VARIOUS CHARACTERAZATION OF (i,j)-vp CLOSED SETS IN BITOPOLOGICAL SPACES

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ABSTRACT: In this paper we introduce some new bitopological spaces (i,j)- T_{vp},(i,j)-g^*pT_{vp},(i,j)-sgT_{vp},(i,j)-ΨT_{vp} spaces as applications.Further we introduce and study vp-continuity in bitopological spaces and investigate their properties.

KEYWORDS: (i,j)-vp closed sets; (i,j)- T_{vp} space,(i,j)-g^*pT_{vp} space,(i,j)-sgT_{vp} space,(i,j)-ΨT_{vp} space;(i,j)-vp continuity.

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INTRODUCTION:

A triple (X,τ_1,τ_2) where X is a non empty set τ_1 and τ_2 are topologies on X is called a bitopological space and Kelly[7] initiated the study of such spaces.In 1985,Fukutake[4] introduced the concepts of g-closed sets in bitopological spaces and after several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces.Veerakumar[18] introduced and studied the concepts of g^*-closed set and g^* continuity in topological spaces. Norman Levine[13],and R.Devi et.al[6] introduced $T_{1/2}$ spaces and T_d spaces respectively.In 2003,M.Sheik john and P.sundaram[14] was introduced the concept of ${T_{1/2}}^*$ and $*_{T_{1/2}}$.

The purpose of this paper is to introduce the concepts of vp-continuity,T_{vp}-spaces,g^*pT_{vp}-spaces,sgT_{vp}-spaces,ΨT_{vp}-spaces for bitopological spaces and investigate some of their properties.

2.PRELIMINARIES

Definition 2.1 A subset A of a topological space (X,τ) is called

1. regular-open set if $A = int(cl(A))$
2. semi-open set if $A \subseteq cl(int(A))$
3. α-closed if $cl\left(int(cl(A))\right) \subseteq A$
4. generalized closed set(g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).
5. $g^\#$-closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is αg-open in (X,τ).

6. a α-generalized closed(briefly αg-closed) if $\alpha cl(\mathrm{A}) \subseteq U$ whenever $\mathrm{A} \subseteq U$ and U is open in (X, τ).
7. g^*p-closed if $pcl(\mathrm{A}) \subseteq U$ whenever $\mathrm{A} \subseteq U$ and U is g-open in (X, τ).

Definition 2.2 A subset A of a bitopological space $(\mathrm{X}, \tau_i, \tau_j)$ is called

1. a (i,j)-g-closed if τ_j-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_i.
2. a (i,j)-g^*-closed if τ_j-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in τ_i.
3. a (i,j)-gs-closed if τ_j-$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_i.
4. a (i,j)-g^*p-closed τ_j-$pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in τ_i.
5. a (i,j)-$g^\#$-closed τ_j-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is αg-open in τ_i.
6. a (i,j)-g^{**}-closed if τ_j-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^*-open in τ_i.
7. a (i, j)-$g^{\#\#}$-closed if τ_j-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $g^\#$-open inτ_i.
8. a (i,j)-sg-closed if τ_j-$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in τ_i.
9. a (i,j)-vp-closed if τ_j-$cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is $semi$-open in τ_i.

Definition 2.3: A bitopological space (X, τ_1, τ_2) is called

1. a (i,j)-$T_{1/2}$ space if every (i,j)-g-closed set in it is τ_j-closed.
2. a (i,j)-$T_{1/2^*}$ space if every (i,j)-g^*-closed set in it is τ_j-closed.
3. a (i,j)-$*_{T_{1/2}}$ space if every (i,j)-g-closed set is (i,j)-g^*-closed.
4. a (i,j)-T_d space if every (i,j)-gs-closed set is (i,j)-g-closed.
5. a (i,j)-$T_{s\alpha^{**}}$ space if every (i,j)-strongly α^{**}-closed set in it is τ_j-closed.
6. a (i,j)-g^*T_g space if every (i,j)-g^*-closed set is $(i,j) - g^*s^*$-closed.

   Where $i,j \in \{1,2\}$ and $i \neq j$.

