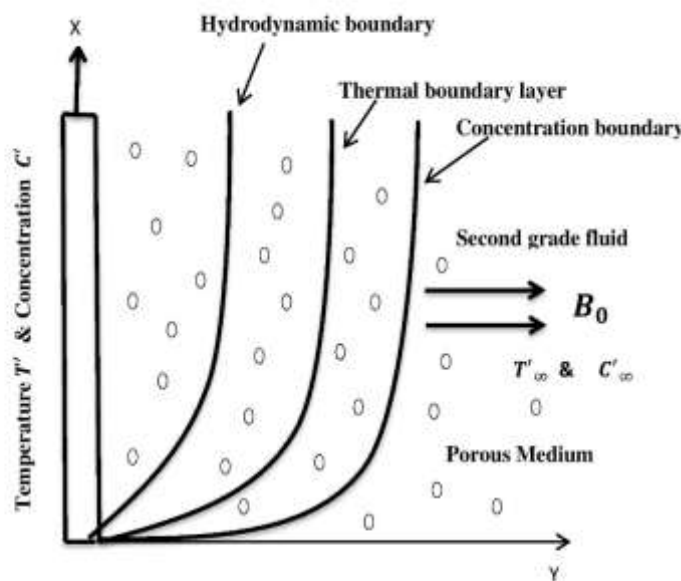
$k'$  Permeability of porous medium  
 $q_r'$  Radiative heat flux  
 $u$  Dimensionless fluid velocity  
 $\theta$  Dimensionless fluid temperature

$C'$ Species concentration	$C$ Dimensionless concentration
$C_p$ Specific heat at constant pressure	$Gm$ Mass Grashof number
$D_M$ Mass diffusion coefficient	$Gr$ Thermal Grashof number
$k$ Permeability parameter	$R$ Thermal Radiation
$Pr$ Prandtl number	$M$ Magnetic parameter
$Sc$ Schmidt number	$t$ Dimensionless time
$H$ Heat generation Parameter	$Sr$ Soret Number

**Greek symbols:**

$\phi$ Porosity of the porous medium	$\rho$ Fluid density
$\nu$ Kinematic viscosity coefficient	$g$ Acceleration due to gravity
$\beta'_T$ Volumetric coefficient of thermal expansion	$\sigma$ Electrical conductivity
$\beta'_c$ Volumetric coefficient of concentration expansion	$k_1$ -Thermal conductivity
$\alpha_1$ One of the material modules of second grade fluids.	$\alpha$ Second grade parameter

**II. MATHEMATICAL FORMULATION OF THE PROBLEM:**



$$\text{Where, } T' = \begin{cases} T'_\infty + (T'_w - T'_\infty) \frac{t'}{t_0} & \text{if } 0 < t' < t_0 \\ T'_w & \text{if } t' \geq t_0 \end{cases}, \quad C' = \begin{cases} C'_\infty + (C'_w - C'_\infty) \frac{t'}{t_0} & \text{if } 0 < t' < t_0 \\ C'_w & \text{if } t' \geq t_0 \end{cases}; y' = 0$$

**Figure 1: Physical sketch of the problem**

In Figure 1, Coordinate system is considered  $x'$  - axis is taken along the wall in the upward direction and  $y'$  - axis is taken normal to it. In  $y'$  - axis direction transverse magnetic field of strength  $B_0$  is applied uniformly. Initially, at time  $t' \leq 0$ , both the fluid and the plate are constant temperature  $T'_\infty$  and concentration near the plate is assumed to be  $C'_\infty$  respectively. At the time  $t' > 0$ , temperature & concentration of the wall is raised or lowered to  $T'_\infty + (T'_w - T'_\infty) \frac{t'}{t_0}$  &  $C'_\infty + (C'_w - C'_\infty) \frac{t'}{t_0}$  when  $t' \leq t_0$  and  $T'_w$  &  $C'_w$  when  $t' > t_0$  respectively which is there after maintained constant  $T'_w$  &  $C'_w$ . It is assumed that the effects of viscous dissipation in energy equation, induce magnetic and electrical field are negligible also assume that flow is incompressible, laminar, uni-direction, one dimensional. Under above assumptions and taking into account the Boussinesq's approximation, governing equations are given below:

$$\frac{\partial u'}{\partial t'} = \left( \nu + \frac{\alpha_1}{\rho} \frac{\partial}{\partial t'} \right) \frac{\partial^2 u'}{\partial y'^2} + g\beta'_T (T' - T'_\infty) - \frac{\sigma B_0^2}{\rho} u' - \frac{\phi}{k'} \left( \nu + \frac{\alpha_1}{\rho} \frac{\partial}{\partial t'} \right) u' + g\beta'_c (C' - C'_\infty) \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k_1}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{Q_0}{\rho c_p} (T' - T'_\infty) \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D_M \frac{\partial^2 C'}{\partial y'^2} + D_T \frac{\partial^2 T'}{\partial y'^2} \tag{3}$$

Where following initial and boundary condition:

$$u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty; \text{ as } y' \geq 0 \text{ and } t' \leq 0$$

$$u' = e^{at'}, \quad T' = \begin{cases} T'_\infty + (T'_w - T'_\infty) \frac{t'}{t_0} & \text{if } 0 < t' \leq t_0 \\ T'_w & \text{if } t' \geq t_0 \end{cases},$$

$$C' = \begin{cases} C'_\infty + (C'_w - C'_\infty) \frac{t'}{t_0} & \text{if } 0 < t' \leq t_0 \\ C'_w & \text{if } t' \geq t_0 \end{cases}; \text{ as } t' > 0 \text{ and } y' = 0$$

$$u' \rightarrow 0, T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty; \text{ as } y' \rightarrow \infty \text{ and } t' \geq 0 \tag{4}$$

Introducing the following dimensionless quantities:

$$y = \frac{y'}{\sqrt{vt_0}}, u = \sqrt{\frac{t_0}{v}} u', t = \frac{t'}{t_0}, \alpha = \frac{\alpha_1}{\rho v t_0}, Gr = \frac{t_0^{3/2} g \beta' T'(T'_w - T'_\infty)}{\sqrt{v}}, M^2 = \frac{\sigma B_0^2 t_0^2}{\rho}, \frac{1}{k} = \frac{vt_0^2 \phi}{k'},$$

$$Gm = \frac{t_0^{3/2} g \beta'_c (C'_w - C'_\infty)}{\sqrt{v}}, \theta = \frac{(T'_w - T'_\infty)}{(T'_w - T'_\infty)}, C = \frac{(C'_w - C'_\infty)}{(C'_w - C'_\infty)}, Pr = \frac{\rho v c_p t_0}{k_1}, H = \frac{Q_0 v^2}{\rho c_p k U_0^2},$$

$$Sc = \frac{vt_0}{D_M}, Sr = \frac{D_T(T'_w - T'_\infty)}{v(C'_w - C'_\infty)}$$

$$c = 1 + \frac{\alpha}{k}, b = M^2 + \frac{1}{k}$$

In the equations (1-4) dropping out the " ' " notation (for simplicity) we get

$$\frac{\partial^2 u}{\partial y^2} + \alpha \frac{\partial^3 u}{\partial y^2 \partial t} - c \frac{\partial u}{\partial t} - bu + Gr\theta + GmC = 0 \tag{5}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + H\theta \tag{6}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + Sr \frac{\partial^2 \theta}{\partial y^2} \tag{7}$$

