COMPRESSED SENSING BASED SECRET IMAGE SHARING SCHEME WITH MINIMUM NUMBER OF SHARES AND SHADOW SIZE

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Abstract: In all the secret sharing schemes the total data of the secret is to be processed and if some of the information is lost then the secret cannot be revealed. The algorithm execution time also increased such that there is a burden to the network transmission. This can be overcome by using compressed sensing technique and extended this to use of minimum number of shares and shadow size Compressed sensing technology measures the original image perceptually through a proper measurement matrix and the measured data covers the vast majority of the useful information of the original image. By using traditional secret image sharing the image is divided into 'n' shadow images and by using any 'r' shadow images we can reconstruct the original image. Less than 'r' the secret cannot be revealed. Here we combine the traditional secret image sharing with compressed sensing technology where only 25% of the shares has been used for reconstructing the image in contrast to that of previous threshold schemes where more than or equal to 50% shares has been used and also reduced the size of shadow image by increasing the threshold value in contrast to previous method. The experimental results reveal that the size of shadow images is reduced to 154 X 128 has reduced to 154 X 64 under the compression ratio 0.6.

IndexTerms - Compressed Sensing, Secret Image Sharing, Measurement Matrix, Wavelets.

I. INTRODUCTION

In the field of information security, the main aim of secret image sharing is to decompose a secret image into many insignificant images or to camouflage a secret image in many substantive images for the sake of storage and transmission. It also eliminates the problem of information loss about the secret image. In communication transfer of images, if any individual information is lost it would not lead to loss all the image information. The method of secret image sharing originates from secret sharing algorithm proposed by Shamir and Blakley in 1979[4]. The main view of secret sharing is to construct the algorithm which is having a threshold value of (r, n) by using r-1 $(0 < r \le n)$ degree polynomial with

 $f(x) = a_0 + \sum_{i=1}^{r-1} a_i x^i \mod p$ in the finite field GF(p).

According to the concept of Lagrange interpolation polynomial, the prime p is selected randomly which is open to public and it should be larger than the secret a_0 and total number of participants n. The coefficients $a_1, a_2, \ldots, a_{r-1}$ are randomly selected from the integer in the range of [0, P-1] and it should be kept secret. The polynomial $s_i = f(x_i) \mod p$, $1 \le i \le n$, is calculated and distributed to n different participants.

Every (x_i, s_i) is a point on the curve f, and any r points can be uniquely decide the r-1 degree polynomial. Hence the secret a_0 can be reconstructed easily r secret shares. From any r shares $s_{i1}, s_{i2}, \ldots, s_{ir}$, the result $a_0 = f(0)$ is obtained according to Lagrange interpolation polynomial $f(x) = \sum_{k=1}^r s_{ik} \prod_{j \neq k, j=1}^r \frac{x-x_{ik}}{x_{ik}-x_{ij}} \mod p$. In secret sharing method with (r,n) threshold scheme, the secret sender divides the

shared secret a_0 into n number of sub-secrets $s_i (1 \le i \le n)$ and with the combination of any r sub-secrets the original secret a_0 can be restored. If the number of combinations is less than r sub-secrets there would be no information about the secret is revealed. The same idea is applying to the secret image sharing, the information of an image is divided into n shares, each share is called a shadow image.

The two important performance indicators of the secret image sharing algorithm is visual quality of the restored image and size of the shadow image. There are many existing methods two improve these parameters but cannot found to achieve an ideal effect in both parameters simultaneously. This is because the character in the image data volume is huge and unchangeable.

In order to reduce the data volume to be shared and to obtain a good visual of restored image, we combined secret image sharing with compressed sensing technology.

Compressed sensing is also known as compressive sensing, compressive sampling or sparse sampling proposed by Donoho, Candes, Romberg and Tao et al. in 2006. CS technique is signal processing technique to acquire and reconstruct the signal efficiently, by finding the solutions to underdetermined systems [2-3]. The focus of CS theory is not on the bandwidth of the signal but on the sparsity or compressibility. CS theory says that when the signal is sparse or compressible in transform domain, the transform coefficients can linearly projected into low dimensional observation vector through the measurement matrix, which is uncorrelated with transformation matrix. At the same time this projection maintains the necessary information to reconstruct the signal. Similarly solution of sparse optimization problem makes the reconstruction of high-dimensional original signal from low-dimensional vector possible. Perceptually by measuring the signal through measurement matrix and obtained measured signal dimension is usually much lower than that of original signal. Like this CS theory not only performs sampling but also done compressing. Though the amount of data obtained is quite small but it is enough for reconstructing.

