

# FUZZY $b$ -BAIRE SPACE, FUZZY $\omega$ -BAIRE SPACE AND FUZZY $\alpha$ - $g^*$ BAIRE SPACES

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## Abstract:

In this paper we introduce fuzzy  $b$ -Baire Spaces, fuzzy  $\omega$ -Baire space, fuzzy  $\alpha$ - $g^*$ Baire space and discuss about some properties with suitable examples.

## Key words:

Fuzzy  $b$ -open, fuzzy  $\omega$ -open, fuzzy  $\alpha$ -open, fuzzy  $g$ -open, fuzzy  $\alpha$ - $g^*$ open, fuzzy  $b$ -Baire Spaces, fuzzy  $\omega$ -Baire space, fuzzy  $\alpha$ - $g^*$ Baire space.

## **I. Introduction:**

The theory of fuzzy sets was initiated by L.A.Zadeh in his classical paper [9] in the year 1965 as an attempt to develop a mathematically precise framework in which to treat systems or phenomena which cannot themselves be characterized precisely. The potential of fuzzy notion was realized by the researchers and has successfully been applied for investigations in all the branches of Science and Technology. The paper of C.L.Chang [2] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.

The concepts of  $\alpha$ -generalized closed sets have been studied in classical topology in [3]. The notion of fuzzy topology on fuzzy sets was introduced by Chakraborty and Ahsanullah as one of treatments of the problem which may be called the subspace problem in fuzzy topological spaces. One of the advantages of defining topology on a fuzzy set lies in the fact that subspace topologies can now be developed on fuzzy subsets of a fuzzy set. Later Chaudhury and Das studied several fundamental properties of such fuzzy topologies.

The concept of separation axioms is one of most important concepts in topology. In fuzzy setting, it had been studied by many authors such as [3,5,6,7,10]. However, the separation and regularity axioms has not yet been studied in the new setting, only they introduced the concept of Hausdorff, regular and normal spaces. In this paper we introduce the fuzzy  $b$ -nowhere dense set, fuzzy  $\omega$ -nowhere dense set, and fuzzy  $\alpha$ - $g^*$ -nowhere dense sets and introduce fuzzy  $b$ -Baire Spaces, fuzzy  $\omega$ -Baire space, fuzzy  $\alpha$ - $g^*$ Baire space with suitable examples.

## **II. Preliminaries:**

Now review of some basic notions and results used in the sequel. In this work by  $(X, T)$  or simply by  $X$ , we will denote a fuzzy topological space due to Chang [2].

### **Definition 2.1 [2]:**

Let  $\lambda$  and  $\mu$  be any two fuzzy sets in a fuzzy topological space  $(X, T)$ . Then we define:

$$\lambda \vee \mu : X \rightarrow [0, 1] \text{ as follows: } \lambda \vee \mu (x) = \max \{ \lambda(x), \mu(x) \};$$

$$\lambda \wedge \mu : X \rightarrow [0, 1] \text{ as follows: } \lambda \wedge \mu (x) = \min \{ \lambda(x), \mu(x) \};$$

$$\mu = \lambda^c \leftrightarrow \mu(x) = 1 - \lambda(x).$$

For a family  $\lambda_i \in I$  of fuzzy sets in  $(X, T)$ , the union  $\psi = \bigcup \lambda_i$  and intersection  $\delta = \bigcap \lambda_i$

are defined respectively as  $\psi(x) = \text{Sup}_i\{\lambda_i(x), x \in X\}$ , and  $\delta(x) = \text{Inf}_i\{\lambda_i(x), x \in X\}$ .

**Definition 2.2 [3]:**

Let  $(X, T)$  be a fuzzy topological space. For a fuzzy set  $\lambda$  of  $X$ , the interior and the closure of  $\lambda$  are defined respectively as  $\text{int}(\lambda) = \vee \{\mu \mid \mu \leq \lambda, \mu \in T\}$  and  $\text{cl}(\lambda) = \wedge \{\mu \mid \lambda \leq \mu, 1 - \mu \in T\}$ .

**Definition 2.3[4]:**

Let  $(X, T)$  be a topological space. For a fuzzy set  $\lambda$  of  $X$  is a  $\alpha$ -generalized closed set (briefly  $\alpha$ -closed) if  $\alpha\text{cl}(\lambda) \leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is fuzzy open in  $X$ .

**Definition 2.4 [9]:**

A fuzzy set  $\lambda$  in a fuzzy topological space  $(X, T)$  is called fuzzy dense if there exists no fuzzy closed set  $\mu \in (X, T)$  such that  $\lambda < \mu < 1$ . That is  $\text{cl}(\lambda) = 1$ .

**Definition 2.5 [8]:**

A fuzzy set  $\lambda$  in a fuzzy topological space  $(X, T)$  is called fuzzy nowhere dense if there exists no non-zero fuzzy open set  $\mu$  in  $(X, T)$  such that  $\mu < \text{cl}(\lambda)$ . That is,  $\text{intcl}(\lambda) = 0$ .