With initial and boundary conditions

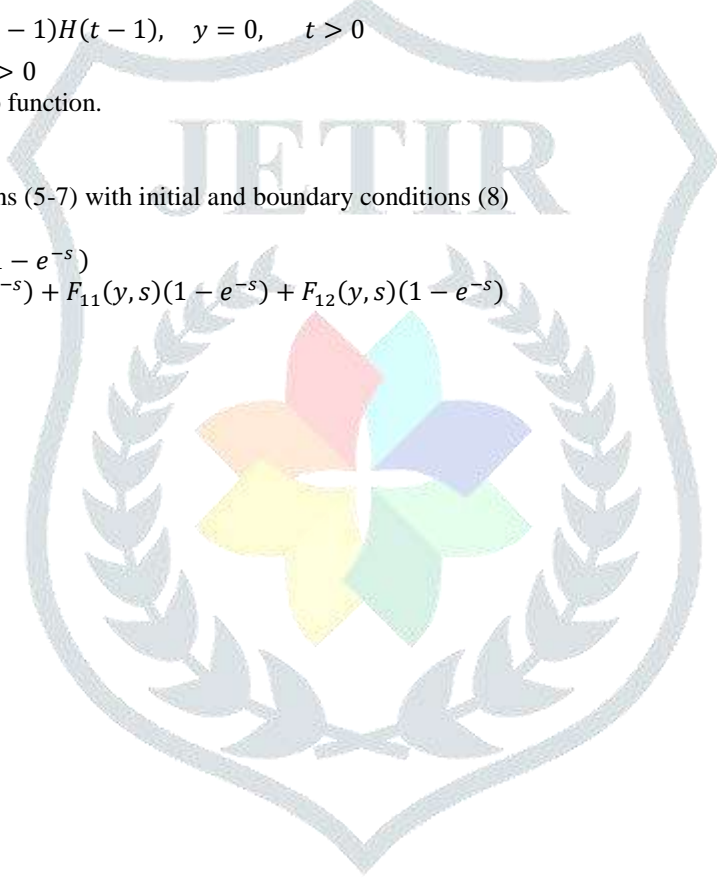
$$u = \theta = C = 0, \quad y \geq 0, t = 0$$

$$u = e^{at}, \theta = \begin{cases} t, & 0 < t \leq 1 \\ 1, & t > 1 \end{cases} = tH(t) - (t-1)H(t-1),$$

$$C = \begin{cases} t, & 0 < t \leq 1 \\ 1, & t > 1 \end{cases} = tH(t) - (t-1)H(t-1), \quad y = 0, \quad t > 0$$

$$u \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 \quad \text{as } y \rightarrow \infty, t > 0 \tag{8}$$

Where H(.) is Heaviside unit step function.



**III. SOLUTION:**

Taking Laplace transform of equations (5-7) with initial and boundary conditions (8)

$$\bar{\theta} = F_{15}(y, s)(1 - e^{-s}) \tag{9}$$

$$\bar{C} = H_1(y, s)(1 - e^{-s}) + H_2(y, s)(1 - e^{-s}) \tag{10}$$

$$\bar{u}(y, s) = F_1(y, s) + F_{10}(y, s)(1 - e^{-s}) + F_{11}(y, s)(1 - e^{-s}) + F_{12}(y, s)(1 - e^{-s}) \tag{11}$$

Where

$$F_1(y, s) = \frac{1}{s-a} e^{-y\sqrt{\frac{cs+b}{as+1}}} \tag{12}$$

$$F_2(y, s) = \frac{1}{s} e^{-y\sqrt{\frac{cs+b}{as+1}}} \tag{13}$$

$$F_3(y, s) = \frac{1}{s} e^{-y\sqrt{Pr(s-H)}} \tag{14}$$

$$F_4(y, s) = \frac{1}{s} e^{-y\sqrt{Scs}} \tag{15}$$

$$F_5(y, s) = F_2(y, s) - F_3(y, s) \tag{16}$$

$$F_6(y, s) = F_2(y, s) - F_4(y, s) \tag{17}$$

$$F_7(y, s) = \frac{a_{20}}{s} + \frac{a_{21}}{s-b_4} + \frac{a_{22}}{s-b_5} \tag{18}$$

$$F_8(y, s) = \frac{a_{23}}{s} + \frac{a_{24}}{s-b_9} + \frac{a_{25}}{s-b_{10}} + \frac{a_{26}}{s-a_5} \tag{19}$$

$$F_9(y, s) = \frac{a_{27}}{s} + \frac{a_{28}}{s-b_4} + \frac{a_{29}}{s-b_5} + \frac{a_{30}}{s-a_{16}} \tag{20}$$

$$F_{10}(y, s) = F_5(y, s)F_7(y, s) \tag{21}$$

$$F_{11}(y, s) = F_6(y, s)F_8(y, s) \tag{22}$$

$$F_{12}(y, s) = F_5(y, s)F_9(y, s) \tag{23}$$

$$F_{13}(y, s) = \frac{1}{s^2} e^{-y\sqrt{Scs}} \tag{24}$$

$$F_{14}(y, s) = \frac{1}{s-a_5} e^{-y\sqrt{Scs}} \tag{25}$$

$$F_{15}(y, s) = \frac{1}{s^2} e^{-y\sqrt{Pr(s-H)}} \tag{26}$$

$$F_{16}(y, s) = \frac{1}{s-a_{16}} e^{-y\sqrt{Pr(s-H)}} \tag{27}$$

$$H_1(y, s) = a_{10}F_4(y, s) + a_8F_{13}(y, s) + a_9F_{14}(y, s) \tag{28}$$

$$H_2(y, s) = a_{19}F_3(y, s) + a_{17}F_{15}(y, s) + a_{18}F_{16}(y, s) \tag{29}$$

Taking Inverse Laplace transform of equation (9) to (11), we get

$$\theta(y, t) = f_{15}(y, t) - f_{15}(y, t-1)H(t-1) \tag{30}$$

$$C(y, t) = h_1(y, t) - h_1(y, t-1)H(t-1) + h_2(y, t) - h_2(y, t-1)H(t-1) \tag{31}$$

$$u = f_1(y, t) + f_{10}(y, t) - f_{10}(y, t-1)H(t-1) + f_{11}(y, t) - f_{11}(y, t-1)H(t-1) + f_{12}(y, t) - f_{12}(y, t-1)H(t-1) \tag{32}$$

Where

$$f_1(y, t) = L^{-1} \left[ \frac{1}{s-a} e^{-y\sqrt{\frac{cs+b}{as+1}}} \right] \tag{33}$$

$$f_2(y, t) = \frac{c}{\alpha} e^{-t/\alpha} \int_0^\infty \operatorname{erfc} \left( \frac{y}{2\sqrt{z}} \right) e^{-c/\alpha z} I_0 \left( \frac{2}{\alpha} \sqrt{(c-ab)zt} \right) dz + \frac{b}{\alpha} \int_0^\infty \int_0^t \operatorname{erfc} \left( \frac{y}{2\sqrt{z}} \right) e^{-\frac{cz+s}{\alpha}} I_0 \left( \frac{2}{\alpha} \sqrt{(c-ab)zs} \right) ds dz \tag{34}$$

