THE ANALOGY OF COUPLED OSCILLATORS

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Abstract : ***An oscillator is any system that exhibits periodic behavior. When two or more oscillators are coupled, however the ranges of possible behaviours become much more complex. The equations governing their behaviour also tend to become obstinate. Each oscillator may be coupled only to a few immediate neighbours or to all the oscillators in an enormous community.***

Mathematically when two nonlinear oscillators are coupled they entrain (i.e., synchronize to a common frequency) only when certain conditions are met. A strong coupling constant (lower resistance) and a small discrepancy between the two intrinsic frequencies increase the chances of entrainment. Synchrony is the most familiar mode of organization for coupled oscillators. Synchronization now means that two non identical oscillators start to oscillate with the same frequency (or, more generally, with rationally related frequencies). This common frequency usually lies between their intrinsic frequencies f1 and f2. It is notable that locking of the phases and frequencies implies no restrictions on the amplitudes, indeed the synchronized oscillators may have very different amplitudes and waveforms. The phenomena of synchronization among several coupled oscillators including van der Pol oscillators are carried out in real time and the results interpreted.

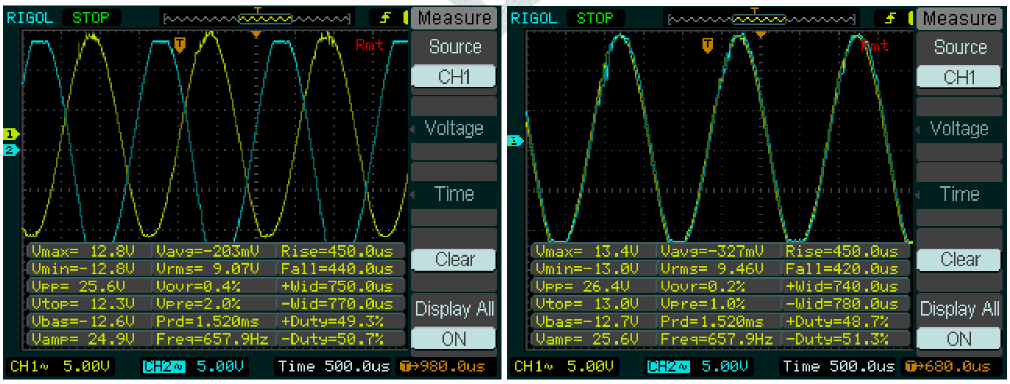
Index Terms - Oscillator, Synchrony, coupling conductance, phase difference, intrinsic frequency.

I. INTRODUCTION

A dynamical system exhibiting a stable periodic orbit is often called as an oscillator. A single oscillator traces out a simple path in phase space. Two oscillators form a basic building block for the case of many mutually coupled systems. Two nonlinear systems each exhibiting self-sustained periodic oscillations, generally with different amplitudes and frequencies can interact, with the strength of the interaction being the key parameter. Each oscillator may be coupled only to a few immediate neighbours or to all the oscillators in an enormous community. Synchrony is the most familiar mode of organization for coupled oscillators. When two identical oscillators are coupled, there are exactly two possibilities: synchrony, a phase difference of zero, and anti synchrony, a phase difference of one half. If the network has more than two oscillators the range of possibilities also increases [1]. If two oscillators are weakly coupled they remain close to their limit cycle at all times which provided the insight to ignore the variations in its amplitude and to consider only the variations in the phase. It can also be assumed that each oscillator is influenced only by the collective rhythm produced by all the others [2]. A limit cycle represents a steady state oscillation, to which or from which all trajectories nearby will converge or diverge. If a few oscillators happen to synchronize, their combined coherent signal rises above the background in exerting a stronger effect on the other. When additional oscillators are pulled into the synchronized nucleus they amplify its signal. This positive feedback leads to an accelerating outbreak of synchrony. Some oscillators nonetheless remain unsynchronized because their frequencies are too far from the value at which the others synchronize for the coupling to pull them in [1].

