APPLICATIONS OF STAR COLOURING AND DOMINATION IN LINE GRAPH

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Abstract:

In this paper we discussed the applications of Star colouring and Domination of line graph. This paper deals how star colouring and domination are used in flight routes of places in India and is illustrated with an example. The relation between chromatic number and star chromatic number are compared and also the relation between some domination parameters are also illustrated with this examples.

Key words:

Star colouring, Line graph, Domination number, Dominating set

1. Introduction and Preliminaries:

The mainstream of mathematics basically involves Graph Theory because its applications is used in numerous ways. Graph theory has grown rapidly in recent times with a lot of research activities. In the last few decades at the international level one third of the mathematics research papers are from graph theory and combinatorics. It is one of the basic field of discrete mathematics which has a wide application in computing natural and social sciences.

Graph theory is one of the important branch of mathematics and it was founded by Leonhard Euler on the paper 'Seven Bridges of Konigsberg'. It was published in the year 1736 which was regarded as the first paper in the history of graph theory. This paper as well as the paper written by Vandermonde on the knight problem carried on which the analysis sites initiated by Leibnitz. The theory of trees and their applications in electrical networks was developed by G.R.Kirchoff in 1847 [34]. One of the lectures in 1840, A.F. Mobius first presented the four colour problem [8]. The study generalized by Cauchy and L'Huillier on Euler's formula dealing with the number of edges, vertices and faces of a convex polyhedron represents the origin of another branch of mathematics known as Topology. The graph concepts are mainly used in Operation Research.It is also used in modelling transport networks, activity networks and theory of games.

Graph Theory is used in many fields such as Biochemistry, Electrical engineering, Computer science, Operation Research etc., In chemistry, graphs are used for representation of chemical compounds. In Computer Science graphs are used for representing data in terms of network. Likewise it is used in many fields. Nowadays cell phones are used by all peoples even also in rural areas. The mobile phone network operates with GSM .Since GSM operates only in four different frequencies here colouring is used.It applies almost everywhere to things that are connected to other things.

Definition 1.1:

The *line graph* of a simple graph G is obtained by means of associating a vertex with every edge of the graph and connecting two vertices with an edge iff the corresponding edges of G have a vertex in common. The Line graph of G is denoted by L(G).

Definition 1.2:

A subset D of V(G) is said to be a *dominating set* of graph G if every vϵV-D is adjacent to atleast one vertex in D. The minimum cardinality of a dominating set D is called domination number. It is denoted by γ(G).

Definition 1.3:

A proper vertex colouring of a graph G is called *star colouring*, if every path of G on four vertices (i.e) every path on length 3 is not bicoloured.

Definition 1.4 :

If D is a dominating set in G and if a dominating set exists in V\D say D' then D' is called the *inverse dominating set*. The inverse domination number denoted by γ'(G) is the smallest size of the inverse dominating set.

Definition 1.5:

If every vertex of V is adjacent to some vertex in D then the dominating set D is said to be *total dominating set*. The minimum cardinality of total dominating set is called the total domination number and is denoted by γ_t (G).

Definition 1.6 :

The *star chromatic number* is the minimum number of colours needed to star colour G and is denoted by χ_s(G).

2. USES OF STAR COLOURING AND DOMINATION:

In today's world Colouring and Domination are the two major areas which has a wide applications. Channel assignment and receiving signals from radio and television transmitting stations are the modern applications of colouring. Also colouring is used to find shortest route which makes travel more easier.