Where

$I_0$ - Bessel function and  $\operatorname{erfc}(g)$ - complementary error function.

$$f_3(y, t) = \frac{1}{2} \left[ e^{-y\sqrt{-HP_r}} \operatorname{erfc} \left( \frac{y}{2\sqrt{t/Pr}} - \sqrt{-Ht} \right) + e^{y\sqrt{-HP_r}} \operatorname{erfc} \left( \frac{y}{2\sqrt{t/Pr}} + \sqrt{-Ht} \right) \right] \quad (35)$$

$$f_4(y, t) = \operatorname{erfc} \left( \frac{y\sqrt{s_c}}{2\sqrt{t}} \right) \quad (36)$$

$$f_5(y, t) = f_2(y, t) - f_3(y, t) \quad (37)$$

$$f_6(y, t) = f_2(y, t) - f_4(y, t) \quad (38)$$

$$f_7(y, t) = a_{20} + a_{21}e^{b_4t} + a_{22}e^{b_5t} \quad (39)$$

$$f_8(y, t) = a_{23} + a_{24}e^{b_9t} + a_{25}e^{b_{10}t} + a_{26}e^{a_5t} \quad (40)$$

$$f_9(y, t) = a_{27} + a_{28}e^{b_4t} + a_{29}e^{b_5t} + a_{30}e^{a_{16}t} \quad (41)$$

$$f_{10}(y, t) = f_5(y, t) * f_7(y, t) \quad (42)$$

$$f_{11}(y, t) = f_6(y, t) * f_8(y, t) \quad (43)$$

$$f_{12}(y, t) = f_5(y, t) * f_9(y, t) \quad (44)$$

$$f_{13}(y, t) = \left( \frac{y^2 s_c}{2} + t \right) \operatorname{erfc} \left( \frac{y\sqrt{s_c}}{2\sqrt{t}} \right) - \frac{y\sqrt{s_c}}{2\sqrt{\pi}} e^{-\frac{y^2 s_c}{4t}} \quad (45)$$

$$f_{14}(y, t) = \frac{e^{a_8t}}{2} \left[ e^{-y\sqrt{s_c a_5}} \operatorname{erfc} \left( \frac{y\sqrt{s_c}}{2\sqrt{t}} - \sqrt{a_5 t} \right) + e^{y\sqrt{s_c a_5}} \operatorname{erfc} \left( \frac{y\sqrt{s_c}}{2\sqrt{t}} + \sqrt{a_5 t} \right) \right] \quad (46)$$

$$f_{15}(y, t) = \frac{1}{2} \left[ \left( t - \frac{y}{2\sqrt{-H/Pr}} \right) e^{-y\sqrt{-HP_r}} \operatorname{erfc} \left( \frac{y}{2\sqrt{t/Pr}} - \sqrt{-Ht} \right) + \left( t + \frac{y}{2\sqrt{-H/Pr}} \right) e^{y\sqrt{-HP_r}} \operatorname{erfc} \left( \frac{y}{2\sqrt{t/Pr}} + \sqrt{-Ht} \right) \right] \quad (47)$$

$$f_{16}(y, t) = \frac{e^{a_{16}t}}{2} \left[ e^{-y\sqrt{Pr(a_{16}-H)}} \operatorname{erfc} \left( \frac{y}{2\sqrt{t/Pr}} - \sqrt{(a_{16}-H)t} \right) + e^{y\sqrt{Pr(a_{16}-H)}} \operatorname{erfc} \left( \frac{y}{2\sqrt{t/Pr}} + \sqrt{(a_{16}-H)t} \right) \right] \quad (48)$$

$$h_1(y, t) = a_{10}f_4(y, t) + a_8f_{13}(y, t) + a_9f_{14}(y, t) \quad (49)$$

$$h_2(y, t) = a_{19}f_3(y, t) + a_{17}f_{15}(y, t) + a_{18}f_{16}(y, t) \quad (50)$$

**Solutions for Plate with Constant Temperature:** In order to understand effects of ramped temperature of the plate on the fluid flow, we must compare our results with constant temperature. In this case, the initial and boundary conditions are the same excluding Eq. (8) that becomes  $\theta = 1$  at  $y = 0, t \geq 0$ .

$$\theta(y, t) = \frac{1}{2} \left[ e^{-y\sqrt{-HP_r}} \operatorname{erfc} \left( \frac{y}{2\sqrt{t/Pr}} - \sqrt{-Ht} \right) + e^{y\sqrt{-HP_r}} \operatorname{erfc} \left( \frac{y}{2\sqrt{t/Pr}} + \sqrt{-Ht} \right) \right] \quad (51)$$

Similarly, Velocity profile for isothermal temperature is

$$u = f_1(y, t) + f_{21}(y, t) + f_{22}(y, t) - f_{22}(y, t-1)H(t-1) - f_{23}(y, t) + f_{24}(y, t) \quad (52)$$

Where

$$F_{17}(y, s) = \frac{a_{32}}{s-b_4} + \frac{a_{33}}{s-b_5} \quad (53)$$

$$F_{18}(y, s) = \frac{a_{34}}{s} + \frac{a_{35}}{s-b_9} + \frac{a_{36}}{s-b_{10}} \quad (54)$$

$$F_{19}(y, s) = \frac{a_{37}}{s-b_9} + \frac{a_{38}}{s-b_{10}} + \frac{a_{39}}{s-a_{16}} \quad (55)$$

$$F_{20}(y, s) = \frac{a_{40}}{s-b_4} + \frac{a_{41}}{s-b_5} + \frac{a_{42}}{s-a_{16}} \quad (56)$$

$$F_{21}(y, s) = F_5(y, s)F_{17}(y, s) \quad (57)$$

$$F_{22}(y, s) = F_6(y, s)F_{18}(y, s) \quad (58)$$

$$F_{23}(y, s) = F_6(y, s)F_{19}(y, s) \quad (59)$$

$$F_{24}(y, s) = F_5(y, s)F_{20}(y, s) \quad (60)$$

Inverse Laplace transform of equation (53) to (60) are

$$f_{17}(y, t) = a_{32}e^{b_4t} + a_{33}e^{b_5t} \quad (61)$$

$$f_{18}(y, t) = a_{34} + a_{35}e^{b_9t} + a_{36}e^{b_{10}t} \quad (62)$$

$$f_{19}(y, t) = a_{37}e^{b_9t} + a_{38}e^{b_{10}t} + a_{39}e^{a_{16}t} \quad (63)$$

$$f_{20}(y, t) = a_{40}e^{b_4t} + a_{41}e^{b_5t} + a_{42}e^{a_{16}t} \quad (64)$$

$$f_{21}(y, t) = f_5(y, t) * f_{13}(y, t) \quad (65)$$

$$f_{22}(y, t) = f_6(y, t) * f_{14}(y, t) \quad (66)$$

$$f_{23}(y, t) = f_6(y, t) * f_{15}(y, t) \quad (67)$$

$$f_{24}(y, t) = f_5(y, t) * f_{16}(y, t) \quad (68)$$

From velocity, temperature and concentration fields, the expressions for Nusselt number, skin friction and Sherwood number can easily be determined. They are measures of the heat transfer rate and shear stress at the boundary.

