

FRACTIONAL INTEGRAL OPERATORS ASSOCIATED WITH GENERALIZED MAINARDI FUNCTION

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Abstract: In this paper, we obtain the image of generalized Mainardi function under the certain fractional integral operators (Saigo, Erdélyi, Kober, Riemann-Liouville and Weyl).

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1. Introduction and Definitions:-

The Generalized Mainardi function

$${}_pS_q \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) \\ (a_1, A_1) \dots (a_p, A_p); (\delta - \alpha, -\alpha); -z \end{matrix} \right]$$

was introduced and investigated by J.Daiya and R.K.Saxena [4]. It is represented in the following for

$${}_pS_q \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) \\ (a_1, A_1) \dots (a_p, A_p); (\delta - \alpha, -\alpha); -z \end{matrix} \right] \\ = \sum_{n=0}^{\infty} \left[\frac{\Gamma(b_1+nB_1) \dots \Gamma(b_q+nB_q)}{\Gamma(a_1+nA_1) \dots \Gamma(a_p+nA_p)} \right] \cdot \frac{(-1)^n}{\Gamma[-\alpha(n+1)+\delta]} \cdot \frac{z^n}{n!}, \quad (1)$$

Where $a_i, b_j \in \mathbb{C}$ and $A_i, B_j \in \mathbb{R}$ ($i = 1, 2, \dots, p; j = 1, 2, \dots, q$), $\alpha \in \mathbb{C}$, $\Re(\alpha) > 0$, $z \in \mathbb{C}$ with \mathbb{C} being the set of complex number and the given series (1) converges for

$$\sum_{j=1}^q B_j - \sum_{i=1}^p A_i - \alpha > -1.$$

If we set $p = q = 0$ and $\delta = 1$ in equation (1) it yields Mainardi function defined by Mainardi [4] as

$$M(z, \alpha) = \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma[-\alpha(n+1)+1]} \cdot \frac{z^n}{n!}, \quad \alpha \in \mathbb{C}, \Re(\alpha) > 0, z \in \mathbb{C} \quad (2)$$

with \mathbb{C} being the set of complex number.

Definition 1 Saigo hypergeometric fractional integral operator of $f(t)$ is defined as [6]

$$\left(I_{0,x}^{\lambda, \nu, \eta} f(t) \right) (x) = \frac{x^{-\lambda-\nu}}{\Gamma(\lambda)} \int_0^x (x-t)^{\lambda-1} {}_2F_1 \left(\lambda + \nu, -\eta; \lambda; 1 - \frac{t}{x} \right) f(t) dt \quad (3)$$

$(x > 0 \text{ and } \lambda, \nu, \eta \in \mathbb{C})$

And

$$(j_{x,\infty}^{\lambda,\nu,\eta} f(t))(x) = \frac{1}{\Gamma(\lambda)} \int_x^\infty (t-x)^{\lambda-1} t^{-\lambda-\nu} {}_2F_1(\lambda+\nu, -\eta; \lambda; 1-\frac{x}{t}) f(t) dt \tag{4}$$

$(x > 0 \text{ and } \lambda, \nu, \eta \in \mathbb{C}).$

Definition 2 Erdélyi fractional integral operator of $f(t)$ is defined as [6]

$$(E_{0,x}^{\lambda,\eta} f(t))(x) = \frac{x^{-\lambda-\nu}}{\Gamma(\lambda)} \int_0^x (x-t)^{\lambda-1} t^\eta f(t) dt, (\Re(\lambda) > 0, \Re(\eta) > 0) \tag{5}$$

Definition 3 Kober fractional integral operator of $f(t)$ is defined as [6]

$$(K_{x,\infty}^{\lambda,\eta} f(t))(x) = \frac{x^\eta}{\Gamma(\lambda)} \int_x^\infty (t-x)^{\lambda-1} t^{-\lambda-\eta} f(t) dt, (\Re(\lambda) > 0, \Re(\eta) > 0). \tag{6}$$

Definition 4 Riemann Liouville fractional integral operator of $f(t)$ is defined as [6]

$$(R_{0,x}^\lambda f(t))(x) = \frac{1}{\Gamma(\lambda)} \int_0^x (x-t)^{\lambda-1} f(t) dt, (\Re(\lambda) > 0). \tag{7}$$

Definition 5 Weyl fractional integral operator of $f(t)$ is defined as [6]

$$(W_{x,\infty}^\lambda f(t))(x) = \frac{1}{\Gamma(\lambda)} \int_x^\infty (t-x)^{\lambda-1} t^{-\lambda-\nu} f(t) dt, (\Re(\lambda) > 0). \tag{8}$$

Theorem 1.1 If $x > 0, \lambda, \nu, \eta \in \mathbb{C}$ be parameters such that $(\Re(\rho) > 1, \Re(\rho + \eta - \nu) > 1), \Re(\rho - \nu) > 1$. Then the following saigo fractional integral formula holds:

$$\begin{aligned} I_{0,x}^{\lambda,\nu,\eta} t^{\rho-1} {}_pS_q \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) \\ (a_1, A_1) \dots (a_p, A_p); (\delta - \alpha, -\alpha); -t \end{matrix} \right] (x) \\ = {}_{p+2}S_{q+2} \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q)(\rho, 1)(\rho + \eta - \nu, 1) \\ (a_1, A_1) \dots (a_p, A_p)(\rho - \nu, 1)(\rho + \eta + \lambda, 1); (\delta - \alpha, -\alpha); -x \end{matrix} \right] \cdot x^{(\rho-\nu-1)} \end{aligned} \tag{9}$$