Definition 2.4 A function $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is called

1. (i,j)-g-continuous if $f^{-1}(V)$ is a (i,j)-g-closed in (X, τ_1, τ_2) for every closed set V of (Y, σ_1, σ_2).
2. (i,j)-$g^{\#\#}$-continuous if $f^{-1}(V)$ is a (i,j)-$g^{\#\#}$-closed in (X, τ_1, τ_2) for every closed set V of (Y, σ_1, σ_2).
3. (i,j)-$\hat{g}$-continuous if $f^{-1}(V)$ is a (i,j)-$\hat{g}$-closed in (X, τ_1, τ_2) for every closed set V of (Y, σ_1, σ_2).
4. (i,j)-g^*p-continuous if $f^{-1}(V)$ is a $(i,j) - g^*p$ -closed in (X, τ_1, τ_2) for every closed set V of (Y, σ_1, σ_2).
5. (i,j)-αg-continuous if $f^{-1}(V)$ is a (i,j)-αg-closed in (X, τ_1, τ_2) for every closed set V of (Y, σ_1, σ_2).
6. (i,j)-(Ψ) continuous if $f^{-1}(V)$ is a (i,j)-Ψ-closed in (X, τ_1, τ_2) for every closed set V of (Y, σ_1, σ_2).

3.APPLICATIONS OF (i,j)-vp CLOSED SETS IN BITOPOLOGICAL SPACES

We introduce the following definition:

Definition 3.1: A space (X, τ_1, τ_2) is called a (i,j)-T_{vp}-space if every (i,j)-vp closed set is τ_j-closed.

Example 3.2: Let $X = \{a, b, c\}$,τ_1=$\{\varphi, \{a\}, \{b\}, \{a, b\}, X\}$,$\tau_2$=$\{\varphi, \{a\}, \{a, b\}, \{a, c\}, X\}$

τ_2-closed sets are $\{\varphi, \{b, c\}, \{c\}, \{b\}, X\}$

(i,j)-vp closed sets are $\{\varphi, \{b\}, \{c\}, \{b, c\}, X\}$

Therefore (i,j)-vp closed set is τ_j-closed.

Definition 3.3: A space (X, τ_1, τ_2) is called (i,j)-g^*pT_{vp}-space if every (i,j)-vp closed set is (i,j)-g^*p closed.

Example 3.4: Let $X = \{a, b, c\}$,τ_1=$\{\varphi, \{a\}, X\}$,τ_2=$\{\varphi, \{a\}, \{a, b\}, X\}$

Hence our condition holds.

Therefore (i,j)-vp closed set is (i,j)-g^*p closed.

Definition 3. 5: A space (X, τ_1, τ_2) is called (i,j)-sgT_{vp}-space if every (i,j)-vp closed set is (i,j)-sg closed.

Example 3.6: Let $X = \{a, b, c\}$,τ_1=$\{\varphi, \{c\}, \{a, c\}, X\}$,$\tau_2$=$\{\varphi, \{a\}, X\}$

(i,j)-vp closed sets are $\{\varphi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, X\}$

(i,j)-sg closed sets are $\{\varphi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, X\}$

Definition 3.7: A space (X, τ_1, τ_2) is called (i,j)-ΨT_{vp}-space if every (i,j)-vp closed set is (i,j)-sg^* closed.

Example 3.8: Let $X = \{a, b, c\}$,τ_1=$\{\varphi, \{a\}, \{b\}, \{a, b\}, X\}$,$\tau_2$=$\{\varphi, \{a\}, \{a, b\}, X\}$

(i,j)-vp closed sets are $\{\varphi, \{b\}, \{c\}, \{b, c\}, X\}$=$(i,j)$-$sg^*$ closed set.

Proposition 3.9: Every (i,j)-T_{vp}-space is (i,j)-$T_{1/2^*}$-space.

Proof: Let A be a (i,j)-g^* closed.

Then A is (i,j)-vp closed.Since (X, τ_1, τ_2) is a (i,j)-T_{vp}-space,A is τ_j-closed.