**Definition 2.6 [8]:**

Let  $(X, T)$  be a fuzzy topological space. A fuzzy set  $\lambda$  in  $(X, T)$  is called fuzzy first category set if  $\lambda = \bigcup_{i=1}^{\infty} \lambda_i$ , where  $\lambda_i$ 's are fuzzy nowhere dense sets in  $(X, T)$ . A fuzzy set which is not fuzzy first category set is called a fuzzy second category set in  $(X, T)$ .

**Definition 2.7 [7]:**

A fuzzy topological space  $(X, T)$  is called fuzzy first category space if  $1 = \bigcup_{i=1}^{\infty} \lambda_i$ , where  $\lambda_i$ 's are fuzzy nowhere dense sets in  $(X, T)$ . A topological space which is not of fuzzy first category is said to be of fuzzy second category space.

**Definition 2.8 [8]:**

Let  $(X, T)$  be a fuzzy topological space. Then  $(X, T)$  is called a fuzzy Baire space if  $\text{int}(\bigcup_{i=1}^{\infty} (\lambda_i)) = 0$ , where  $(\lambda_i)$ 's are fuzzy nowhere dense sets in  $(X, T)$ .

**Definition 2.9[1]:**

A fuzzy topological space  $(X, T)$  is called fuzzy  $\alpha$ -generalized Baire space if  $\alpha - \text{int}(\bigcup_{i=1}^{\infty} (\lambda_i)) = 0$ , where  $(\lambda_i)$ 's are fuzzy  $\alpha$ -nowhere dense sets in  $(X, T)$ .

**Definition 2.10 [1]:**

A fuzzy topological space  $(X, T)$  is called fuzzy generalized  $\alpha$ -Baire space if  $\text{int}(\bigcup_{i=1}^{\infty} (\lambda_i)) = 0$ , where  $(\lambda_i)$ 's are fuzzy nowhere dense sets in  $(X, T)$ .

### III. Fuzzy $b$ -nowhere dense sets, Fuzzy $\omega$ - nowhere dense sets, Fuzzy $\alpha$ - $g^*$ nowhere dense sets:

We introduce the fuzzy  $b$ , fuzzy  $\omega$  and fuzzy  $\alpha$ - $g^*$ nowhere dense sets with suitable examples.

**Definition 3.1:**

A fuzzy set  $\lambda$  in a fuzzy topological space  $(X, T)$  is called fuzzy  $b$ - nowhere dense if there exists no non-zero fuzzy  $b$ -open set  $\mu$  in  $(X, T)$  such that  $b\text{-cl}(\lambda) \leq \mu$ . That is,  $b\text{-int cl}(\lambda) = 0$ .

**Example 3.1:**

Let  $X = \{a, b, c\}$ . The fuzzy sets  $\lambda$ ,  $\mu$  and  $\gamma$  are defined on  $X$  as follows:

$\lambda : X \rightarrow [0, 1]$  defined as  $\lambda(a) = 0.8$  ;  $\lambda(b) = 0.6$  ;  $\lambda(c) = 0.7$ .

$\mu : X \rightarrow [0, 1]$  defined as  $\mu(a) = 0.9$  ;  $\mu(b) = 0.8$  ;  $\mu(c) = 0.7$ .

$\gamma : X \rightarrow [0, 1]$  defined as  $\gamma(a) = 0.7$  ;  $\gamma(b) = 0.6$  ;  $\gamma(c) = 0.5$ .

Then  $T = \{0, \lambda, \mu, \gamma, 1\}$  is fuzzy topology on  $X$ . The fuzzy sets  $\lambda, \mu, \gamma$  are fuzzy  $b$ -open sets.

Since  $\lambda \leq [\text{int cl}(\lambda)] \vee [\text{cl int}(\lambda)] \leq \lambda \vee 1 \leq 1$ ,

$\mu \leq [\text{int cl}(\mu)] \vee [\text{cl int}(\mu)] \leq \mu \vee 1 \leq 1$ ,

$\gamma \leq [\text{int cl}(\gamma)] \vee [\text{cl int}(\gamma)] \leq \gamma \vee 1 \leq 1$ .

$\lambda, \mu, \gamma$ 's are fuzzy  $b$ -open.

Now  $b\text{-int } b\text{-cl}(1-\lambda) = b\text{-int}(1-\lambda) = 0$ ,

$b\text{-int } b\text{-cl}(1-\mu) = b\text{-int}(1-\mu) = 0$ ,

$b\text{-int } b\text{-cl}(1-\gamma) = b\text{-int}(1-\gamma) = 0$ ,

Therefore  $1-\lambda$ ,  $1-\mu$ ,  $1-\gamma$ 's are fuzzy  $b$ - nowhere dense sets.

**Example 3.2:**

Let  $X = \{a,b,c\}$ . The fuzzy sets  $\lambda$ ,  $\mu$  and  $\gamma$  are defined on  $X$  as follows:

$\lambda : X \rightarrow [0,1]$  defined as  $\lambda(a) = 0.6$  ;  $\lambda(b) = 0.7$ .

$\mu : X \rightarrow [0,1]$  defined as  $\mu(a) = 0.7$  ;  $\mu(b) = 0.7$ .