#### Nusselt Number:

The Nusselt number  $Nu$  can be written as

$$Nu = - \left( \frac{\partial \theta}{\partial y} \right)_{y=0} \quad (69)$$

Using the equation (30), we obtained the Nusselt number for Ramped wall temperature

$$Nu = -[J_{15}(t) - J_{15}(t-1)H(t-1)] \quad (70)$$

Using the equation (42), we obtained the Nusselt number for isothermal temperature

$$Nu = -J_3(t) \quad (71)$$

#### Sherwood Number:

Sherwood Number is defined and denoted by the formula

$$Sh = - \left( \frac{\partial c}{\partial y} \right)_{y=0} \quad (72)$$

Using the equation (31), we obtained the Sherwood Number for Ramped wall temperature

$$Sh = -[J_{25}(t) - J_{25}(t-1)H(t-1) + J_{26}(t) - J_{26}(t-1)H(t-1)] \quad (73)$$

#### Skin Friction:

Skin friction, in dimensionless form, is

$$\tau_w(t) = \tau(y, t) \text{ at } y = 0 \quad (74)$$

Where the shear stresses  $\tau(y, t)$

$$\tau(y, t) = \left(1 + \alpha \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial y} \quad (75)$$

Using the equation (32) we find the shear stress for ramped temperature as

$$\tau(y, t) = J_1(t) + J_{10}(t) - J_{10}(t-1)H(t-1) + J_{11}(t) - J_{11}(t-1)H(t-1) + J_{12}(t) - J_{12}(t-1)H(t-1) \quad (76)$$

Where

$$J_i(t) = \left. \frac{df_i}{dy} \right|_{y=0}, i = 1 \text{ to } 24 \quad (77)$$

$$J_i(t) = \left. \frac{dh_i}{dy} \right|_{y=0}, i = 25, 26 \quad (78)$$

#### IV. RESULT AND DISCUSSION:

To get perfect understanding for physics of the problem, we find numerical solution of velocity, temperature and concentration using Matlab software, obtained results presented graphically. Effects of several involved parameters are described in Figure 2 to Figure 10.

Fig. 2 shows effect of second grade fluid parameter  $\alpha$  on velocity profile for both thermal condition, ramped wall temperature with ramped surface concentration and isothermal temp with ramped surface concentration. It is seen that second grade parameter tends to reduce velocity throughout region. It is also clear that, the velocity approaches to zero at the far away from the plate. For ramped temperature on the plate, fluids flow slower than for the constant plate temperature.

Fig.3 depicts the velocity profile for various values of  $k$ . It is observed that, for both thermal cases, velocity increases with increase in  $k$ . A similar behavior was also expected, because when we increase the permeability, due to decreasing effects of drag force wholes becomes large in porous medium and hence velocity increases. Fig.4 shows the effect of the magnetic field on velocity. It is seen that, for both heating cases, velocity decreases with increase in magnetic parameter  $M$ . The present phenomena occur when magnetic field can induce current in the conductive fluid and create Lorentz force on the fluid in the boundary layer, which slow down the velocity of the fluid. Therefore, the magnetic field acts like a drag force. The practice of magnetic fields has effectively been applied to monitoring melt convection in solidification systems. Fig. 5 show temperature profile for different values of Prandtl number  $Pr$ , when the other parameters are fixed. For both thermal conditions, it is observed that temperature of the fluid decreases with increasing Prandtl number  $Pr$ . Fig.6 illustrates that concentration profile is displayed with the variations in Schmidt number  $Sc$ . It is observed that concentration decreases with increase in Schmidt number  $Sc$ . We also observe that, due to enhancement in the values of  $Sc$ , the concentration near the plate gets reducing the thickness this leads minimizing the mass buoyancy force. The mass Grashof number  $Gm$  means ratio of buoyancy force and viscous hydrodynamic force and thermal Grashof number  $Gr$  indicates the ratio of thermal buoyancy force to viscous hydrodynamic force. It is observed that velocity rises with increase in  $Gm$  or  $Gr$  in Fig. 7-8 respectively. This implies that motion of fluid accelerated due to improvement in either temperature buoyancy force or mass buoyancy force. Physically, Increase in  $Gr$  indicates increase in the strength of the flow, small viscous effects in the momentum equation and thus, reasons the increase in velocity profiles. Fig. 9 shows that temperature profile for different values of heat generation  $H$ . It is seen that, heat generation  $H$  parameter is the important role in MHD fluid flow problem. It is observe that heat generation tends to improve temperature profile because of that when the amount of heat generated the bond holding the components of the fluid particles is easily broken and the fluid temperature will increased. Fig. 10 shows that concentration profile for different values of Soret number  $Sr$ . It is observe that as soret number increase concentration also increase.

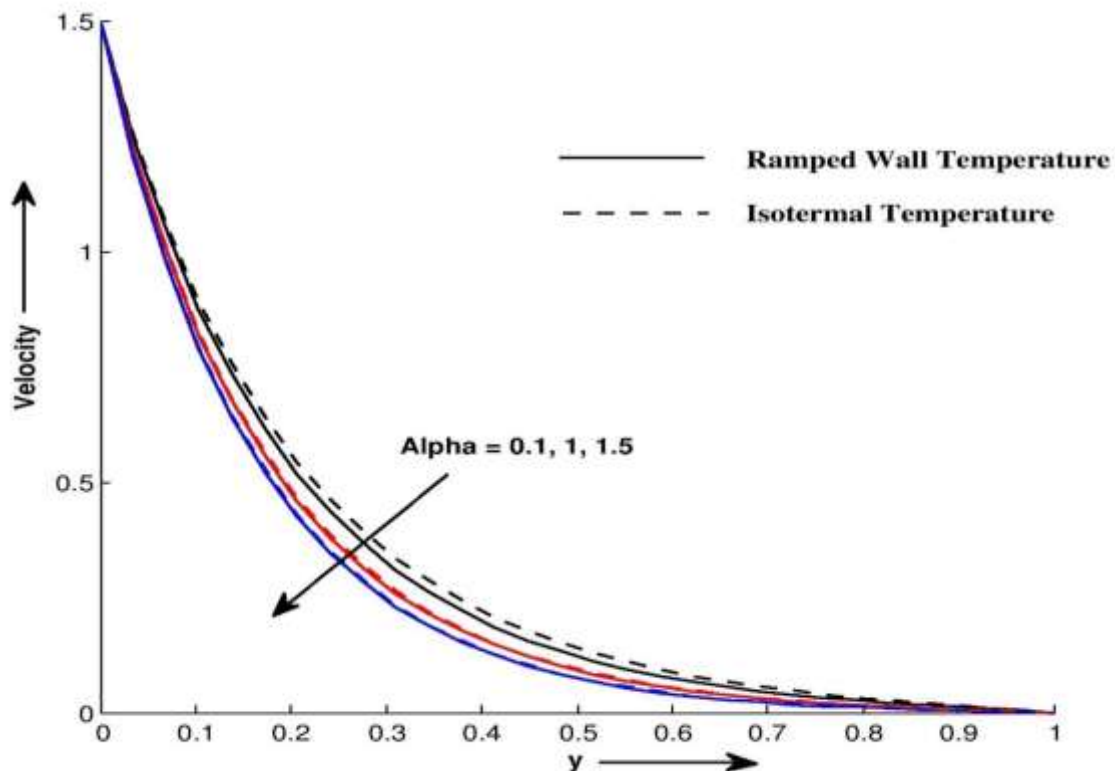


Figure 2: Velocity profile  $u$  for different values of  $y$  and  $\alpha$  at  $M = 5, k = 0.5, Pr = 7, Sc = 0.66, Gm = 3, Gr = 2, H = 5, Sr = 5$  and  $t = 0.4$