Therefore (X, τ_1, τ_2) is a (i,j)-$T_{1/2^*}$-space.

Remark 3.10: Converse of the above proposition need not be true as seen from the following example:

Example 3.11: Let $X = \{a, b, c\}$,τ_1=$\{\varphi, \{a\}, X\}$,τ_2=$\{\varphi, \{a\}, \{a, b\}, X\}$

(i,j)-g^* closed closed set is τ_j-closed and (X, τ_1, τ_2) is a (i,j)-$T_{\frac{1}{2}}{}^*$-space.

(i,j)-vp closed sets are $\{\varphi,\{b\},\{c\},\{b,c\},X\}$

A=$\{b\}$ is (i,j)-vp closed but not τ_j-closed.

Thus we proved ,every (i,j)-$T_{1/2^*}$-space need not be (i,j)-T_{vp}-space .

Proposition 3.12: Every (i,j)-T_{vp}-space is (i,j)-sgT_{vp}-space.

Proof: Let A be (i,j)-vp closed set.

Then A is τ_j-closed,since the space is a (i,j)-T_{vp}-space.

Then A is (i,j)-sg closed and hence (X,τ_1,τ_2) is a (i,j)-sgT_{vp}-space.

Remark 3.13: Converse of the above proposition need not be true as seen from the following example:

Example 3.14: Let $X=\{a,b,c\}$,τ_1=$\{\varphi,\{c\},\{a,c\},X\}$,$\tau_2$=$\{\varphi,\{a\},X\}$

(i,j)-vp closed sets are $\{\varphi,\{b\},\{c\},\{a,b\},\{b,c\},X\}$

A=$\{a,b\}$ is (i,j)-vp closed but not τ_j-closed, (X,τ_1,τ_2) is not a (i,j)-T_{vp}-space.

(i,j)-sg closed sets are $\{\varphi,\{b\},\{c\},\{a,b\},\{b,c\},X\}$

Since every (i,j)-vp closed set is (i,j)-sg closed, (X,τ_1,τ_2) is a (i,j)-sgT_{vp}-space.

Therefore every (i,j)-sgT_{vp}-space need not be (i,j)-T_{vp}-space.

Proposition 3.15: Every (i,j)-T_{vp}-space is (i,j)-ΨT_{vp}-space.

Proof: Let A be (i,j)-vp closed set.

Then A is τ_j-closed and hence,A is (i,j)- sg^*closed.

Therefore (X,τ_1,τ_2) is a (i,j)-ΨT_{vp}-space.

Remark 3.16: Converse of the above proposition need not be true as seen from the following example:

Example 3.17: Let $X=\{a,b,c\}$,τ_1=$\{\varphi,\{a\},\{a,b\},X\}$,$\tau_2$=$\{\varphi,\{a\},\{b\},\{a,b\},X\}$

(i,j)-vp closed closed sets are $\{\varphi,\{b,c\},\{a,c\},\{c\},X\}$

(i,j)-vp closed closed set is τ_j-closed and (X,τ_1,τ_2) is a (i,j)-T_{vp}-space.

But (i,j)- sg^*closed sets are $\{\varphi,\{a\},\{b\},\{c\},\{a,c\},\{b,c\},X\}$

A=$\{a\}$ is (i,j)- sg^*closed but not (i,j)-vp closed closed.

Therefore (X, τ_1, τ_2) is not a (i, j)-sg^*T_{vp}-space.

Hence (i, j)-sg^*T_{vp}-space need not be (i, j)-T_{vp}-space.

Proposition 3.18: Every (i, j)-sgT_{vp}-space is (i, j)-ΨT_{vp}-space.

Proof: Let A be (i, j)-vp closed set.

Then A is (i, j)-sg closed set and hence A is (i, j)-sg^* closed.

Therefore A is a (i, j)-ΨT_{vp}-space.

Proposition 3.19: Every (i, j)-T_{vp} space is (i, j)-$*_{T_{1/2}}$ space.

Proof: follows from the definition.