The rest of the paper is explained as follows: in next section two we introduced about related work on secret image sharing and compared with existing ones. In section three we explained about design of our scheme and implementation process in detail. In section four we explained the experimental setup and results of images to show the effectiveness of the scheme. In section five the performance metrics of the scheme is evaluated. In the last section six the conclusion of the paper is explained and discussed about the future work.

II. RELATED WORK

Many schemes have been introduced in secret image sharing. Thien proposed secret image sharing scheme with steganography. In this method the shadow images are generated by embedding secret image into cover images and obtained shadow images are different from cover images which are meaningless [5]. Lin and Tsai introduces secret image sharing with steganography and authentication. In this scheme they used watermarking technique to embed watermark signal into camouflaged images [6]. Yang improved the scheme of Lin and Tsai but he noticed that the size expansion of no additional pixels from new method but the produced restored image is of low quality [7]. Lin proposed secret image sharing scheme using modular arithmetic and produced high quality images, restores the cover and secret images with no distortion [8].

In all the traditional methods as explained above the size expansion of shadow images remained unsolved. To reduce data expansion following schemes have been proposed. Thien and Lin introduced secret image sharing scheme based on (r, n) threshold method [9]. In this method they inserted secret image pixels in all the coefficients of (r-1) degree polynomial to produce shadow images. In restoration process they used Lagrange interpolation principle that is to restore original image any r shadow images are computed. So the size of shadow image is reduced to $\frac{1}{r}$ times of original image. Wang and Su introduced secret image sharing with Huffman coding technique to reduce the size of the shadow image [10].

Here we combined compressed sensing technique to secret image sharing scheme [1] and performed this method for different thresholds and different formats of the images. The main view of this method is measure the original image perceptually by choosing proper measurement matrix. By this process the original image is compressed from high dimensional to low dimensional image so that the data to be processed is reduced. According to different requirements in restoring the size of measured image can be changed with different compression ratios. When the compression ratio is changed the size of the shadow image is also varies. But compared to traditional methods the size expansion of the shadow image is small in our proposed method.

III. DESIGN

Secret image sharing scheme is proposed based on compressed sensing technique. Fig 1 explains the flow chart of our proposed method. Measure the original image perceptively with proper measurement matrix. By using this measuring process on to the original image of size $N \times N$ is reduced to $M \times N$ where $M \ll N$. The obtained image meaningless image and numerical range of the resulted image is quite large. The order of magnitude of measurement image is 10³ and there exits both positive and negative values. In order to process the image by image sharing algorithm the values are adjusted by using sigmoid conversion. Now we apply the image sharing algorithm to converted image and divided into n shares. In reconstruction side by using any r shadow images that suits the threshold value to obtain the converted image. Now obtain the result of perceptual measurement through inverse sigmoid transformation. Finally by applying the reconstruction algorithm of compressed sensing the original is obtained.

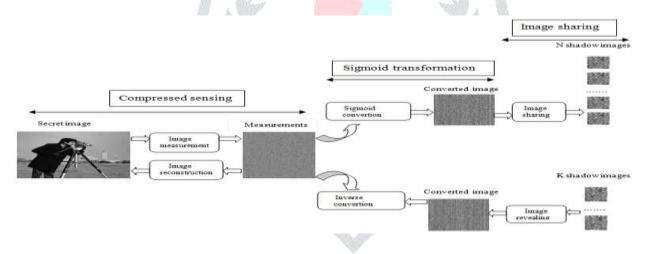


Fig 1: flow chart of compressed sensing with secret image sharing scheme

To perform the entire method the following parameters should be considered. This is followed from reference [1].

A. Signal sparsity:

A signal is said to be sparse if it contains most of the elements as zeroes and only few non-zeroes elements. If the signal has s non-zero elements then it is said to be s-sparse. The signal with less sparsity contains a quality output.