$\gamma : X \rightarrow [0,1]$  defined as  $\gamma(a) = 0.9$ ;  $\gamma(b) = 0.8$ .

Then  $T = \{0, \lambda, \mu, \gamma, 1\}$  is fuzzy topology on  $X$ . The fuzzy sets  $\lambda$ ,  $\mu$ , and  $\gamma$  are fuzzy  $B$ -open sets.

Now  $\lambda \leq [\text{int cl}(\lambda)] \vee [\text{cl int}(\lambda)] \leq \lambda \vee 1 \leq 1$ ,

$\mu \leq [\text{int cl}(\mu)] \vee [\text{cl int}(\mu)] \leq \mu \vee 1 \leq 1$ ,

$\gamma \leq [\text{int cl}(\gamma)] \vee [\text{cl int}(\gamma)] \leq \gamma \vee 1 \leq 1$ .

$\lambda, \mu, \gamma$ 's are fuzzy  $b$ -open.

we define the fuzzy sets  $\beta, \delta$  and  $\eta$  on  $X$  as follows:

$\beta : X \rightarrow [0,1]$  defined as  $\beta(a) = 0.6$  ;  $\beta(b) = 0.6$ .

$\delta : X \rightarrow [0,1]$  defined as  $\delta(a) = 0.7$  ;  $\delta(b) = 0.6$ .

$\eta : X \rightarrow [0,1]$  defined as  $\eta(a) = 0.9$  ;  $\eta(b) = 0.7$ .

The fuzzy subsets  $\beta$ ,  $\delta$  and  $\eta$  are not fuzzy  $b$ -nowhere dense sets. Since  $b\text{-int}b\text{-cl}(\beta) = 1 \neq 0$ ,  $b\text{-int}b\text{-cl}(\delta) = 1 \neq 0$  and  $b\text{-int}b\text{-cl}(\eta) = 1 \neq 0$ . Therefore  $\beta, \delta$  and  $\eta$  are not of fuzzy  $b$ - nowhere dense set.

**Definition 3.3:**

A fuzzy set  $\lambda$  in a fuzzy topological space  $(X, T)$  is called fuzzy  $\omega$ -nowhere dense if there exists no non-zero fuzzy  $\omega$ - open set  $\mu$  in  $(X, T)$  such that  $\omega\text{-cl}(\lambda) \leq \mu$ . That is,  $\omega\text{-int}\omega\text{-cl}(\lambda) = 0$ .

**Example 3.3:**

Let  $X = \{a,b,c\}$ . The fuzzy sets  $\lambda$ ,  $\mu$  and  $\gamma$  are defined on  $X$  as follows:

$\lambda : X \rightarrow [0,1]$  defined as  $\lambda(a) = 0.6$  ;  $\lambda(b) = 0.7$ ;  $\lambda(c) = 0.5$ .

$\mu : X \rightarrow [0,1]$  defined as  $\mu(a) = 0.7$ ;  $\mu(b) = 0.8$ ;  $\mu(c) = 0.6$ .

$\gamma : X \rightarrow [0,1]$  defined as  $\gamma(a) = 0.8$ ;  $\gamma(b) = 0.8$  ;  $\gamma(c) = 0.8$

Then  $T = \{0, \lambda, \mu, \gamma, 1\}$  is fuzzy topology on  $X$ . The fuzzy sets  $\lambda$ ,  $\mu$ ,  $\gamma$  are semi open in  $X$ .

Now  $\lambda \leq 1, \mu \leq 1, \gamma \leq 1$ . Where  $\lambda, \mu, \gamma$  are semi open.

$(1-\lambda) < \gamma$ ,  $\gamma$  is semi open  $\Rightarrow \omega\text{-cl}(1-\lambda) \leq \gamma$

$(1-\mu) < \gamma$ ,  $\gamma$  is semi open  $\Rightarrow \omega\text{-cl}(1-\mu) \leq \gamma$ ,

$(1-\gamma) < \gamma$ ,  $\gamma$  is semi open  $\Rightarrow \omega\text{-cl}(1-\gamma) \leq \gamma$

Where  $1-\lambda, 1-\mu, 1-\gamma$ 's are fuzzy  $\omega$ - closed set.

Then the complement of  $\lambda, \mu, \gamma$  are  $\omega$ - open.

Now  $\omega\text{-Int}\omega\text{-cl}(1-\lambda) = \omega\text{-int}(1-\lambda) = 0$ ,

$\omega\text{-Int}\omega\text{-cl}(1-\mu) = \omega\text{-int}(1-\mu) = 0$ ,

$\omega\text{-Int}\omega\text{-cl}(1-\gamma) = \omega\text{-int}(1-\gamma) = 0$ ,

Therefore  $1-\lambda, 1-\mu, 1-\gamma$ 's are fuzzy  $\omega$ -nowhere dense set.

**Example 3.4**

Let  $X = \{a,b,c\}$ . The fuzzy sets  $\lambda, \mu$  and  $\gamma$  are defined on  $X$  as follows:

$\lambda : X \rightarrow [0,1]$  defined as  $\lambda(a) = 0.9$  ;  $\lambda(b) = 0.6$ .