II. COUPLED WIEN BRIDGE OSCILLATORS IN REAL TIME

Two low frequency Wien Bridge oscillators that have good stability at their resonant frequency as well as low distortion are realized. Two such identical oscillators are resistively coupled to one another satisfying the requirements for oscillation with the frequency of oscillations 'F' fixed at 658 Hz (F1 = F2) by proper value of circuit components. When coupled the oscillators fail to synchronize as shown in Fig. 1(a). Suitable variations made in the coupling conductance (G_C) in the range of 125-200 µS (Siemens)

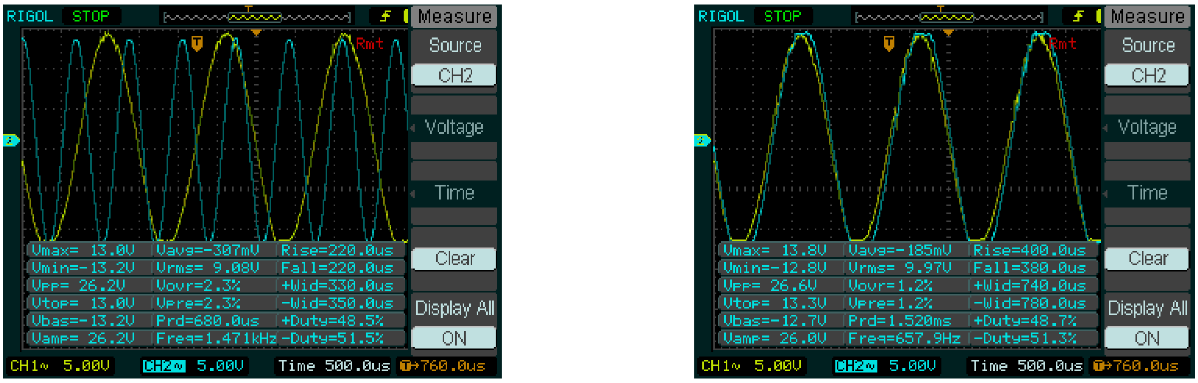


(a) 2 identical oscillators not in synchrony **b) 2 identical oscillators in synchrony**

Fig. 1. Response of Coupled identical Wien bridge oscillators with F1 = F2

eventually synchronize the two oscillators with a very small phase difference to a common frequency of 657.9 Hz as presented in Fig. 1(b).

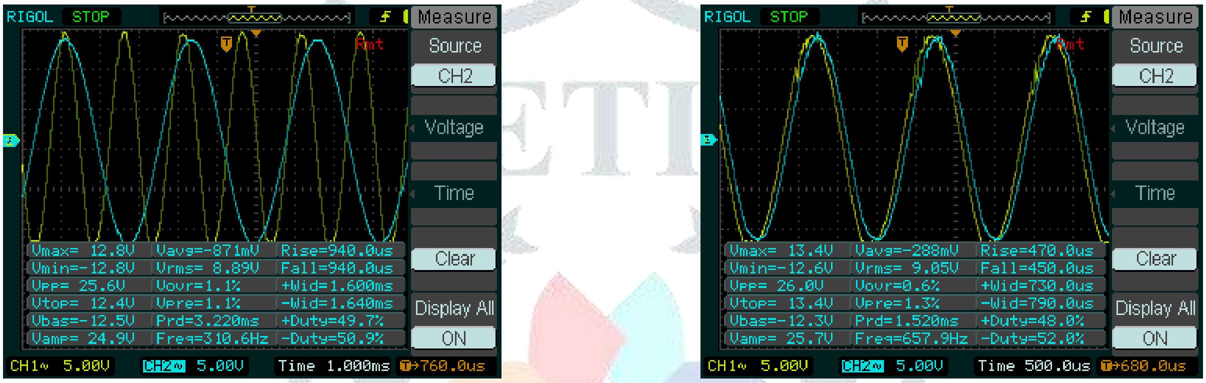
The frequency of the first oscillator 'F1' is maintained as before and with proper values of circuit components the frequency of oscillation of the second oscillator 'F2' is fixed at a higher value of 1.47 KHz. When coupled the two oscillators initially fail to entrain each other as in Fig. 2(a). Variations made in the coupling conductance (G_C) eventually synchronize the two oscillators to a common frequency of 657.9 Hz with a small phase difference for a range of conductance values of 250-500 µS as in Fig. 2(b).