Let us take the signal X in one-dimensional having the length N. Now expand X in standard orthogonal basis as $\Psi = \{\Psi_1 \Psi_2 \dots \Psi_N\}$ having N X N dimension is:

$$X = \sum_{k=1}^{N} \Psi_k \theta_k = \Psi \theta \tag{1}$$

Where X and θ are N X 1 dimensional vectors, if k is number of non-zero elements in θ then the signal X is K-sparse with respect to Ψ . To apply CS theory to a signal it should satisfy sparsity or compressibility. This can be achieved by using wavelet transformation. Because the coefficients of natural images in sparse representation are almost equal to zero. To obtain the enough sparse to orthogonal basis Ψ we use wavelet basis [11-12] function.

B. Measurement matrix:

Let us project NX1 input signal X onto the measurement matrix ϕ having length MXN. The size of measurement matrix is far less than input signal. By projection the sampling data Y is obtained of length MX1. This process is described as

$$X = \Psi\theta$$
 and $Y =$

 ΦX

then by substituting X in Y we get

where matrix Θ satisfies K rank RIP.

The obtained measurement matrix should satisfy restricted isometric property (RIP) because Tao, Donoho and Candes have proved that L0-norm and L1-norm have same optimal solutions under RIP criteria. RIP is defined as, for every integer K = 123 having isometric constant $\delta_K \ge 0$ of matrix θ , has a minimum value that ensures for K-sparse vector V:

Here we use Gaussian random matrix as measurement matrix which satisfies RIP criteria and most of the signals are sparse and uncorrelated.

C. Sigmoid transformation:

After projecting original image onto the measurement matrix the numerical values of the original image is quite large. The order of magnitude is 10³ and exits both positive and negative values. We use sigmoid conversion to adjust the range of values to [0-250] because the modulus p in image sharing algorithm is 251. Formula of sigmoid function is:

$$y = \frac{a_1}{(1+e^{-a_2(x-a_3)})}$$

D. Reconstruction algorithm:

In reconstruction process to restore original signal X we have to use measurement matrix ϕ sparse basis Ψ and measurement value Y. i.e., in CS theory reconstruct the signal X from Y. The formula is $Y = \phi X = \phi \Psi \theta$ for an under determined systems. As the signal X is sparse it is solved by minimum L0-norm optimization and gives exact solution from previous results. The formula of L0-norm is $\min_{\theta} ||\theta||_0$ such that $Y = \theta \theta$.

For the reason that optimization of the L0-norm requires exhausting C_K^N kinds of possibilities of K non zero values in θ , it is NP-hard problem. This is because when the obtained measurement matrix satisfies RIP, then the minimization problems of L0 and L1 norms are equivalent. So L0-norm is replaced by L1-norm [3] $min_{\theta}||\theta||_1$ such that $Y = \theta\theta$.

The problem of transforming non-convex optimization into convex optimization is done by L0 norm and L1 norm, to find the solutions to linear programming method.

Currently the reconstruction algorithm in CS theory is done by three major algorithms. They are non-convex algorithm, greedy algorithm and relaxed algorithm. In this paper orthogonal matching pursuit(OMP) is used[14].

Image sharing flow chart:

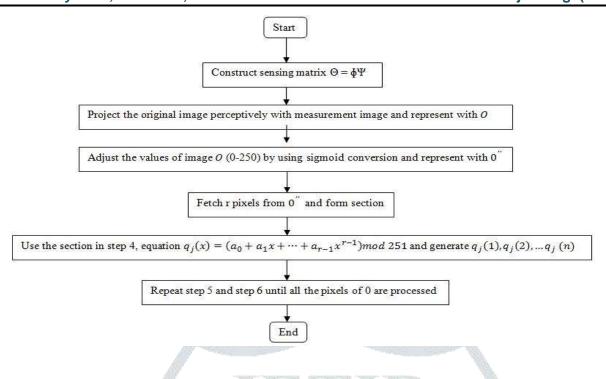
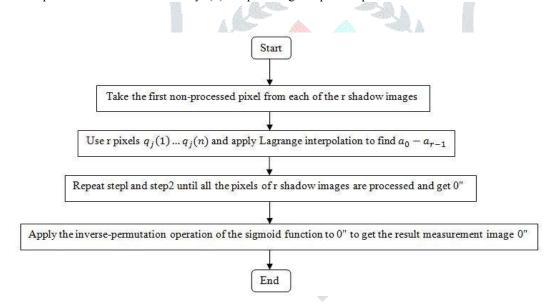
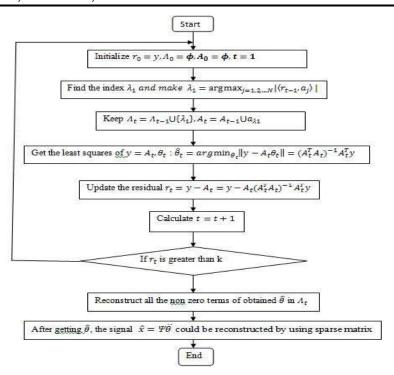