$\mu : X \rightarrow [0,1]$  defined as  $\mu(a) = 0.7$  ;  $\mu(b) = 0.6$ .

$\gamma : X \rightarrow [0,1]$  defined as  $\gamma(a) = 0.8$ ;  $\gamma(b) = 0.6$ .

Then  $T = \{0, \lambda, \mu, \gamma, 1\}$  is fuzzy topology on  $X$ . The fuzzy sets  $\lambda, \mu$ , and  $\gamma$  are semi open sets.

Now  $\lambda \leq 1, \mu \leq 1, \gamma \leq 1$ . Where  $\lambda, \mu, \gamma$  are semi open.

$(1-\lambda) < \lambda$ ,  $\lambda$  is semi open  $\Rightarrow \omega\text{-cl}(1-\lambda) \leq \lambda$

$(1-\mu) < \lambda$ ,  $\lambda$  is semi open  $\Rightarrow \omega\text{-cl}(1-\mu) \leq \lambda$ ,

$(1-\gamma) < \lambda$ ,  $\lambda$  is semi open  $\Rightarrow \omega\text{-cl}(1-\gamma) \leq \lambda$ ,

Now we define the fuzzy sets  $\beta, \delta$  and  $\eta$  on  $X$  as follows:

$\beta : X \rightarrow [0,1]$  defined as  $\beta(a) = 0.9$  ;  $\beta(b) = 0.5$ .

$\delta : X \rightarrow [0,1]$  defined as  $\delta(a) = 0.7$  ;  $\delta(b) = 0.5$ .

$\eta : X \rightarrow [0,1]$  defined as  $\eta(a) = 0.8$  ;  $\eta(b) = 0.5$ .

The fuzzy subsets  $\beta$ ,  $\delta$  and  $\eta$  are not a fuzzy  $\omega$ -nowhere dense sets. Since  $\omega\text{-int}\omega\text{-cl}(\beta) = 1 \neq 0$ ,  $\omega\text{-int}\omega\text{-cl}(\delta) = 1 \neq 0$  and  $\omega\text{-int}\omega\text{-cl}(\eta) = 1 \neq 0$ . Therefore  $\beta$ ,  $\delta$  and  $\eta$  are not of fuzzy  $\omega$ - nowhere dense set.

**Definition 3.5:**

A fuzzy set  $\lambda$  in a fuzzy topological space  $(X, T)$  is called fuzzy  $\alpha - g^*$  nowhere dense if there exists no non-zero fuzzy  $\alpha - g^*$  pre-open set  $\mu$  in  $(X, T)$  such that  $\alpha - g^*\text{-cl}(\lambda) \leq \mu$ . That is,  $\alpha - g^*\text{int}\alpha - g^*\text{cl}(\lambda) = 0$ .

**Example 3.5:**

Let  $X = \{a, b, c\}$ . The fuzzy sets  $\lambda$ ,  $\mu$  and  $\gamma$  are defined on  $X$  as follows:

$\lambda : X \rightarrow [0, 1]$  defined as  $\lambda(a) = 0.9$ ;  $\lambda(b) = 0.9$ ;  $\lambda(c) = 0.7$ .

$\mu : X \rightarrow [0, 1]$  defined as  $\mu(a) = 0.8$ ;  $\mu(b) = 0.8$ ;  $\mu(c) = 0.6$ .

$\gamma : X \rightarrow [0, 1]$  defined as  $\gamma(a) = 0.7$ ;  $\gamma(b) = 0.7$ ;  $\gamma(c) = 0.5$ .

Then  $T = \{0, \lambda, \mu, \gamma, 1\}$  is fuzzy topology on  $X$ . The fuzzy sets  $\lambda$ ,  $\mu$ ,  $\gamma$ 's are  $\alpha - g^*$  pre-open sets.

Now  $1 - \lambda < \lambda \Rightarrow \mu$  is  $\alpha - g^*$  open  $\Rightarrow \alpha - g^*\text{cl}(1 - \lambda) \leq \lambda$ ,

$1 - \mu < \lambda \Rightarrow \mu$  is  $\alpha - g^*$  open  $\Rightarrow \alpha - g^*\text{cl}(1 - \mu) \leq \lambda$ ,

$1 - \gamma < \lambda \Rightarrow \mu$  is  $\alpha - g^*$  open  $\Rightarrow \alpha - g^*\text{cl}(1 - \gamma) \leq \lambda$ .

$1 - \lambda$ ,  $1 - \mu$ ,  $1 - \gamma$ 's are  $\alpha - g^*$  set.

Now  $\alpha - g^*\text{Int}\alpha - g^*\text{cl}(1 - \lambda) = \text{int}(1 - \lambda) = 0$ ,

$\alpha - g^*\text{Int}\alpha - g^*\text{cl}(1 - \mu) = \text{int}(1 - \mu) = 0$ ,

$\alpha - g^*\text{Int}\alpha - g^*\text{cl}(1 - \gamma) = \text{int}(1 - \gamma) = 0$ .