(a) Not in synchrony **(b) In synchrony**

Fig. 2. Response of Coupled Wien bridge oscillators with F1 < F2

The frequency of the oscillation 'F2' for the second oscillator is now fixed at a lower value of 311 Hz while that of the first oscillator 'F1' is maintained as before. When coupled the two oscillators initially fail to synchronize as in Fig. 3(a). Suitable adjustments made in the coupling conductance (G_C) in the range of 50-100 µS ultimately establish synchrony among the oscillators at a common frequency of 657.9 Hz though with a phase shift as illustrated in Fig. 3(b).

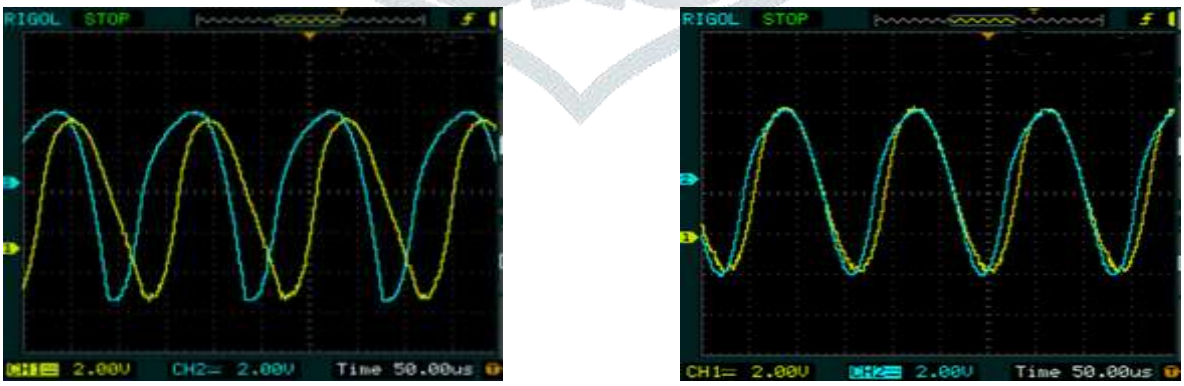


(a) Not in synchrony **(b) In synchrony**

Fig. 3. Response of Coupled Wien bridge oscillators with F1 > F2

III. COUPLED RC PHASE SHIFT OSCILLATORS IN REAL TIME

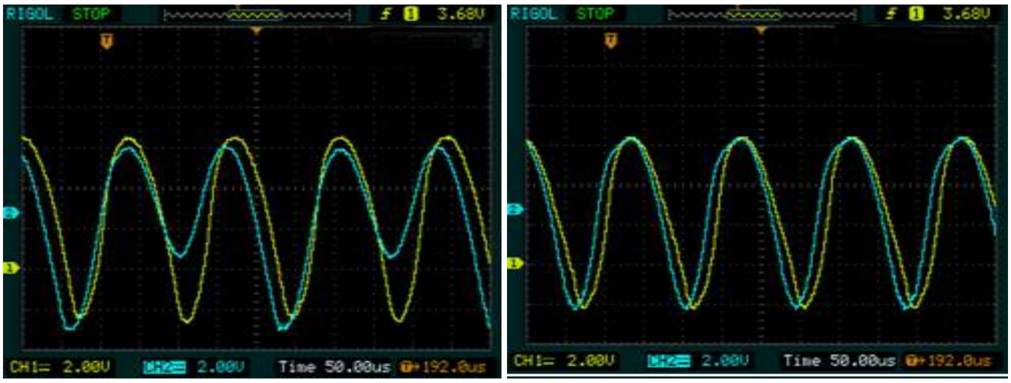
The RC phase shift oscillator, another low frequency oscillator possesses less distortion than the Wien bridge oscillator, coupled with good frequency stability [3]. With proper component values, the frequency of oscillations 'F' is fixed at 6.8 KHz for both the oscillators (F1=F2=6.8 KHz). When resistively coupled with another oscillator of the same frequency, the two oscillators are initially not in synchronization as seen in Fig. 4(a). The G_C is varied to establish mutual entrainment between the coupled oscillators and finally at a range of value around 17µS, the oscillators synchronize to a common frequency of 6.8 KHz but with a phase shift as seen in Fig. 4(b).