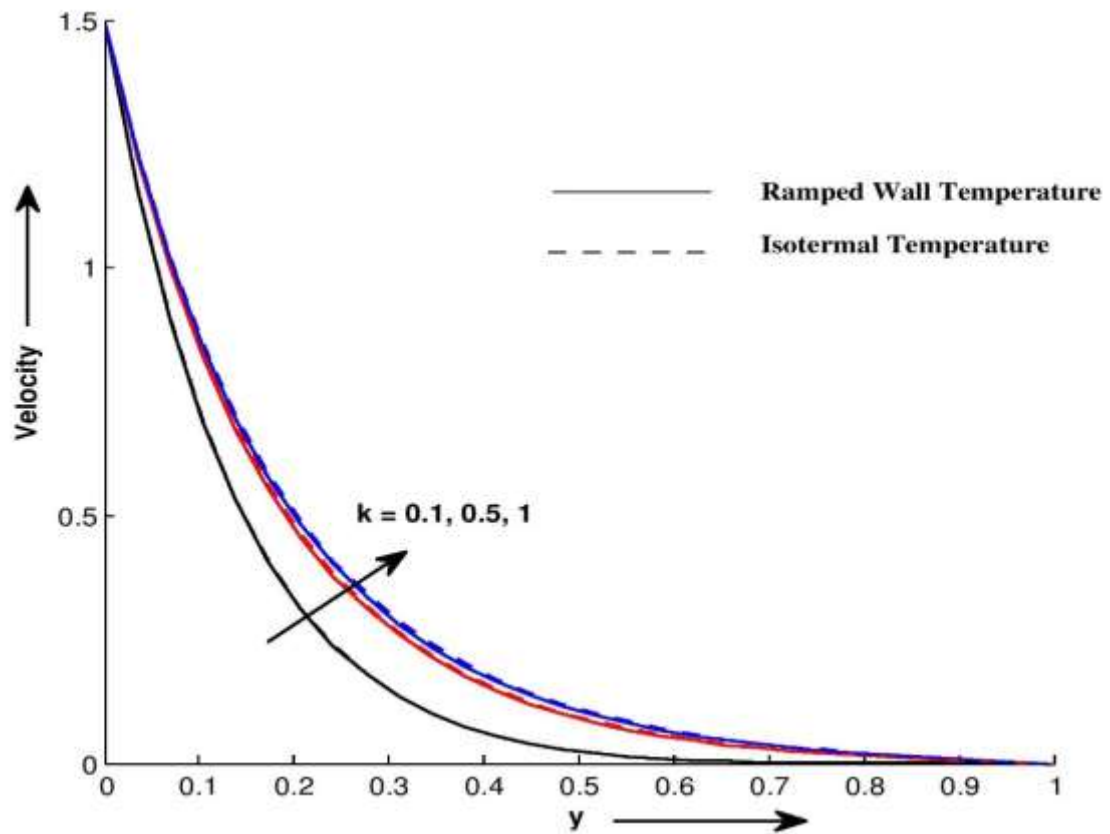


Figure 3: Velocity profile  $u$  for different values of  $y$  and  $k$  at  $M = 5, \alpha = 1, Pr = 7, Sc = 0.66, Gm = 3, Gr = 2, H = 5, Sr = 5$  and  $t = 0.4$

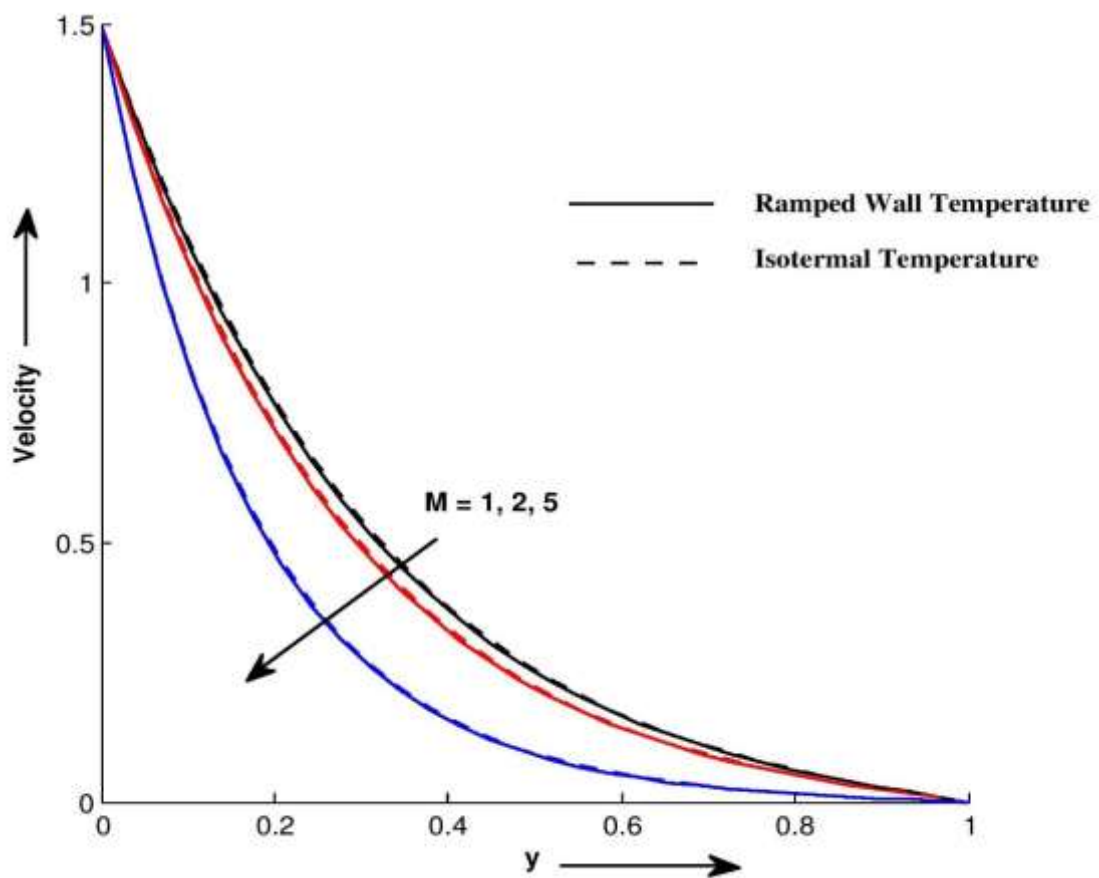


Figure 4: Velocity profile  $u$  for different values of  $y$  and  $M$  at  $k = 0.5, \alpha = 1, Pr = 7, Sc = 0.66, Gm = 3, Gr = 2, H = 5, Sr = 5$  and  $t = 0.4$