Remark 3.20: Converse of the above theorem need not be true as seen from the following example:

Example 3.21: Let $X = \{a, b, c\}$,τ_1=$\{\varphi, \{a\}, \{b\}, \{a, b\}, X\}$,$\tau_2$=$\{\varphi, \{a\}, \{c\}, \{a, c\}, X\}$

(i, j)-vp closed closed sets are $\{\varphi, \{b\}, \{a, b\}, \{b, c\}, X\}$

(i, j)-vp closed closed set is τ_j-closed and (X, τ_1, τ_2) is a (i, j)-T_{vp}-space.

But (i, j)- g^*closed sets are $\{\varphi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$

And we have (i, j)- g^*closed sets are (i, j)-g closed.

Therefore (X, τ_1, τ_2) is (i, j)-$*_{T_{1/2}}$ space.

A=$\{a, c\}$ is (i, j)- g^*closed but not (i, j)-vp closed closed.

Therefore (X, τ_1, τ_2) is not a (i, j)-T_{vp}-space.

Hence (i, j)-$*_{T_{1/2}}$ space need not be (i, j)-T_{vp}-space.

Remark 3.22: (i, j)-$T_d$ness is independent of (i, j)- T_{vp}-space.

Example 3.23: Let X=$\{a, b, c\}$, τ_1=$\{\varphi, \{a\}, X\}$,τ_2=$\{\varphi, \{a\}, \{a, b\}, X\}$

(i, j)-gs closed sets are $\{\varphi, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$

Since Every (i, j)-gs closed set is (i, j)-g closed.

Therefore (X, τ_1, τ_2) is a (i, j)-T_d space.

(i, j)-vp closed sets are $\{\varphi, \{b\}, \{c\}, \{b, c\}, X\}$

A={b} is (i,j)-vp closed set but not τ_2-closed.

Therefore (X,τ_1,τ_2) is not (i,j)- T_{vp}-space.

Let $X=\{a,b,c\}$,τ_1={$\varphi,\{a\},\{a,b\},X$},τ_2={$\varphi,\{a\},\{b\},\{a,b\},X$}

(i,j)-vp closed closed sets are $\{\varphi,\{c\},\{a,c\},\{b,c\},X\}$

(i,j)-vp closed closed set is τ_j-closed and (X,τ_1,τ_2) is a (i,j)-T_{vp}-space.

(i,j)-gs closed sets are $\{\varphi,\{b\},\{c\},\{a,c\},\{b,c\},X\}$

(i,j)-g closed sets are $\{\varphi,\{c\},\{a,c\},\{b,c\},X\}$

A={b} is (i,j)-gs closed but not (i,j)-g closed.

Hence (X,τ_1,τ_2) is not (i,j)-T_d space.

Therefore (i,j)-T_d space is independent of (i,j)-T_{vp}-space.

Remark 3.24: (i,j)-$T_d$ness is independent of (i,j)- sgT_{vp}-space.

Example 3.25: Let $X=\{a,b,c\}$,τ_1={$\varphi,\{a\},\{b,c\},X$},τ_2={$\varphi,\{a,c\},X$}

Then (X,τ_1,τ_2) is a (i,j)-T_d space.

(i,j)-sg closed sets are $\{\varphi,\{b\},\{a,b\},\{a,c\},X\}$

(i,j)-vp closed closed sets are $\{All\ the\ subsets\ of\ X\}$

A={b,c} is not (i,j)-sg closed.

Therefore (X,τ_1,τ_2) is not (i,j)- sgT_{vp}-space.

Let $X=\{a,b,c\}$,τ_1={$\varphi,\{a\},\{b\},\{a,b\},X$},$\tau_2$={$\varphi,\{a\},\{a,b\},X$}

Then (X,τ_1,τ_2) is a (i,j)- sgT_{vp} space.