(a) 2 identical oscillators not in synchrony **b) 2 identical oscillators in synchrony**

Fig. 4. Response of Coupled identical RC phase shift oscillators with F1 = F2

With the aid of the circuit components the frequency of oscillation of the second oscillator 'F2' is increased to 9.9 KHz while that of the first oscillator 'F1' maintained as before. When coupled the two fail to entrain each other initially as seen in Fig. 5(a). Subtle variations in the G_C at one point of time allow the oscillators to eventually synchronize with a phase shift at an entrained frequency of 7.14 KHz as seen from Fig. 5(b) with the range of the conductance values being around 50 µS.



(a) Not in synchrony **(b) In synchrony**

Fig. 5. Response of Coupled RC phase shift oscillators with F1 < F2

The frequency of the oscillation 'F2' for the second oscillator is fixed at a lower value of 4.3 KHz while that of the first oscillator 'F1' is maintained as before. Upon coupling resistively the two oscillators initially fail to synchronize as in Fig. 6(a). The adjustments made in the coupling conductance G_C ultimately establish synchrony among the oscillators at a common frequency of 7.7 KHz for conductance values in the range around 33 µS as presented in Fig. 6(b).

(a) Not in synchrony **(b) In synchrony**

Fig. 6. Response of Coupled RC phase shift oscillators with F1 > F2

In all the above schematics, bidirectional coupling is used where both the oscillators (subsystems) are coupled with each other, and the coupling factor induces an adjustment of the rhythms onto a common synchronized manifold, thus inducing mutual synchronization behaviour [4]. It is seen that when both the systems (oscillators) are connected in such a way they mutually influence each other's behaviour. When not coupled, each oscillator beats independently at its own intrinsic frequency. However, when they are coupled together through an ohmic resistor, the current flowing from one oscillator to the other (i.e., the coupling current) is strictly a function of the differences in their voltages. At any given instant the coupling current for one oscillator is equal and opposite to that of the other. By means of this coupling current, they mutually influence one another. The faster oscillator does indeed increase the frequency of the slower, but the reverse is also true. The slower oscillator slows down the faster one; thereby they arrive at a mutual consensus [5].

The output frequency at some point exactly matches with the input frequency and continue to remain as such thereafter. The phenomenon which results in a synchronization or matching of the output frequency with the input frequency is called frequency entrainment or synchronization.

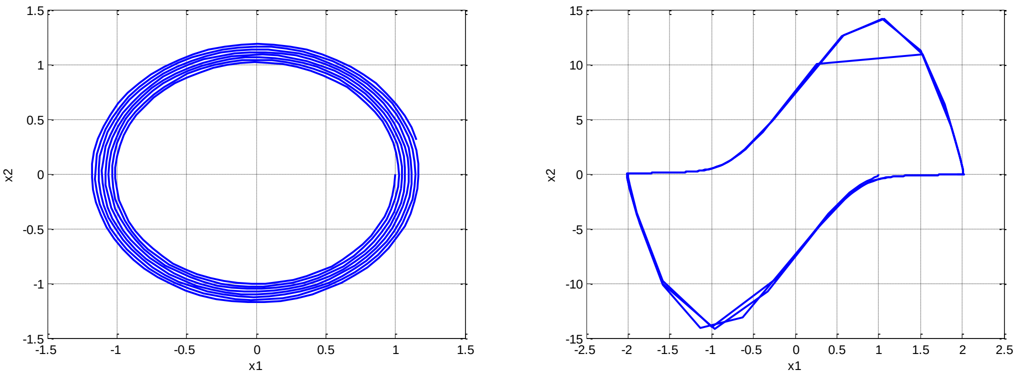
IV. VAN DER POL OSCILLATOR

Many important physical, chemical and biological systems are composed of coupled nonlinear oscillators. Van der Pol's equation is used as a model for numerous biological oscillators. The differential equation

$$\ddot{x} + \varepsilon(x^2 - 1)\dot{x} + x = 0, \quad \varepsilon > 0 \qquad (1)$$

is called the van der Pol oscillator, where x is the position coordinate; a dynamical variable, and ε is a scalar parameter which controls the nonlinearity and the strength of the damping [6]. In a van der Pol oscillator, the stability is very much dependent on the input and also the initial state with the system exhibiting limit cycles which are self-sustained oscillations of fixed frequency and amplitude.