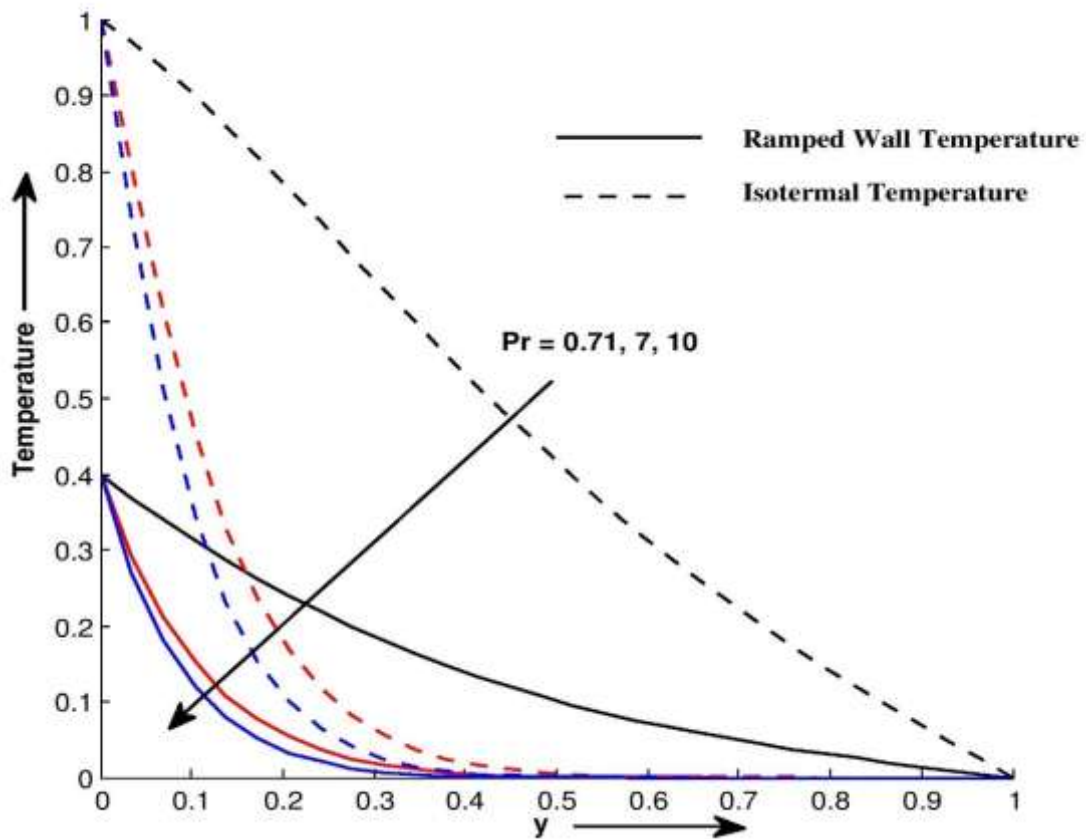


Figure 5: Temperature profile  $\theta$  for different values of  $y$  and  $Pr$  at  $H = 5$  and  $t = 0.4$

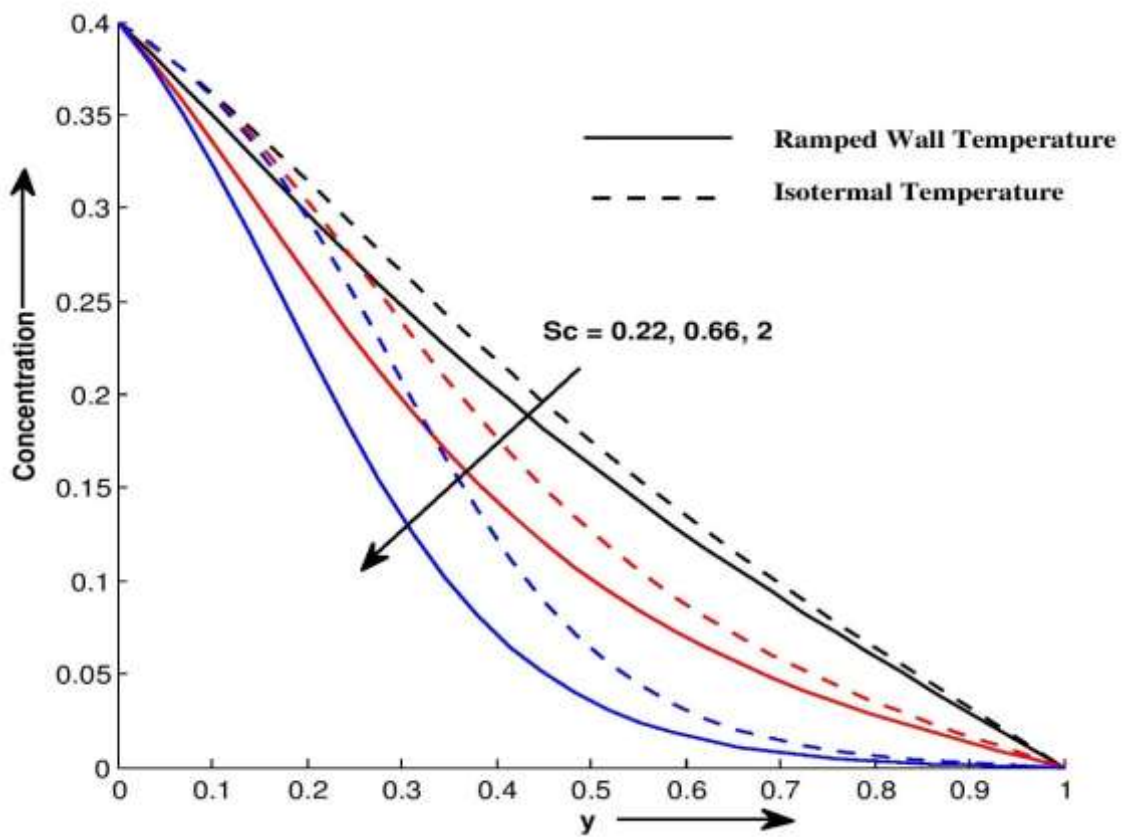


Figure 6: Concentration profile  $C$  for different values of  $y$  and  $Sc$  at  $Sr = 5$  and  $t = 0.4$