In real time applications Colouring is used in

- Clustering
- Capturing of image
- Modelling of network
- Time management of scheduling process
- Scheduling of sports
- Preparation of timetable
- Designing of seating arrangement

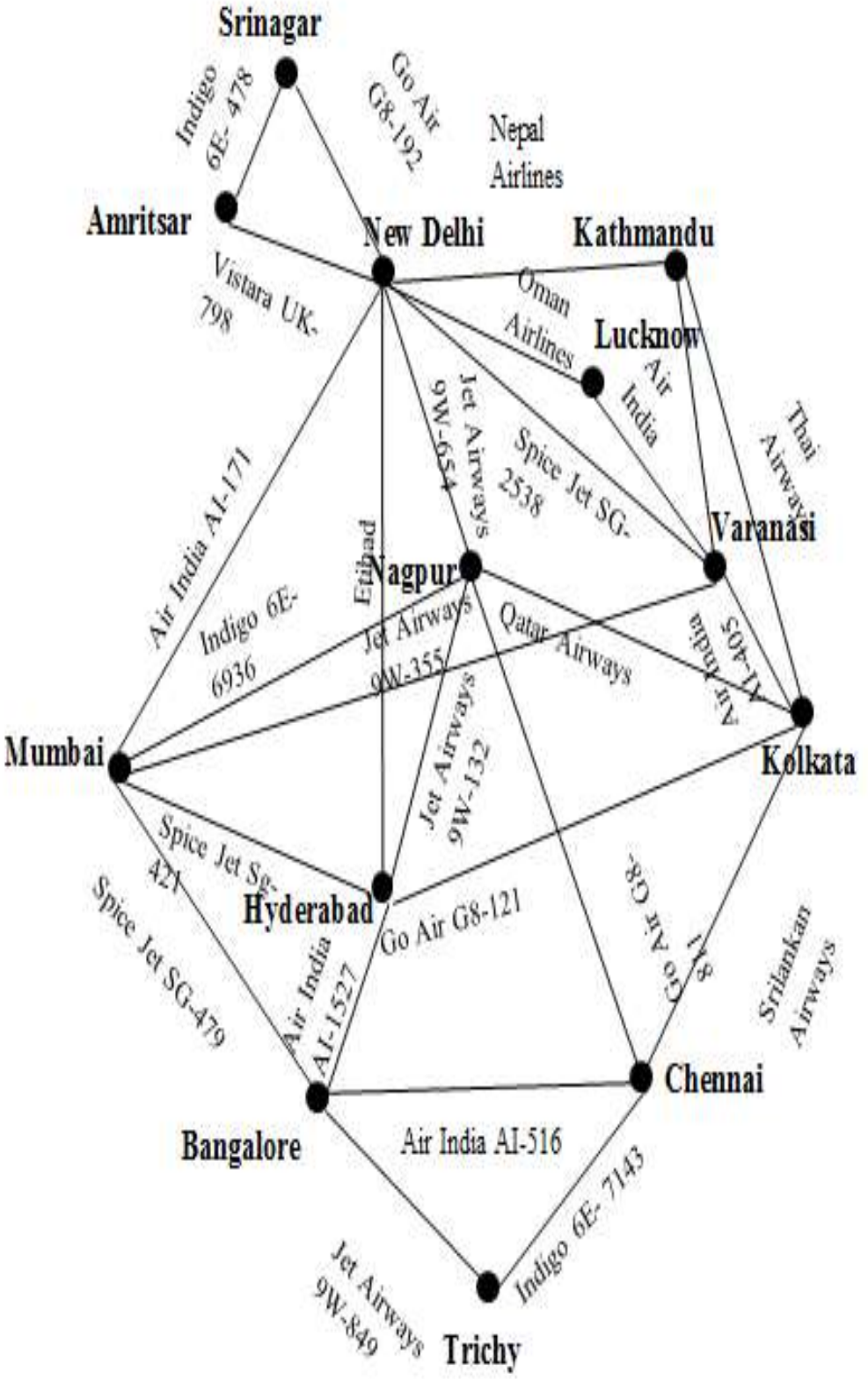
Also colouring is used in solving Sudoku puzzles and also in Google maps. For example in an art gallery for placing the cameras so that the full gallery is covered colouring is used.