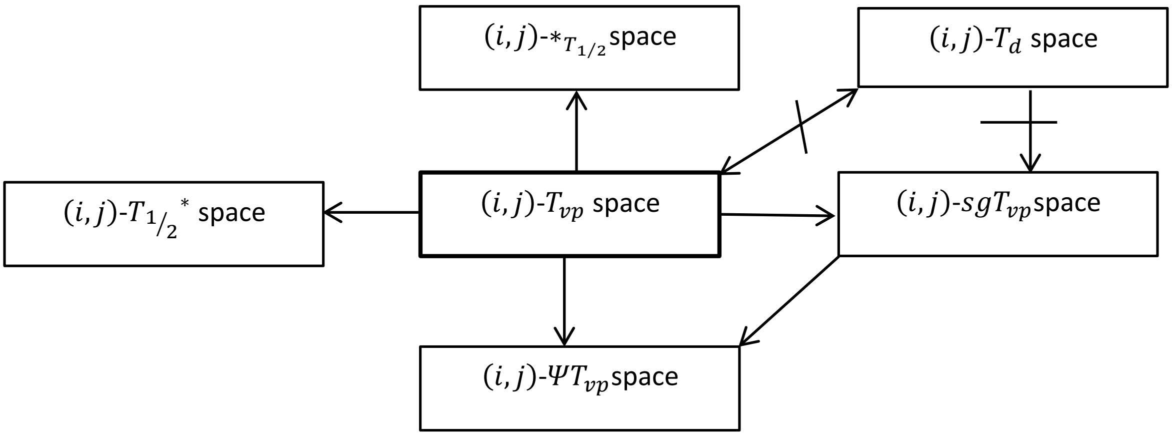
(i,j)-gs closed sets are $\{\varphi,\{b\},\{c\},\{a,c\},\{b,c\},X\}$

$\{i,j\}$-g closed sets are $\{\varphi,\{c\},\{a,c\},\{b,c\},X\}$

A={b} is not (i,j)-gs closed and (X,τ_1,τ_2) is not (i,j)-T_d space.

There fore (i,j)-$T_d$ness is independent of (i,j)- sgT_{vp}-space.

The above results can be shown in the following figure:



4.(i,j)-VP CONTINUOUS MAPS IN BITOPOLOGICAL SPACES

We introduce the following definition:

Definition 4.1: A map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ from a topological space (X,τ_1,τ_2) to a topological space (Y,σ_1,σ_2) is called a (i,j)-vp continuous if the inverse image of every closed set in (Y,σ_1,σ_2) in (i,j)-vp closed set in (X,τ_1,τ_2).

Proposition 4.2: If $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is continuous then it is (i,j)-vp continuous.

Proof : Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a continuous map.

To prove : f is (i,j)-vp continuous.

Let V be a closed set in (Y,σ_1,σ_2).

Since f is continuous,$f^{-1}(V)$ is closed in (X,τ_1,τ_2).

Therefore $f^{-1}(V)$ is (i,j)-vp closed.

Hence f is (i,j)-vp continuous.

Remark 4.3: The converse of the above proposition need not be true as seen from the following example:

Example 4.4: Let X={a,b,c},τ_1={φ,{a},{a,b},X},τ_2={φ,{a},{b,c},X} and

Let Y={p,q} , $\sigma_1=\{\varphi,Y,\{p\}\}$; $\sigma_2=\{\varphi,Y,\{q\}\}$

Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be defined by $f(b)=f(c)=\{p\}$, $f(a)=\{q\}$

Let us prove that f is (i,j)-vp continuous but not continuous.

(i,j)-vp closed sets are $\{All\ the\ subsets\ of\ X\}$

$f^{-1}(p) = \{b, c\}$ and $f^{-1}(q) = \{a\}$ are (i,j)-vp closed set in (X, τ_1, τ_2)

Therefore f is (i,j)-vp continuous but not continuous in (X, τ_1, τ_2).

Proposition 4.5: If $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i,j)-$g^{\#\#}$ continuous then it is (i,j)-vp continuous.

Proof : Let V be a closed set in (Y, σ_1, σ_2).

Then $f^{-1}(V)$ is (i,j)-$g^{\#\#}$ closed and every (i,j)-$g^{\#\#}$ closed set is (i,j)-vp closed.

Therefore $f^{-1}(V)$ is (i,j)-vp closed.

Hence f is (i,j)-vp continuous.