Now  $1 - \lambda$ ,  $1 - \mu$ ,  $1 - \gamma$ 's are fuzzy  $\alpha - g^*$  nowhere dense set.

**Example 3.6:**

Let  $X = \{a, b, c\}$ . The fuzzy sets  $\lambda$ ,  $\mu$  and  $\gamma$  are defined on  $X$  as follows:

$\lambda : X \rightarrow [0, 1]$  defined as  $\lambda(a) = 0.7$ ;  $\lambda(b) = 0.5$ .

$\mu : X \rightarrow [0, 1]$  defined as  $\mu(a) = 0.8$ ;  $\mu(b) = 0.5$ .

$\gamma : X \rightarrow [0, 1]$  defined as  $\gamma(a) = 0.9$ ;  $\gamma(b) = 0.6$ .

Then  $T = \{0, \lambda, \mu, \gamma, 1\}$  is fuzzy topology on  $X$ . The fuzzy sets  $\lambda$ ,  $\mu$ , and  $\gamma$  are fuzzy  $\alpha - g^*$  pre-open sets.

Now  $1 - \lambda < \gamma \Rightarrow \mu$  is  $\alpha - g^*$  open  $\Rightarrow \alpha - g^*\text{cl}(1 - \lambda) \leq \gamma$ ,

$1 - \mu < \gamma \Rightarrow \mu$  is  $\alpha - g^*$  open  $\Rightarrow \alpha - g^*\text{cl}(1 - \mu) \leq \gamma$ ,

$1 - \gamma < \gamma \Rightarrow \mu$  is  $\alpha - g^*$  open  $\Rightarrow \alpha - g^*\text{cl}(1 - \gamma) \leq \gamma$ .

$1 - \lambda$ ,  $1 - \mu$ ,  $1 - \gamma$ 's are  $\alpha - g^*$  closed set.

Now  $\alpha - g^*\text{Int}\alpha - g^*\text{cl}(1 - \lambda) = \text{int}(1 - \lambda) = 0$ ,

$\alpha - g^*\text{Int}\alpha - g^*\text{cl}(1 - \mu) = \text{int}(1 - \mu) = 0$ ,

$\alpha - g^*\text{Int}\alpha - g^*\text{cl}(1 - \gamma) = \text{int}(1 - \gamma) = 0$ .

Now  $1 - \lambda$ ,  $1 - \mu$ ,  $1 - \gamma$ 's are fuzzy  $\alpha - g^*$  nowhere dense set

Now we define the fuzzy sets  $\beta$ ,  $\delta$  and  $\eta$  on  $X$  as follows:

$\beta : X \rightarrow [0, 1]$  defined as  $\beta(a) = 0.7$ ;  $\beta(b) = 0.4$ .

$\delta : X \rightarrow [0, 1]$  defined as  $\delta(a) = 0.8$ ;  $\delta(b) = 0.4$ .

$\eta : X \rightarrow [0, 1]$  defined as  $\eta(a) = 0.9$ ;  $\eta(b) = 0.5$ .

The fuzzy subsets  $\beta$ ,  $\delta$  and  $\eta$  are not fuzzy  $\alpha - g^*$  nowhere dense sets. Since  $\alpha - g^*\text{int}\alpha - g^*\text{cl}(\beta) = 1 \neq 0$ ,  $\alpha - g^*\text{int}\alpha - g^*\text{cl}(\delta) = 1 \neq 0$  and  $\alpha - g^*\text{int}\alpha - g^*\text{cl}(\eta) = 1 \neq 0$ . Therefore  $\beta$ ,  $\gamma$  and  $\eta$  are not of fuzzy  $\alpha - g^*$  nowhere dense set.

#### IV. Fuzzy $b$ -Baire spaces, Fuzzy $\omega$ -Baire space, Fuzzy $\alpha - g^*$ Baire space:

We introduce fuzzy  $b$ -open Baire space, fuzzy  $\omega$ -Baire space, fuzzy  $\alpha - g^*$  Baire space with suitable examples.

**Definition 4.1:**

A fuzzy topological space  $(X, T)$  is called fuzzy  $b$ -Baire space if  $b - \text{int}(\cup_{i=1}^{\infty} (\lambda_i)) = 0$ , where  $(\lambda_i)$ 's are fuzzy  $b$ -nowhere dense sets in  $(X, T)$ .

In example 3.1, The fuzzy sets  $1 - \lambda$ ,  $1 - \mu$ ,  $1 - \gamma$ 's are fuzzy  $b$ - nowhere dense sets. Now  $b - \text{int}[(1 - \lambda) \vee (1 - \mu) \vee (1 - \gamma)] = b - \text{int}(1 - \gamma) = 0$ . Therefore the fuzzy topological space  $(X, T)$  is fuzzy  $b$ -Baire space.

**Definition 4.2:**

A fuzzy topological space  $(X, T)$  is called fuzzy  $\omega$ -Baire space if  $\omega - \text{int}(\cup_{i=1}^{\infty} (\lambda_i)) = 0$ , where  $(\lambda_i)$ 's are fuzzy  $\omega$ -nowhere dense sets in  $(X, T)$ .