Domination also has several applications. It is used in hospitals, fire stations, school bus, roading etc.. It is also in land surveying for minimizing the number of places a surveyor must stand in order to take height measurements for an entire region. Also it is used in Nuclear power plant problem, modelling biological networks, loading theory etc..

In today's world, traffic occur in every places. This is because of increase in population. There are many ways to travel to the places where we decide. But the route to travel matters a lot. In some places traffic occurs a lot. So we have to select the route where traffic is less and also we have to reach in short time. This can be achieved with the help of star colouring and Domination.

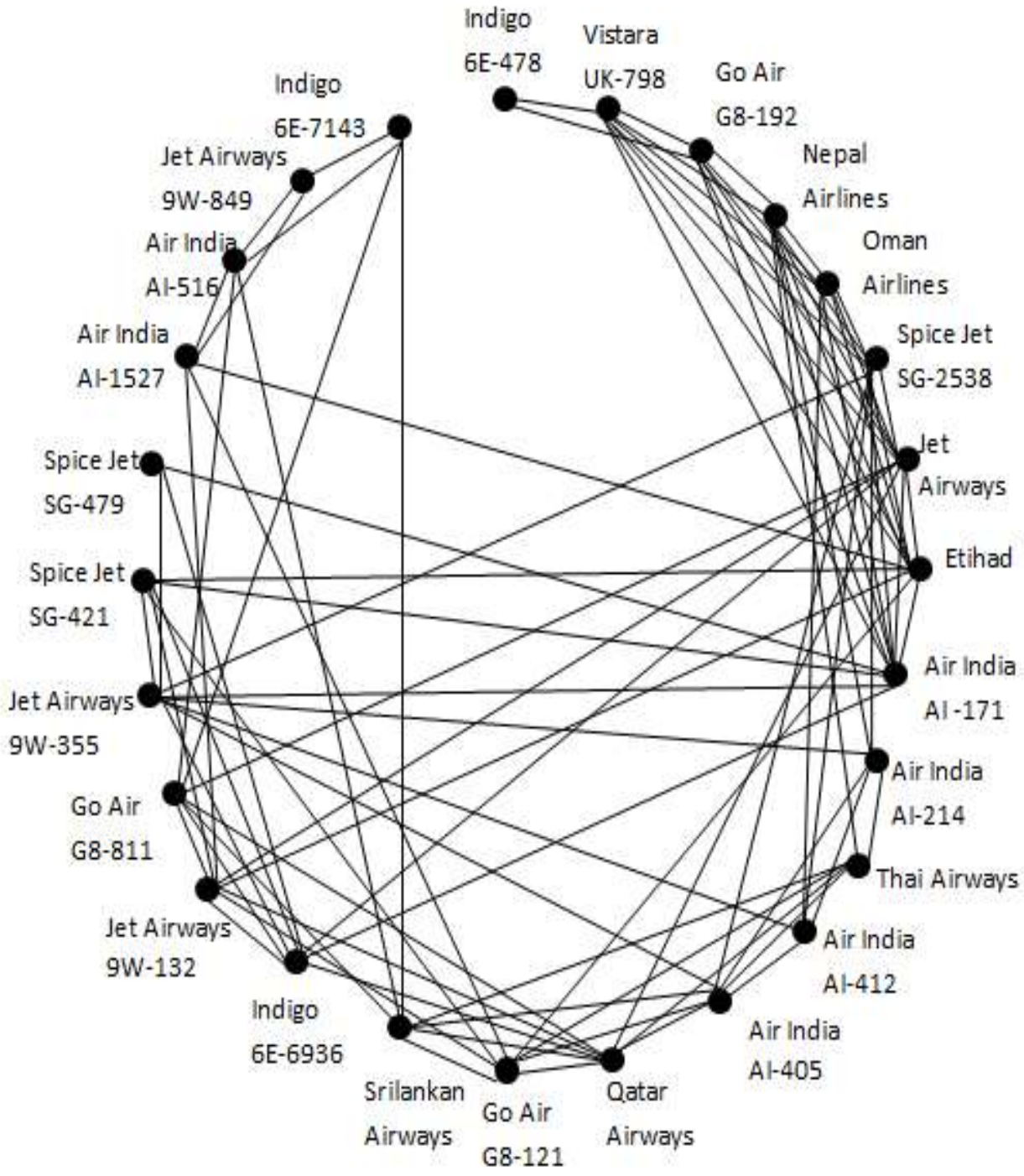
This chapter deals how star colouring and domination are used in flight routes of many places in India. The flight routes from Srinagar to Chennai are observed and represented by a graph. In this graph the places are considered as vertices and the name of the flights between the places are taken as nodes. From this graph a line graph is constructed by taking the name of the flights as vertices and the edges are drawn based on the adjacencies between them.