Remark 4.6: The converse of the above proposition need not be true as seen from the following example:

Example 4.7: Let $X = \{a, b, c\}$,τ_1=$\{\varphi, \{a\}, \{a, c\}, X\}$,$\tau_2$=$\{\varphi, \{a\}, X\}$ and

Y=$\{p, q\}$,σ_1=$\{\varphi, Y, \{p\}\}$,σ_2=$\{\varphi, Y, \{q\}\}$

Define a mapping $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by $f(a)$=$f(c)$=p;$f(b)$=q

We have that (i,j)-vp closed sets of (X, τ_1, τ_2) are all the subsets of X.

$f^{-1}(p)$=$\{a, c\}$;$f^{-1}(q)$=$\{b\}$ are (i,j)-vp closed sets in (X, τ_1, τ_2)

Hence f is a (i,j)-vp continuous.

Also we have that (i,j)-$g^{\#\#}$ closed sets are $\{\varphi, \{b\}, \{a, b\}, \{b, c\}, X\}$

$f^{-1}(p)$=$\{a, c\}$ is not (i,j)-$g^{\#\#}$ closed set in (X, τ_1, τ_2).

Therefore f is not (i,j)-$g^{\#\#}$ continuous.

Hence f is (i,j)-vp continuous but not (i,j)-$g^{\#\#}$ continuous.

Proposition 4.8: If $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i,j)-$\hat{g}$ continuous then it is (i,j)-vp continuous.

Proof : Let V be a closed set in (Y, σ_1, σ_2).

Then $f^{-1}(V)$ is (i,j)-$\hat{g}$ closed and every (i,j)-$\hat{g}$ closed set is (i,j)-vp closed.

Therefore $f^{-1}(V)$ is (i,j)-vp closed.

Hence f is (i,j)-vp continuous.

Remark 4.9: The **c**onverse of the above proposition need not be true as seen from the following example:

Example 4.10: Let $X = \{a, b, c\}$,τ_1=$\{\varphi, \{b\}, X\}$,τ_2=$\{\varphi, \{c\}, X\}$ and

Y=$\{p, q\}$,σ_1=$\{\varphi, Y, \{p\}\}$,σ_2=$\{\varphi, Y, \{q\}\}$

Define a mapping $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by $f(b)$=$f(c)$=p;$f(a)$=q

We have that (i, j)-vp closed sets of (X, τ_1, τ_2) are $\{\varphi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$

$f^{-1}(p)$=$\{b, c\}$;$f^{-1}(q)$=$\{b\}$ are (i, j)-vp closed sets in (X, τ_1, τ_2)

Hence f is a (i, j)-vp continuous.

Also we have that (i, j)-$\hat{g}$ closed sets are $\{\varphi, \{a\}, \{a, b\}, \{a, c\}, X\}$

$f^{-1}(p)$=$\{b, c\}$ is not (i, j)-$\hat{g}$ closed set in (X, τ_1, τ_2).

Therefore f is not (i, j)-$\hat{g}$ continuous.

Hence f is (i, j)-vp continuous but not (i, j)-$\hat{g}$ continuous.

Proposition 4.11: If $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i, j)-vp continuous then it is (i, j)-αg continuous.

Proof : Let V be a closed set in (Y, σ_1, σ_2).

Then $f^{-1}(V)$ is (i, j)-vp closed and every (i, j)-vp closed set is (i, j)-αg closed.

Therefore $f^{-1}(V)$ is (i, j)-αg closed.

Hence f is (i, j)-αg continuous.

Remark 4.12: The **c**onverse of the above proposition need not be true as seen from the following example:

Example 4.13: Let $X = \{a, b, c\}$,τ_1=$\{\varphi, \{b\}, X\}$,τ_2=$\{\varphi, \{c\}, X\}$ and

Y=$\{p, q\}$,σ_1=$\{\varphi, Y, \{p\}\}$,σ_2=$\{\varphi, Y, \{q\}\}$

Define a mapping $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by $f(a)$=$f(b)$=p;$f(c)$=q

We have that (i, j)-αg closed sets of (X, τ_1, τ_2) are $\{\varphi, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$

$f^{-1}(p)$=$\{a, b\}$;$f^{-1}(q)$=$\{c\}$ are (i, j)-αg closed sets in (X, τ_1, τ_2)

Hence f is a (i, j)-αg continuous.