It is seen from the simulated results that for small values of ε (0.01) the limit cycle in van der Pol's equation (1) is nearly a circle of radius 2 in the phase plane, and its frequency is approximately equal to unity as depicted in Fig. 7(a). The phase plane is a state plane where the two state variables x1 and x2 are analyzed which may be the output variable and its derivative. In the Fig.7, x1 and x2 corresponds to the two states of the van der Pol oscillator. The character of the limit cycle gradually changes as ε is increased, until for very large values of ε (10) it becomes a relaxation oscillation as illustrated in Fig. 7(b).



(a) (b)

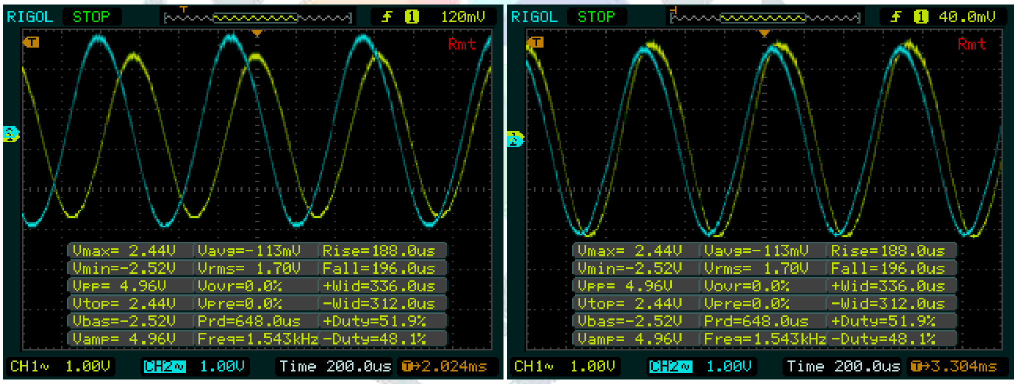
Fig. 7. Limit cycle representation for a van der Pol oscillator; (a) ε=0.01; (b) ε=10

4.1 TWO COUPLED LIMIT CYCLE OSCILLATORS

A limit cycle oscillator, such as the van der Pol oscillator, can autonomously generate an attractive periodic motion. If two such oscillators are coupled together, i.e., operate physically near one another, the output from either one of them can influence the behavior of the other. Although both oscillators may in general have different frequencies, the effect of the coupling acts to produce a motion which is phase and frequency locked. The three states of a system of two coupled limit cycle oscillators can be strongly locked, weakly locked or unlocked [7].

4.2 COUPLED VAN DER POL OSCILLATORS IN REAL TIME

Two van der Pol oscillators are realized using TL082 wide bandwidth dual JFET input Operational Amplifiers and with low cost, four-quadrant analog multipliers AD633 and other allied circuit components. When resistively coupled with another oscillator of the same frequency (F1=F2=1.55 KHz) the two fail to synchronize initially with varying phase and amplitudes between them as seen in Fig. 8(a). The coupling conductance is varied accordingly and finally at a value around G_C=1/83 S the coupled oscillators synchronize to a common frequency of 1.54 KHz with equality in the amplitude but with a small phase difference as represented in Fig. 8(b). The G_C is initially set to a known minimal value and is gradually increased by observing the output till both the oscillators responses are in synchrony both in phase as well as amplitude. It is worth to note that the frequencies of the realized van der Pol oscillators are scaled accordingly with the aid of the circuit component values and the parameter 'ε' is maintained at a value of 0.1 throughout.