Some important places of international Airports and the international route and domestic route between them are represented as a graph which is shown in Fig(1).



Fig(1): G

From the graph G, the line graph which is given in Fig(2) is constructed by taking the edges of G as the vertices in L(G) and the edges in L(G) are drawn on the adjacencies between them.



Fig(2): L (G)

Now we apply definition of star colouring that no path on four vertices is bicoloured. In line graph any path is considered with vertices and accordingly star colour is applied. Likewise every path on the line graph is assigned a colour. Finally eight colours are assigned and the flight which receive each colour is given below:

- Indigo 6E- 478 , Nepal Airlines, Air India AI- 405, Jet Airway 9W-132,Spice Jet SG- 479 is assigned colour1.
- Vistara UK- 798, Air India AI-214, Qatar Airways, Spice JetSG- 421,Air India AI-516 is assigned colour 2.

- Colour 3 is assigned to Go Air G8- 192, Thai Airways, Indigo 6E- 6936, Air Asia 15-1527, Indigo 6E-7143
- Oman Airlines, Go Air G8-121, Go Air G8-811, Jet Airways 9W-355, Jet Airways 9W-849 is assigned colour 4.
- Colour 5 is assigned to Spice Jet SG-2538 and Sri Lankan Airways
- Jet Airways 9W-654 and Air India AI-412 are assigned colour 6
- Etihad Airways is assigned colour 7 and finally
- Air India 6E-171 is assigned colour 8.

From this we conclude chromatic number = star chromatic number

(i.e) $\chi(G) = \chi_s(G)$

Next we apply domination to the line graph we have

- The dominating set D= {Vistara UK-798, Spice Jet SG- 2538, Go Air G8- 121, Spice Jet SG-421,Jet Airways 9W- 849, Indigo 6E-7143}.
  Therefore the domination number is $\gamma[L(G)]=6$.
- The inverse dominating set is D'={ Go Air G8-192, Air India AI-171, Air India AI-405, Qatar Airways, Air India AI-1527, Air India AI-516}.
  Hence the inverse domination number is $\gamma'(G)=6$.
- Since every vertex in the L(G) are adjacent to any vertex in the dominating set of L(G) therefore the total domination number is $\gamma_t(G)=6$.

From this we have $\gamma[L(G)]=\gamma'[L(G)]=\gamma_t[L(G)]$.

Finally we conclude that all flights receive different colours which is used for identifying different ways for travelling and all peoples who travel from Srinagar to Trichy must use any one of the flights in the dominating set. There are many ways to travel from Srinagar to Trichy. But without the use of the flights in the dominating set no people can travel from Srinagar to Trichy. Also by avoiding traffic and by using any one of the flights in the dominating set we can travel from Srinagar to Trichy in shortest time.

Therefore Star Colouring and domination are used in real life to find different routes to the destination places and also shortest routes which saves time in today's busy world.

3. CONCLUSION:

The applications of Star colouring and Domination of Line graph are described with examples. This helps people to find many route to the place where we travel and also one can select the route to the comfort of them

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