Image revealing flowchart:

Image reconstruction is done in two steps. (1) reconstruct the image by using k shadow images that suit threshold value and apply images sharing restoring algorithm. Now for the obtained image from previous algorithm apply (OMP) algorithm. Here the inputs are (1) sensing matrix $A=\phi\Psi$ and measurement vector y. (2) Output is signal sparse representation coefficient estimation



In the following process: r_t represents residual, t represents iteration times, \emptyset represents empty set, Λ_t represents serial number set of t times' iteration, λ_t represents the obtained index (serial number) of the t-the iteration, a_j represents the t-the column of matrix A , A_t represents the selected column set of matrix A according to index A_t , θ_t is the column vector of $t \times 1$, the symbol \cup represents set and operation and $\langle a,b \rangle$ means to find inner product of vectors.



IV. RESULTS AND DISCUSSIONS

This section explains about the results produced by proposed system. The computer used in this method is laptop with Intel i3 4GB RAM and done experiment in MATLAB 2017a.

This method is applied for three different informative images namely 256 x 256, 512 x 512 and 1024 x 1024 gray scale images in four different formats jpeg, png, bmp and tiff.



Fig 2: camaraman.jpeg



Fig 3: vegetables.jpeg



Fig 4: HD leaf.jpeg

Here we select a 256 X 256 gray scale camaraman.jpeg image and applied (4,8) threshold secret sharing scheme and then out of 50% of shares is reduced to 25% of shares i.e., (2,8) threshold value. Here the amount useful information in the original is varied with different compression ratios. Through experiments it is found that at the compression ratio 0.4 the restoring accuracy of the image is not enough. So as the compression ratio increases the size of the shadow image is increased and the useful data required to restore the image also increases.

Now performed the results when the compression ratio is 0.6 for (4,8) threshold. The results is shown in figure 2, image (a) is original image with 256 X 256 of gray scale image. Image (b) is formed after measurement of original image is of size 154 X 256. Image (c)

is obtained after transformation of sigmoid conversion. Image (d) is eight shadow images and does not reveal any information about original image. The size of shadow image is 154 X 64. Image (e) is the reconstructed image by using 2 shadow images.

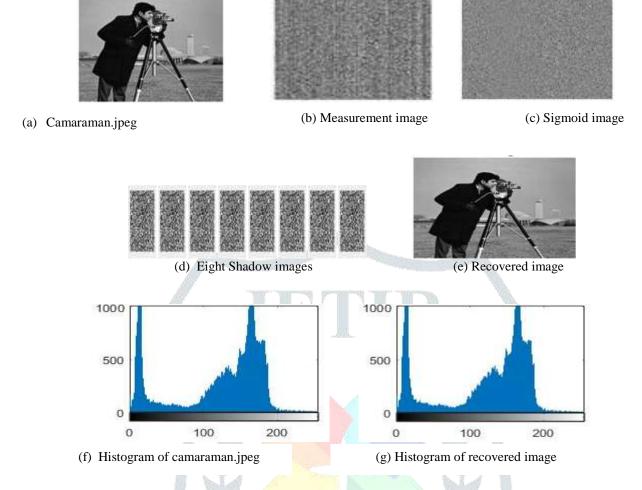
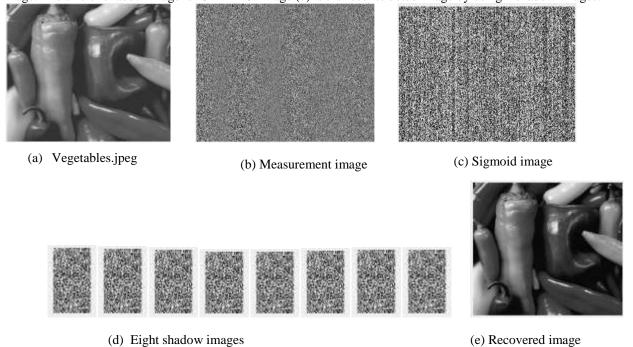
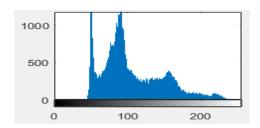