In example 3.3, The fuzzy sets  $1-\lambda, 1-\mu, 1-\gamma$ 's are fuzzy  $\omega$ -nowhere dense sets. Now  $\omega - \text{int}[(1-\lambda) \vee (1-\mu) \vee (1-\gamma)] = \omega - \text{int}(1-\lambda) = 0$ . Therefore the fuzzy topological space  $(X, T)$  is fuzzy  $\omega$ -Baire space.

**Definition 4.3:**

A fuzzy topological space  $(X, T)$  is called fuzzy  $\alpha$ - $g^*$ Baire space if  $\alpha - g^* \text{int}(\cup_{i=1}^{\infty} (\lambda_i)) = 0$ , where  $(\lambda_i)$ 's are fuzzy  $\alpha$ - $g^*$  pre-closed nowhere dense sets in  $(X, T)$ .

In example 3.5, The fuzzy sets  $1-\lambda, 1-\mu, 1-\gamma$ 's are fuzzy  $\alpha$ - $g^*$  nowhere dense sets. Now  $\alpha - g^* \text{int}[(1-\lambda) \vee (1-\mu) \vee (1-\gamma)] = \alpha - g^* \text{int}(1-\gamma) = 0$ . Therefore the fuzzy topological space  $(X, T)$  is fuzzy  $\alpha$ - $g^*$  pre-closed Baire space.

## V. Some relations of fuzzy $b$ -Baire space, fuzzy $\omega$ -Baire space and fuzzy $\alpha$ - $g^*$ Baire space:

**Proposition 5.1:**

**A fuzzy  $b$ -Baire space is also a fuzzy  $\omega$ -Baire space.**

Consider the following example.

Let  $X = \{a, b, c\}$ . The fuzzy sets  $\lambda, \mu$  and  $\gamma$  are defined on  $X$  as follows:

$\lambda : X \rightarrow [0, 1]$  defined as  $\lambda(a) = 0.9 ; \lambda(b) = 0.9 ; \lambda(c) = 0.8$ .

$\mu : X \rightarrow [0, 1]$  defined as  $\mu(a) = 0.8 ; \mu(b) = 0.7 ; \mu(c) = 0.6$ .

$\gamma : X \rightarrow [0, 1]$  defined as  $\gamma(a) = 0.9 ; \gamma(b) = 0.8 ; \gamma(c) = 0.6$ .

Then  $T = \{0, \lambda, \mu, \gamma, 1\}$  is fuzzy topology on  $X$ .

Since  $\lambda \leq [\text{int cl}(\lambda)] \vee [\text{cl int}(\lambda)] \leq \lambda \vee 1 \leq 1$ ,

$\mu \leq [\text{int cl}(\mu)] \vee [\text{cl int}(\mu)] \leq \mu \vee 1 \leq 1$ ,

$\gamma \leq [\text{int cl}(\gamma)] \vee [\text{cl int}(\gamma)] \leq \gamma \vee 1 \leq 1$ .

$\lambda, \mu, \gamma$ 's are fuzzy  $b$ -open.

Now  $b - \text{int} b - \text{cl}(1-\lambda) = b - \text{int}(1-\lambda) = 0$ ,

$b - \text{int} b - \text{cl}(1-\mu) = b - \text{int}(1-\mu) = 0$ ,

$b - \text{int} b - \text{cl}(1-\gamma) = b - \text{int}(1-\gamma) = 0$ ,

Therefore  $1-\lambda, 1-\mu, 1-\gamma$ 's are fuzzy  $b$ - nowhere dense sets.

Now  $b - \text{int}[(1-\lambda) \vee (1-\mu) \vee (1-\gamma)] = 0$

$b - \text{int}(1-\mu) = 0$ .

Therefore  $(X, T)$  is a fuzzy  $b$ -Baire space. Now to say that it is fuzzy  $\omega$ -Baire space we have to show that  $\mu$  is semi open such that  $\omega - \text{cl}(\lambda) \leq \mu$ .

$\lambda \leq \text{cl int}(\lambda) = \text{cl}(\lambda) = 1$ ,

$\mu \leq \text{cl int}(\mu) = \text{cl}(\mu) = 1$ ,

$\gamma \leq \text{cl int}(\gamma) = \text{cl}(\gamma) = 1$ ,

$\lambda, \mu, \gamma$  are semi open in  $X$ .

Now  $(1-\lambda) < \lambda, \lambda$  is semi open  $\Rightarrow \omega - \text{cl}(1-\lambda) \leq \lambda$ ,

$(1-\mu) < \mu, \mu$  is semi open  $\Rightarrow \omega - \text{cl}(1-\mu) \leq \mu$ ,

$(1-\gamma) < \gamma, \gamma$  is semi open  $\Rightarrow \omega - \text{cl}(1-\gamma) \leq \gamma$ ,

Where  $1-\lambda, 1-\mu, 1-\gamma$ 's are fuzzy  $\omega$ - closed set.