Also we have that (i, j)-vp closed sets are $\{\varphi, \{b\}, \{c\}, \{b, c\}, X\}$

$f^{-1}(p)$=$\{a, b\}$ is not (i, j)-vp closed set in (X, τ_1, τ_2).

Therefore f is not (i,j)-vp continuous.

Hence f is (i,j)-αg continuous but not (i,j)-vp continuous.

Proposition 4.14: If $f:(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is (i,j)-g^*p continuous then it is (i,j)-vp continuous.

Proof : Let V be a closed set in (Y,σ_1,σ_2).

Then $f^{-1}(V)$ is (i,j)-g^*p closed and every (i,j)-g^*p closed set is (i,j)-vp closed.

Therefore $f^{-1}(V)$ is (i,j)-vp closed.

Hence f is (i,j)-vp continuous.

Remark 4.15: The converse of the above proposition need not be true as seen from the following example:

Example 4.16: Let $X = Y = \{a,b,c\}$, τ_1=$\{\varphi,\{a\},\{a,b\},X\}$ and τ_2=$\{\varphi,\{a\},\{b,c\},X\}$

σ_1=$\{\varphi,Y,\{b\}\}$,σ_2=$\{\varphi,Y,\{a,c\}\}$

Define a mapping $f:(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ by $f(a)$=$\{a\}$; $f(b)$=$\{b\}$;$f(c)$=$\{c\}$

We have that (i,j)-vp closed sets of (X,τ_1,τ_2) are $\{All\ the\ subsets\ of\ X\}$

$f^{-1}(a,c)$=$\{a,c\}$;$f^{-1}(b)$=$\{b\}$ are (i,j)-vp closed sets in (X,τ_1,τ_2)

Hence f is a (i,j)-vp continuous.

Also we have that (i,j)-g^*p closed sets are $\{\varphi,\{a\},\{b,c\},X\}$

$f^{-1}(p)$=$\{a,c\}$ is not (i,j)-g^*p closed set in (X,τ_1,τ_2).

Therefore f is not (i,j)-g^*p continuous.

Hence f is (i,j)-vp continuous but not (i,j)-g^*p continuous.

Proposition 4.17: If $f:(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is (i,j)-Ψ continuous then it is (i,j)-vp continuous.

Proof : Let V be a closed set in (Y,σ_1,σ_2).

Then $f^{-1}(V)$ is (i,j)-Ψ closed and every (i,j)-Ψ closed set is (i,j)-vp closed.

Therefore $f^{-1}(V)$ is (i,j)-vp closed.

Hence f is (i,j)-vp continuous.

Remark 4.18: The converse of the above proposition need not be true as seen from the following example:

Example 4.19: Let $X = Y = \{a, b, c\}$,τ_1=$\{\varphi, \{b\}, \{b, c\}, X\}$,$\tau_2$=$\{\varphi, \{ab\}, X\}$ and

σ_1=$\{\varphi, Y, \{a\}\}$,σ_2=$\{\varphi, Y, \{b, c\}\}$

Define a mapping $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by $f(a)$=$\{a\}$; $f(b)$=$\{b\}$;$f(c)$=$\{c\}$

We have that (i, j)-vp closed sets of (X, τ_1, τ_2) are $\{\varphi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, X\}$

$f^{-1}(b, c)$=$\{b, c\}$;$f^{-1}(a)$=$\{a\}$ are (i, j)-vp closed sets in (X, τ_1, τ_2)

Hence f is a (i, j)-vp continuous.

Also we have that (i, j)-Ψ closed sets are $\{\varphi, \{c\}, \{a, c\}, X\}$

$f^{-1}(b, c)$=$\{b, c\}$ is not (i, j)-Ψ closed set in (X, τ_1, τ_2).

Therefore f is not (i, j)-Ψ continuous.

Hence f is (i, j)-vp continuous but not (i, j)-Ψ continuous.

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