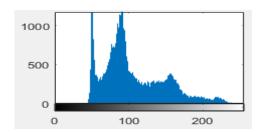
Fig 5: Outputs for (4,8) threshold scheme

We performed the results when the compression ratio is 0.6 for (2,8) threshold. The results is shown in figure 1, image (a) is original image with 256 X 256 of gray scale image. Image (b) is formed after measurement of original image is of size 154 X 256. Image (c) is obtained after transformation of sigmoid conversion. Image (d) is having eight shadow images and does not reveal any information about original image. The size of shadow image is 154 X 128. Image (e) is the reconstructed image by using 2 shadow images.





(f) Histogram of vegetables.jpeg



(g) Histogram of recovered image

Fig 6: Outputs for (4,8) threshold scheme

V. PERFORMANCE METRICS:

In this section evaluation is done based on quantitative evaluation index. Two criteria is evaluated for the proposed method. **Accuracy of the reconstructed image.**

Peak signal to noise ratio (PSNR):

Here the quality of the reconstructed image is evaluated by peak signal to noise ratio (PSNR). Higher PSNR values indicates better fidelity. It is defined as:

$$PSNR = 10log_{10} \left(\frac{255^2}{MSE}\right) db \tag{6}$$

Structural similarity (SSIM):

It measures the similarity between two images. SSIM index satisfies the condition of symmetry: SSIM(x, y) = ssim(y, x).

$$SSIM = \frac{(2\mu_x \mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)}$$
(7)

Where μ_x = the average of x, μ_y = average of y, σ_x^2 = variance of x, σ_y^2 = variance of y, σ_{xy} =covariance of x and y, C_1 = $(K_1, L)^2$ and C_2 = $(K_2, L)^2$ two variables two stabilize the division with weak denominator, L the dynamic range of the pixel value and K_1 = 0.01 and K_2 = 0.03 by default.

Mean square error (MSE):

The mean square is used to describe the difference between two pixels in input and out images. This difference is noted as error. MSE is defined by assuming size of image as H X W then

$$MSE = \frac{1}{HXW} \sum_{j=1}^{HXW} (p_j - p'_j)^2$$
 (8)

Where p_i is the pixel in first image and p'_i is the pixel in second image.

Shadow image based analysis:

To find the working of scheme effectively, is evaluated based on size of shadow images of our proposed scheme is compared with Thien-Lin's scheme. These three evaluation metrics is calculated for different images variying in four formats with different informative images namely "camaraman.jpg, vegetables.jpg, leaf.jpg, camaraman.png, vegetables.png, leaf.png, camaraman.bmp, vegetables.bmp, leaf.bmp, camaraman.tiff, vegetables.tiff, leaf.tiff".

Table 1: Comparison of shadow size

scheme		Shadow image size (2,8)	Shadow image size(4,8)
Thien-Lin		[256,128]	
	0.4	[102,128]	[102,64]
CC	0.5	[128,128]	[128,64]
CS	0.6	[154,128]	[154,64]
	0.7	[179,128]	[179,64]
	0.8	[205,128]	[205,64]
	0.9	[230,128]	[230,64]

0.9915

From Table1 concluded that from (2,8) and (4,8) threshold schemes, as **number of shares** are **increased** to reconstruct the image the **size** of the **shadow images** is **decreased.** Comparing our scheme with thein lin, when the size of input image is 256 X 256 and the obtained shadow image in thein's scheme is 256 X 126 and in our scheme the size is changing with compression ratio. At 0.4 the size is 102 X 128 but the visual quality is not good. At 0.6 the size is 154 X 128 which is less than thein-lin's scheme.

The classification of Shadow Size, PSNR, MSE, and SSIM based on varying compression ratios are tabulated in Table 2 for the images "camarama.jpg", "vegetables.jpg", "leaf.jpg" and performed for (4,8) threshold scheme.