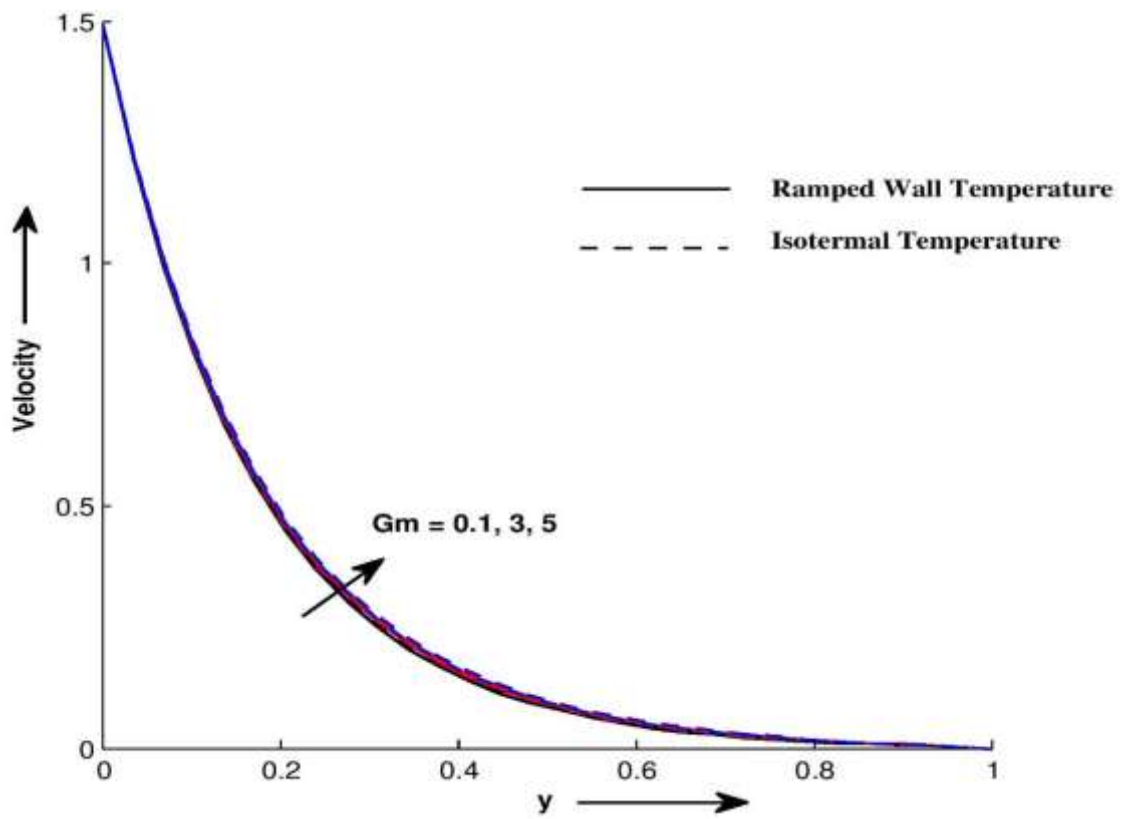


Figure 7: Velocity profile  $u$  for different values of  $y$  and  $G_m$  at  $M = 5, \alpha = 1, k = 0.5, Pr = 7, Gr = 2, Sc = 0.66, H = 5, Sr = 5$  and  $t = 0.4$

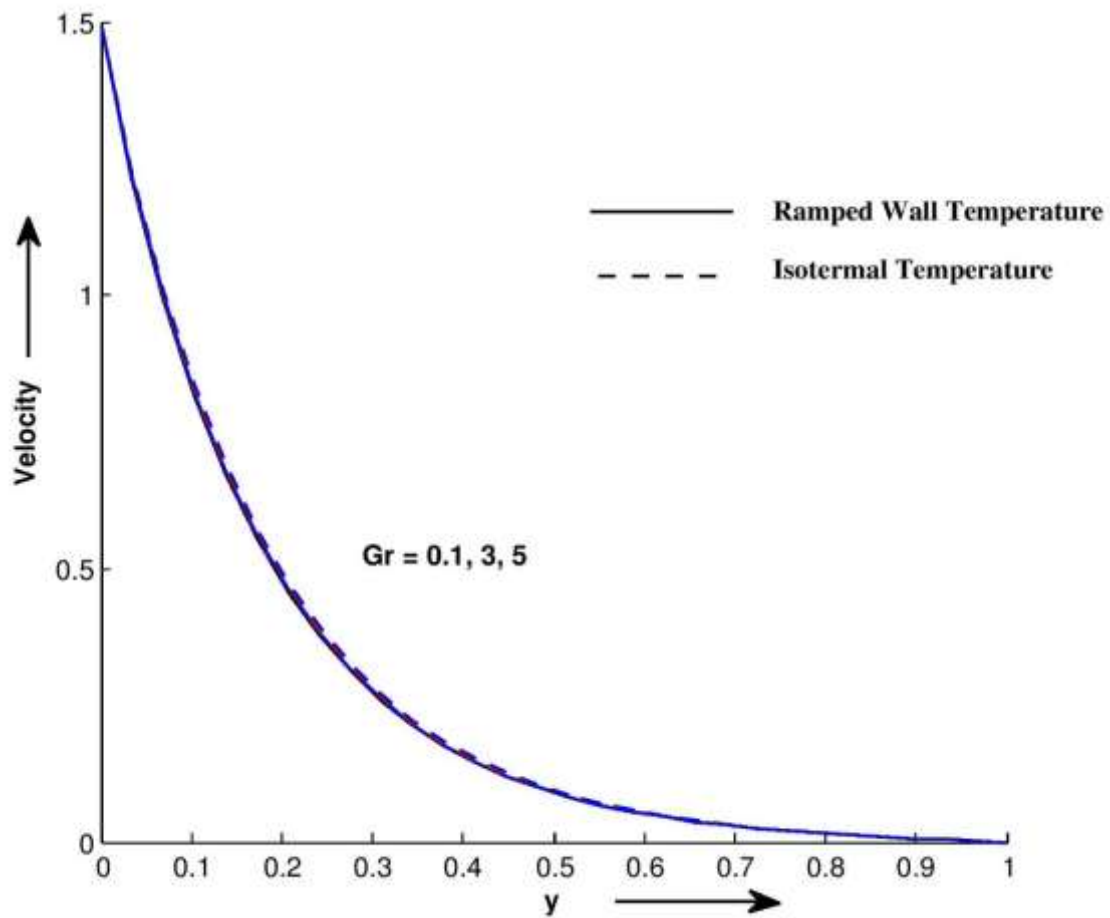


Figure 8: Velocity profile  $u$  for different values of  $y$  and  $Gr$  at  $M = 5, \alpha = 1, k = 0.5, Pr = 7, Sc = 0.66, G_m = 3, H = 5, Sr = 5$  and  $t = 0.4$