Now  $\omega - \text{int cl}(1-\lambda) = \omega - \text{int}(1-\lambda) = 0$ ,

$\omega - \text{int cl}(1-\mu) = \omega - \text{int}(1-\mu) = 0$ ,

$\omega - \text{int cl}(1-\gamma) = \omega - \text{int}(1-\gamma) = 0$ ,

Therefore  $1-\lambda, 1-\mu, 1-\gamma$ 's are fuzzy  $\omega$ - nowhere dense set.

$\omega - \text{int}[(1-\lambda) \vee (1-\mu) \vee (1-\gamma)] = 0$

$\omega - \text{int}(1-\mu) = 0$

Then  $(X, T)$  is a  $\omega$ -Baire space. Thus  $b$ -Baire space is also a  $\omega$ -Baire space.

### Proposition 5.2:

**A  $\alpha$ -g\*Baire space is also a  $b$ -Baire space.**

Consider an example.

Let  $X = \{a, b, c\}$ . The fuzzy sets  $\lambda$ ,  $\mu$  and  $\gamma$  are defined on  $X$  as follows:

$\lambda : X \rightarrow [0, 1]$  defined as  $\lambda(a) = 0.6$ ;  $\lambda(b) = 0.5$ ;  $\lambda(c) = 0.5$ .

$\mu : X \rightarrow [0, 1]$  defined as  $\mu(a) = 0.9$ ;  $\mu(b) = 0.9$ ;  $\mu(c) = 0.9$ .

$\gamma : X \rightarrow [0, 1]$  defined as  $\gamma(a) = 0.9$ ;  $\gamma(b) = 0.8$ ;  $\gamma(c) = 0.8$ .

Then  $T = \{0, \lambda, \mu, \gamma, 1\}$  is fuzzy topology on  $X$ .

$\lambda, \mu, \gamma$ 's are fuzzy  $\alpha$ -g\* openset.

$(1-\lambda) < \mu \Rightarrow \mu$  is  $\alpha$ -g\* open  $\Rightarrow \alpha$ -g\* cl  $(1-\lambda) \leq \mu$ ,

$(1-\mu) < \mu \Rightarrow \mu$  is  $\alpha$ -g\* open  $\Rightarrow \alpha$ -g\* cl  $(1-\mu) \leq \mu$ ,

$(1-\gamma) < \mu \Rightarrow \mu$  is  $\alpha$ -g\* open  $\Rightarrow \alpha$ -g\* cl  $(1-\gamma) \leq \mu$ ,

$(1-\lambda), (1-\mu), (1-\gamma)$ 's are fuzzy  $\alpha$ -g\* closed sets.

$\alpha$ -g\* Int  $\alpha$ -g\* cl  $(1-\lambda) = \alpha$ -g\* int  $(1-\lambda) = 0$ ,

$\alpha$ -g\* Int  $\alpha$ -g\* cl  $(1-\mu) = \alpha$ -g\* int  $(1-\mu) = 0$ ,

$\alpha$ -g\* Int  $\alpha$ -g\* cl  $(1-\gamma) = \alpha$ -g\* int  $(1-\gamma) = 0$ .

$(1-\lambda), (1-\mu), (1-\gamma)$ 's are fuzzy  $\alpha$ -g\* nowhere dense set.

$\alpha$ -g\* Int  $[(1-\lambda) \vee (1-\mu) \vee (1-\gamma)] = 0$

$\alpha$ -g\* Int  $(1-\lambda) = 0$ . Therefore  $(X, T)$  is a fuzzy  $\alpha$ -g\* Baire spaces. Now to say that it is  $b$ -Baire space we have to show that  $\lambda \leq \text{cl int}(\lambda) \vee \text{int cl}(\lambda)$ .

Since  $\lambda \leq [\text{int cl}(\lambda)] \vee [\text{cl int}(\lambda)] \leq \lambda \vee 1 \leq 1$ ,

$\mu \leq [\text{int cl}(\mu)] \vee [\text{cl int}(\mu)] \leq \mu \vee 1 \leq 1$ ,

$\gamma \leq [\text{int cl}(\gamma)] \vee [\text{cl int}(\gamma)] \leq \gamma \vee 1 \leq 1$ .

$\lambda, \mu, \gamma$ 's are fuzzy  $b$ -open.

Now  $b$ -int  $b$ -cl  $(1-\lambda) = b$ -int  $(1-\lambda) = 0$ ,

$b$ -int  $b$ -cl  $(1-\mu) = b$ -int  $(1-\mu) = 0$ ,

$b$ -int  $b$ -cl  $(1-\gamma) = b$ -int  $(1-\gamma) = 0$ ,

Therefore  $1-\lambda, 1-\mu, 1-\gamma$ 's are fuzzy  $b$ -nowhere dense sets. Now

$b$ -int  $[(1-\lambda) \vee (1-\mu) \vee (1-\gamma)] = 0$

$b$ -int  $(1-\lambda) = 0$ . Therefore  $(X, T)$  is a fuzzy  $b$ -Baire spaces. Thus A  $\alpha$ -g\* Baire space is also a  $b$ -Baire space.