Size(Secret)	Compression Ratio	Size(Shadow)	PSNR	MSE	SSIM
256X256	0.4	102x64	23.36	1.5671	0.9643
	0.5	128x64	27.96	0.8953	0.9693
	0.6	154x64	29.35	0.5074	0.9763
	0.7	179x64	32.90	0.3335	0.9803
	0.8	205x64	34.29	0.2421	0.9810
	0.9	230x64	37.92	0.0652	0.9911
512X512	0.4	102x64	28.67	1.8821	0.9461
	0.5	128x64	32.46	1.6883	0.9649
	0.6	154x64	33.13	0.8546	0.9748
	0.7	179x64	34.05	0.4532	0.9702
	0.8	205x64	36.13	0.3465	0.9807
	0.9	230x64	36.49	0.0442	0.9924
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1024X1024	0.4	102x64	22.77	1.6777	0.928
	0.5	128x64	25.88	1.0176	0.9472
	0.6	154x64	28.06	0.9628	0.9754
	0.7	179x64	33.26	0.3067	0.9825
	0.8	205x64	34.45	0.2939	0.9846

Table 2: Comparison of different informative images with compression ratio for (4,8) threshold

The classification of Shadow Size, PSNR, MSE, and SSIM based on varying compression ratios are tabulated in Table 3 for the images "camarama.jpg", "vegetables.jpg", "leaf.jpg" and performed for (2,8) threshold scheme.

36.16

0.2445

COMPRESSION SIZE(SECRET) SIZE(SHADOW) **PSNR** SSIM MSE **RATIO** 256X256 0.4 102X128 25.80 0.4552 0.9409 0.5 128x128 28.17 0.3641 0.9615 0.9850 154x128 29.40 0.2987 0.6 0.2106 0.9862 0.7 179x128 32.73 205x128 35.64 0.1794 0.9871 0.8 0.9 230x128 37.13 0.0725 0.9881 512X512 0.4 102X128 25.39 1.7794 0.9066 28.72 1.4459 0.9588 0.5 128x128 0.9569 0.6 154x128 29.44 0.9686 0.7 179x128 32.81 0.7544 0.9720 0.8 205x128 34.84 0.4502 0.9886 0.9 230x128 35.44 0.1258 0.9892 1024X1024 0.4 102X128 2.1141 0.9519 26.19 0.5 128x128 28.30 1.6251 0.9641 0.6 154x128 29.30 1.0228 0.9627 179x128 33.37 0.9954 0.9740 0.7 0.8 205x128 36.79 0.3607 0.9871 0.9 230x128 37.35 0.0776 0.9922

Table 3: Comparison of different informative images with compression ratio for (2,8) threshold

230x64

0.9

From the tables 2 and 3 concluded that when the **compression ratio is increased** the size of **shadow image, PSNR, SSIM is increased** and **MSE** because the measurement matrix with high compression ratio will obtain more useful information of the original image.

The classification of Shadow Size, PSNR, MSE, and SSIM is tabulated in Table 4 for (4,8) Threshold scheme and performed for all the four formats of the image namely "camaraman.jpg, vegetables.jpg, leaf.jpg, camaraman.png, vegetables.png, leaf.png, camaraman.bmp, vegetables.bmp, leaf.bmp, camaraman.tiff, vegetables.tiff, leaf.tiff" under the compression ratio 0.6.

Table 4: Comparison of different formats of images for (4,8) threshold

Size(Secret)	Format	Size(Shadow)	PSNR	MSE	SSIM
	Camaraman.jpeg	154X64	28.91	1.3238	0.9802
256X256	Camaraman.png	154X64	36.91	0.0342	0.9920
	Camaraman.tiff	154X64	31.79	0.9140	0.9812
	Camaraman.bmp	154X64	33.52	0.0836	0.9862
512X512	Vegetables.jpeg	154X64	28.01	1.1545	0.9586
	Vegetables.png	154X64	38.10	0.0401	0.9905
	Vegetables.tiff	154X64	31.07	1.0529	0.9699
	Vegetables.bmp	154X64	35.91	0.5086	0.9754
1024X1024	Leaf.jpeg	154X64	27.42	1.2057	0.9605
	Leaf.png	154X64	36.41	0.4596	0.9948
	Leaf.tiff	154X64	30.12	1.4872	0.9823
	Leaf.bmp	154X64	33.51	0.6337	0.9845