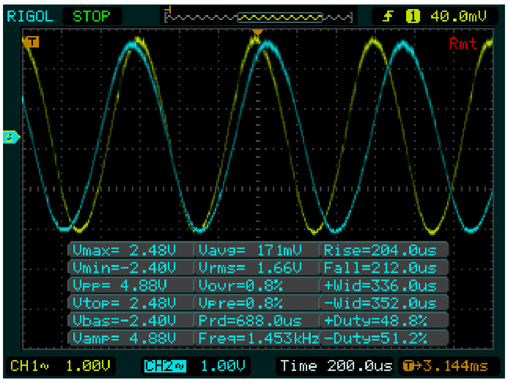


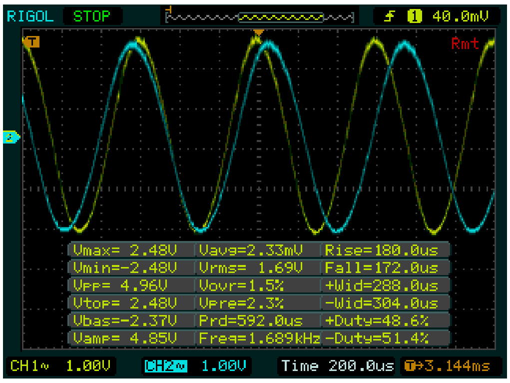
(a) Not in synchrony **(b) In synchrony**

Fig. 8. Response of Coupled identical van der Pol oscillators with F1=F2

The intrinsic frequency of the second van der Pol oscillator 'F2' (1.7 KHz) alone is increased by appropriate variations while maintaining that of the first oscillator 'F1' (1.46 KHz). When coupled, for any range of variations effected in the coupling conductance (G_C), the two oscillators fail to synchronize with varying phase shifts between their states as seen in Fig. 9.

The Fig. 9(a) explicitly illustrates the response of the first oscillator with its frequency 'F1' displayed whereas the Fig. 9(b) depicts the frequency value of the second oscillator 'F2'. In both the responses the coupled pair of oscillators fails to synchronize with varying phase shifts.

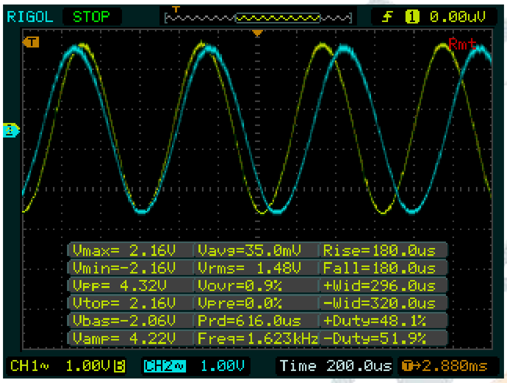


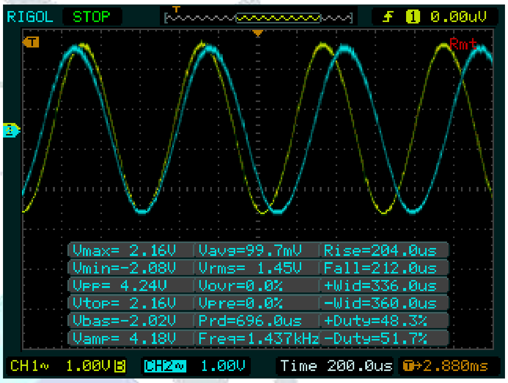


(a) Not in synchrony with display of F1 **(b) Not in synchrony with display of F2**

Fig. 9. Response of Coupled van der Pol oscillators with F1<F2

On the contrary when the intrinsic frequency of the second oscillator 'F2' (1.44 KHz) is appropriately decreased with regard to the first one 'F1' (1.63 KHz) such that (F1>F2) it is again inferred that any range of values of the coupling conductance magnitude effected fail to establish synchrony among the pair with varying phases between their states as evident from Fig. 10. The responses depicted in Fig. 10(a) represents the frequency displayed for the first oscillator and the response Fig. 10(b) depicts the frequency displayed for the second oscillator.





(a) Not in synchrony with display of F1 **(b) Not in synchrony with display of F2**

Fig. 10. Response of Coupled van der Pol oscillators with F1>F2

V. RESULTS AND DISCUSSION

The "adjustment of rhythms of oscillating objects due to their weak interaction" is defined as Synchronization [8]. When synchronization is in view, one generally looks at objects that oscillate alone, without outside influence called self-sustaining oscillators. With more than one of these oscillating objects, interactions can occur called coupling, and the coupling strength describes how strong the interactions are. Even with weak coupling, non identical oscillators can interact in such a way to synchronize to each other (Fig's. 3(b), 5(b) and 6(b)).

Generally, a group of oscillators is said to be synchronized when each oscillator's frequency has locked onto the same value as all the others' [8]. Entrainment is a property of coupled oscillators. The simplest case occurs when the coupling is only one way; the frequency of one oscillator is constant and entrainment occurs when the other matches this frequency (Fig's. 1(b), 2(b), 3(b), 4(b) and 8(b)).