### Proposition 5.3:

**A  $\omega$ -Baire space is also a  $\alpha$ -g\* Baire space.**

Consider an example.

Let  $X = \{a, b, c\}$ . The fuzzy sets  $\lambda$ ,  $\mu$  and  $\gamma$  are defined on  $X$  as follows:

$\lambda : X \rightarrow [0, 1]$  defined as  $\lambda(a) = 0.8$ ;  $\lambda(b) = 0.8$ ;  $\lambda(c) = 0.7$ .

$\mu : X \rightarrow [0, 1]$  defined as  $\mu(a) = 0.8$ ;  $\mu(b) = 0.7$ ;  $\mu(c) = 0.6$ .

$\gamma : X \rightarrow [0, 1]$  defined as  $\gamma(a) = 0.8$ ;  $\gamma(b) = 0.6$ ;  $\gamma(c) = 0.6$ .

Then  $T = \{0, \lambda, \mu, \gamma, 1\}$  is fuzzy topology on  $X$ .

**Semi open:**

$\text{cl int}(\lambda) = \text{cl}(\lambda) = 1 \geq \lambda$ ,

$\text{cl int}(\mu) = \text{cl}(\mu) = 1 \geq \mu$ ,

$\text{cl int}(\gamma) = \text{cl}(\gamma) = 1 \geq \gamma$ ,

$\lambda, \mu, \gamma$ 's are fuzzy semi open in  $X$ .

Now  $(1-\lambda) < \lambda$ ,  $\lambda$  is semi open  $\Rightarrow \omega$ -cl  $(1-\lambda) \leq \lambda$ ,

$(1-\mu) < \mu$ ,  $\mu$  is semi open  $\Rightarrow \omega$ -cl  $(1-\mu) \leq \mu$ ,

$(1-\gamma) < \gamma$ ,  $\gamma$  is semi open  $\Rightarrow \omega$ -cl  $(1-\gamma) \leq \gamma$ ,

Where  $1-\lambda, 1-\mu, 1-\gamma$ 's are fuzzy  $\omega$ -closed set.

Now  $\omega$ -int  $\text{cl}(\lambda) = \omega$ -int  $(1-\lambda) = 0$ ,

$\omega$ -int  $\text{cl}(\mu) = \omega$ -int  $(1-\mu) = 0$ ,

$$\omega\text{-int cl}(1-\gamma) = \omega\text{-int}(1-\gamma) = 0,$$

Therefore  $1-\lambda, 1-\mu, 1-\gamma$ 's are fuzzy  $\omega$ - nowhere dense set.

$$\omega\text{-int}[(1-\lambda) \vee (1-\mu) \vee (1-\gamma)] = 0$$

$$\omega\text{-int}(1-\mu) = 0$$

Then  $(X, T)$  is a  $\omega$ - Baire space. Now to say that it is  $\alpha$ - $g^*$  Baire space we have to show that it is  $\alpha$ - $g^*$  open in  $X$  whenever  $\lambda \leq \mu$  and  $\alpha - g^* \text{cl}(\lambda) \leq \mu$ .

$$\text{Now } 1-\lambda < \lambda \Rightarrow \lambda \text{ is } \alpha - g^* \text{ open} \Rightarrow \alpha - g^* \text{cl}(1-\lambda) \leq \lambda,$$

$$1-\mu < \lambda \Rightarrow \lambda \text{ is } \alpha - g^* \text{ open} \Rightarrow \alpha - g^* \text{cl}(1-\mu) \leq \lambda,$$

$$1-\gamma < \lambda \Rightarrow \lambda \text{ is } \alpha - g^* \text{ open} \Rightarrow \alpha - g^* \text{cl}(1-\gamma) \leq \lambda.$$

$1-\lambda, 1-\mu, 1-\gamma$ 's are  $\alpha - g^*$  pre-closed set.

$$\text{Now } \alpha - g^* \text{Int} \alpha - g^* \text{cl}(1-\lambda) = \alpha - g^* \text{int}(1-\lambda) = 0,$$

$$\alpha - g^* \text{Int} \alpha - g^* \text{cl}(1-\mu) = \alpha - g^* \text{int}(1-\mu) = 0,$$

$$\alpha - g^* \text{Int} \alpha - g^* \text{cl}(1-\gamma) = \alpha - g^* \text{int}(1-\gamma) = 0.$$

Now  $1-\lambda, 1-\mu, 1-\gamma$ 's are fuzzy  $\alpha - g^*$  nowhere dense set.

$$\alpha - g^* \text{Int} [(1-\lambda) \vee (1-\mu) \vee (1-\gamma)] = 0$$

$$\alpha - g^* \text{Int}(1-\gamma) = 0. \text{ Then } (X, T) \text{ is a fuzzy } \alpha - g^* \text{ Baire space.}$$

Thus A  $\omega$ -Baire space is also a  $\alpha - g^*$  Baire space.

## VI. Conclusion:

In this paper, we introduce a concept fuzzy  $b$ -Baire space, fuzzy  $\omega$ -Baire space and fuzzy  $\alpha - g^*$  Baire space, Further some relation between them and illustrated with some examples.

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