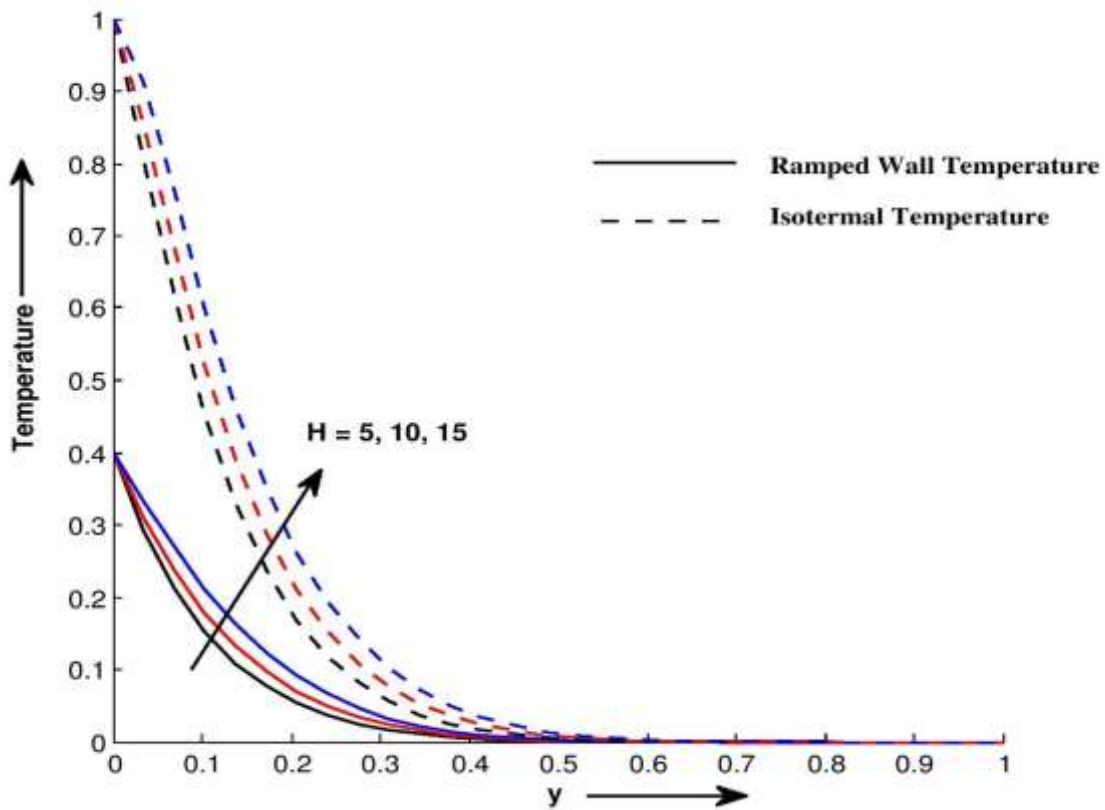


Figure 9: Temperature profile  $\theta$  for different values of  $y$  and  $H$  at  $Pr = 7$  and  $t = 0.4$

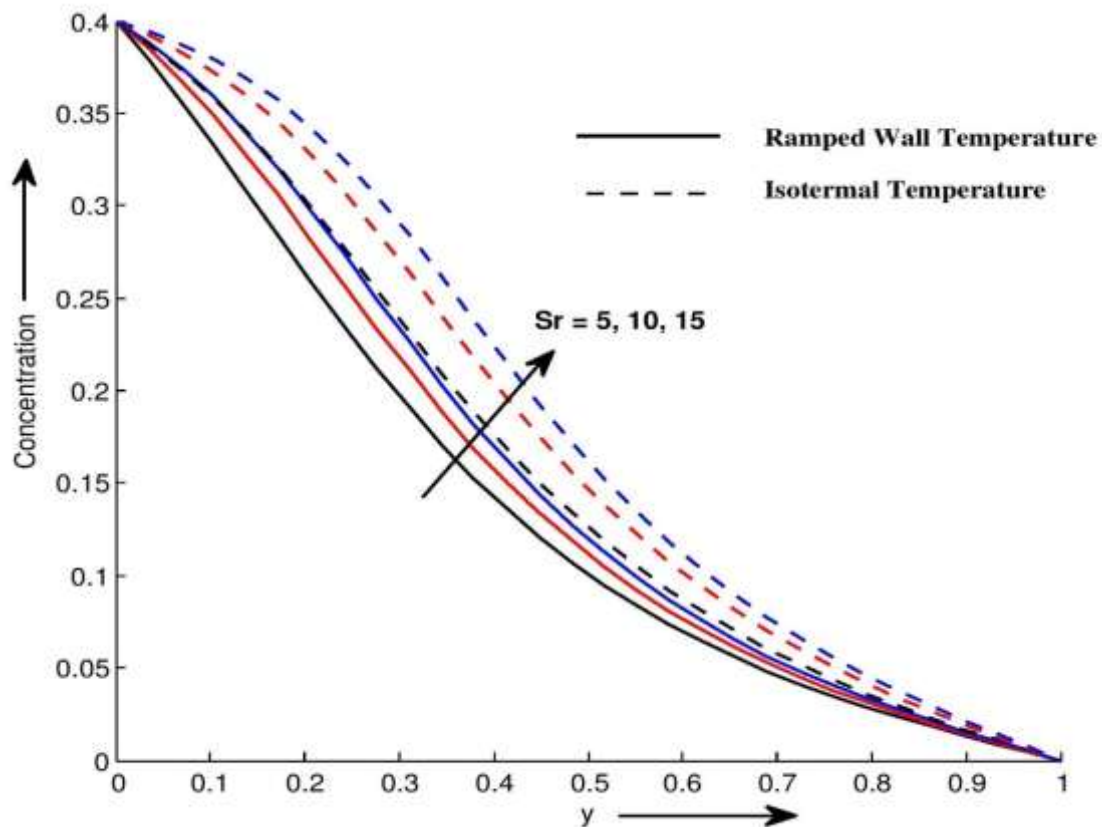


Figure 10: Concentration profile  $C$  for different values of  $y$  and  $Sr$  at  $Sr = 0.66$  and  $t = 0.4$

The numerical values of Nusselt number  $Nu$  and Sherwood number  $Sh$ , calculated from the analytical expressions are exhibited in tabular form in Tables 1 and Table 2 for both thermal cases. It is observed from Tables 1 that, for both thermal cases, Thermal radiation tends to reduced magnitude of Nusselt number whereas Prandtl number  $Pr$  have reverse effect on it. Table 2 illustrates effects of  $Sc$  and  $t$  on rate of mass transfer  $Sh$ . It is seen that, for both thermal condition, Magnitude of Sherwood number increase with increase in Schmidt number  $Sc$ . It is also seen that, for ramped wall temperature, time variable  $t$  tends to improve rate of heat and mass transfer whereas in isothermal plates time variable tends reverse effect on it.

**Table 1: Nusselt number variation**

Pr	T	Nusselt number for Ramped Temperature	Nusselt number for isothermal Temperature
0.71	0.4	0.6013	0.7517
7	0.4	1.8881	2.3602
15	0.4	2.7640	3.4549
7	0.5	2.1110	2.1110
7	0.6	2.3125	1.9271
7	0.7	2.4978	1.7841

**Table 2: Sherwood Number variation**

Sc	t	Sherwood Number Sh for Ramped Surface Concentration	Sherwood Number Sh for Constant Surface Concentration
0.22	0.4	0.3347	0.4184
0.66	0.4	0.5798	0.7247
1.0	0.4	0.7136	0.8921
0.66	0.5	0.6482	0.6482
0.66	0.6	0.7101	0.5917
0.66	0.7	0.7670	0.5478

## V. CONCLUSION:

In this paper, a mathematical model is presented to investigate the thermal radiation, Heat and Mass transfer effects on Natural convective unsteady MHD second grade fluid flow with ramped wall temperature and ramped surface concentration in a porous medium. The governing dimensionless equations are solved using the Laplace transform technique. In order to determine the effect of various parameters on velocity, temperature and concentration profile, we derive the numerical solution using Matlab Software and discuss through several figures. Most important result are defined

- It is observed that velocity and temperature profile in case of ramped temperature are less than that of isothermal temperature.
- Second grade parameter  $\alpha$ , Magnetic field  $M$  tends to de-accelerate motion of the fluid flow throughout the region, whereas permeability of porous medium  $k$ , thermal Grashof number  $Gr$ , mass Grashof number  $Gm$  tends to reverse effect on it.
- Concentration decrease with increase in  $Sc$ .
- Rate of heat transfer increase with  $Pr$ .
- Rate of mass transfer improve with  $Sc$ .

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