Two coupled oscillators can periodically drive each other to a common phase-locked state [9] (Fig's. 1(b), 2(b), 3(b), 4(b), 5(b), 6(b) and 8(b)).

When there is no coupling the states of both the oscillators will almost always asymptotically approach their respective stable limit cycles. As the motions on the respective limit cycles are uncoupled, any phase difference is possible.

For sufficiently small coupling it may be expected that there will exist periodic motions in which both oscillators remain close to their uncoupled limit cycles. Now, however, the phases of the two oscillators may be "locked", i.e., the difference in phase angles may approach a certain constant value regardless of initial conditions (Fig's. 2(b), 3(b), 4(b), 5(b), 6(b) and 8(b)).

Phase entrainment, in which the phase difference between the oscillators varies periodically, is seen as an intermediate state between phase locking and phase drift. Two oscillators are said to be 1:1 phase locked if the phase difference is constant (Fig's. 2(b), 3(b), 4(b), 5(b), 6(b) and 8(b)). Whereas if the oscillators are running at unequal average frequencies, then the phase difference grows unbounded, defining the condition of 1:1 phase drift (Fig's. 9 and 10).

In a generalized system of interacting oscillators when a large number of oscillators with different natural frequencies are allowed to interact strongly enough, they all start to oscillate at the same rate.

There are generally two values that determine how easily a group of oscillators can synchronize: the coupling strength and the frequency detuning. Frequency detuning is a number describing how different the natural frequencies of the oscillators are in the absence of the effects of the other oscillators.

A group with large frequency detuning is harder to synchronize because the individual oscillators want to oscillate at highly varied frequencies. All interacting oscillators have a range of values for the detuning for which they are able to synchronize. Inside this range, the coupling causes the oscillators to have the exact same frequency (not just close, as one would expect even without coupling) [8, 10]; this range typically expands as the coupling strength increases (Fig's. 1(b), 2(b) and 8(b)).

Self-sustained oscillators like the van der Pol oscillators have stable limit cycles, meaning that all trajectories near the limit cycle approach the limit cycle eventually revisiting the same points over and over (Fig. 7(a), (b)). The efficacy of coupling conductance though remains as the major determinant in the synchronization of the coupled oscillators; it fails to synchronize the coupled pair even for larger magnitudes of coupling strengths with the state of the oscillators undergoing an unbounded phase difference (Fig's. 9 and 10). It can be perceived that the value of the coupling conductance can coax the coupled oscillators to synchronize only for a limited range of adjustment. Interacting nonlinear oscillators with different individual frequencies can spontaneously synchronize themselves to a common frequency-if the coupling strength exceeds a certain threshold value (Fig's. 2(b), 3(b), 5(b) and 6(b)).

While synchronization is a delicate issue even in a simple pair of interacting oscillators, it ought to be acute in a large, interacting population where the synchronization occurs by the phase dependant interaction of thousands of pacemaker oscillators. This phenomenon will be of much relevance to the understanding of biological oscillators such as, e.g., coupled heart pacemaker cells [11].

VI. CONCLUSION

This paper deals initially with the real time coupling of two wien bridge oscillators where the intrinsic frequencies of both the oscillators are kept the same. The intrinsic frequencies of the oscillators are later held different with the emphasis on the synchronization with the efficacy of the coupling varied. The same phenomenon is carried out next on a coupled RC phase shift oscillators in real time. Two van der Pol oscillators are realized using appropriate circuit components and resistively coupled with the same intrinsic frequencies and later on with varied intrinsic frequencies as before. The impact of the coupling conductance with varied intrinsic frequencies of the coupled van der Pol oscillators with regard to synchronization has been brought out in real time and the results interpreted. This work can be extended by considering different frequency distributions (that is, one where most oscillators are at the same frequency but a few are not). The synchronization phenomenon in different topologies such as simple ring, or having the scenario where diametrically opposed oscillators alone are coupled can be tried. On the other hand, instead of coupling at the same strength, a gradual decay in coupling (as distance from the oscillator increases rather than only being coupled to the nearest neighbors) can also be considered.

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