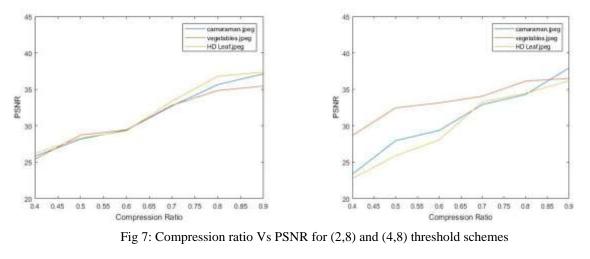
The classification of Shadow Size, PSNR, MSE, and SSIM is tabulated in Table 5 for (2,8) Threshold scheme and performed for all the four formats of the image namely "camaraman.jpg, vegetables.jpg, leaf.jpg, camaraman.png, vegetables.png, leaf.png, camaraman.bmp, vegetables.bmp, leaf.bmp, camaraman.tiff, vegetables.tiff, leaf.tiff" under the compression ratio 0.6

Table 5: comparison of different formats of images for (2,8) threshold

SIZE(SECRET)	FORMAT	SIZE(SHADOW)	PSNR	MSE	SSIM
	Camaraman.jpeg	154X128	27.84	1.4282	0.9664
256X256	Camaraman.png	154X128	36.41	0.4237	0.9839
	Camaraman.tiff	154X128	29.25	0.7456	0.9674
	Camaraman.bmp	154X128	32.86	0.5362	0.9824
			Y A		
512X512	Vegetables.jpeg	154X128	29.55	0.7927	0.9682
	Vegetables.png	154X128	36.52	0.0458	0.9909
	Vegetables.tiff	154X128	31.69	0.3617	0.9795
	Vegetables.bmp	154X128	33.14	0.0698	0.9824
			The same of the sa		
1024X1024	Leaf.jpeg	154X128	28.22	2.0979	0.9477
	Leaf.png	154X128	34.60	0.2556	0.9951
	Leaf.tiff	154X128	31.08	1.8784	0.9532
	Leaf.bmp	154X128	33.54	1.0110	0.9787

From the above tables 4 and 5 concluded that from the four formats of the images tiff, bmp, and png are having better PSNR, SSIM and MSE values than compared to jpeg format because tiff bmp and png are lossless compression formats and jpeg is a lossy compression format.

The graphs are plotted based on varying compression ratios for different threshold schemes for three different informative images, namely "camarama.jpg", "vegetables.jpg", "leaf.jpg".



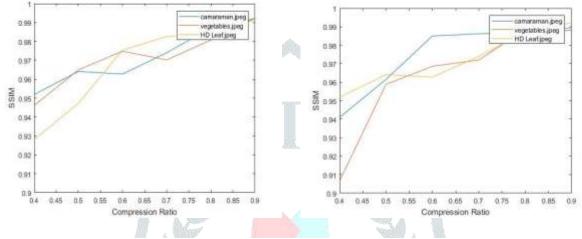


Fig 8: Compression ratio Vs SSIM for (2,8) and (4,8) threshold schemes

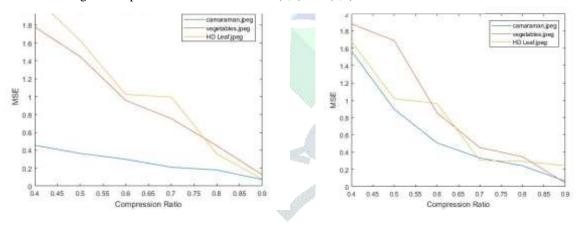


Fig 10: Compression ratio Vs MSE for (2,8) and (4,8) threshold schemes

From fig 8, fig 9, fig 10 shows the PSNR, SSIM, and MSE values of reconstructed image and original image under different compression ratios. It can be seen that with the increasing of compression ratio, the restoring accuracy of the image also increases and MSE decreases. The reason is that the measurement matrix with high compression ratio will obtain more useful information of the original image. From results of experiments, the proper compression ratio should be selected to obtain the accurate restoration of original image, to reduce the data volume that needs to be processed effectively and then to reduce transmission burden, storage overhead.

VI. CONCLUSION:

The problems caused by large amount of data and maintaining secrecy to the information is solved by combining traditional secret image sharing with compressed sensing technology. By constructing suitable measurement matrix and by projecting it with original image, the size greatly reduces. Through experimental results it is found that the size of shadow image greatly reduces than thein-lin scheme. Through parameters found that for png format image gives better results compared to jpeg, bmp and tiff because png is lossless compression and other formats are lossy compression. As no. of shares required to reconstruct an image the size of shadow is reduces and it is observed from (4,8) and (2,8) threshold schemes.

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