Proof: To prove the assertion (9), by the taking Saigo fractional integral operator (3), we obtain

$$\begin{aligned} I_{0,x}^{\lambda,\nu,\eta} t^{\rho-1} {}_pS_q \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) \\ (a_1, A_1) \dots (a_p, A_p); (\delta - \alpha, -\alpha); -t \end{matrix} \right] (x) \\ = \frac{x^{-\lambda-\nu}}{\Gamma(\lambda)} \int_0^x (x-t)^{\lambda-1} {}_2F_1 \left(\lambda + \nu, -\eta; \lambda; 1 - \frac{t}{x} \right) t^{\rho-1} \\ {}_pS_q \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) \\ (a_1, A_1) \dots (a_p, A_p); (\delta - \alpha, -\alpha); -t \end{matrix} \right] dt \\ = \frac{x^{-\lambda-\nu}}{\Gamma(\lambda)} \int_0^x (x-t)^{\lambda-1} {}_2F_1 \left(\lambda + \nu, -\eta; \lambda; 1 - \frac{t}{x} \right) t^{\rho-1} \\ \sum_{n=0}^\infty \left[\frac{\Gamma(b_1+nB_1) \dots \Gamma(b_q+nB_q)}{\Gamma(a_1+nA_1) \dots \Gamma(a_p+nA_p)} \right] \cdot \frac{(-1)^n}{\Gamma[-\alpha(n+1)+\delta]} \cdot \frac{t^n}{n!} dt \end{aligned}$$

$$= \frac{x^{-\lambda-\nu}}{\Gamma(\lambda)} \sum_{n=0}^{\infty} \left[\frac{\Gamma(b_1+nB_1) \dots \Gamma(b_q+nB_q)}{\Gamma(a_1+nA_1) \dots \Gamma(a_p+nA_p)} \right] \frac{(-1)^n}{\Gamma[-\alpha(n+1)+\delta]} \cdot \frac{1}{n!} \int_0^x (x-t)^{\lambda-1} {}_2F_1\left(\lambda+\nu, -\eta; \lambda; 1-\frac{t}{x}\right) t^{n+\rho-1} dt .$$

Now, with the help of [2], we obtain

$$= \sum_{n=0}^{\infty} \left[\frac{\Gamma(b_1+nB_1) \dots \Gamma(b_q+nB_q)}{\Gamma(a_1+nA_1) \dots \Gamma(a_p+nA_p)} \right] \frac{(-1)^n}{\Gamma[-\alpha(n+1)+\delta]} \cdot \frac{1}{n!} \cdot I_{0,x}^{\lambda,\nu,\eta} t^{n+\rho-1}$$

$$= \sum_{n=0}^{\infty} \left[\frac{\Gamma(b_1+nB_1) \dots \Gamma(b_q+nB_q)}{\Gamma(a_1+nA_1) \dots \Gamma(a_p+nA_p)} \right] \frac{(-1)^n}{\Gamma[-\alpha(n+1)+\delta]} \cdot \frac{1}{n!} \frac{\Gamma(\rho+n)\Gamma(\rho+n+\eta-\nu)}{\Gamma(\rho+n-\nu)\Gamma(\rho+n+\eta+\lambda)} \cdot x^{(\rho+n-\nu-1)}$$

$$= \sum_{n=0}^{\infty} \left[\frac{\Gamma(b_1+nB_1) \dots \Gamma(b_q+nB_q)\Gamma(\rho+n)\Gamma(\rho+n+\eta-\nu)}{\Gamma(a_1+nA_1) \dots \Gamma(a_p+nA_p)\Gamma(\rho+n-\nu)\Gamma(\rho+n+\eta+\lambda)} \right] \cdot \frac{(-1)^n}{\Gamma[-\alpha(n+1)+\delta]} \cdot \frac{x^n}{n!} \cdot x^{(\rho-\nu-1)}$$

$$= {}_{p+2}S_{q+2} \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) (\rho, 1) (\rho + \eta - \nu, 1) \\ (a_1, A_1) \dots (a_p, A_p) (\rho - \nu, 1) (\rho + \eta + \lambda, 1); (\delta - \alpha, -\alpha); -x \end{matrix} \right] \cdot x^{(\rho-\nu-1)}.$$

Finally, with the help of (1), we get the desired result (9).

Theorem 1.2 If $x > 0, \lambda, \nu, \eta \in \mathbb{C}$ be parameters such that $\Re(\rho) > 1, \Re(\rho + \eta - \nu) > 1, \Re(\rho - \nu) > 1$.

Then the following saigo fractional integral formula holds:

$$\left(j_{x,\infty}^{\lambda,\nu,\eta} \right) t^{-\rho} {}_pS_q \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) \\ (a_1, A_1) \dots (a_p, A_p); (\delta - \alpha, -\alpha); -\frac{1}{t} \end{matrix} \right] (x)$$

$$= {}_{p+2}S_{q+2} \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) (\rho + \nu, 1) (\rho + \eta, 1) \\ (a_1, A_1) \dots (a_p, A_p) (\rho, 1) (\rho + \eta + \nu + \lambda, 1); (\delta - \alpha, -\alpha); -\frac{1}{x} \end{matrix} \right] \cdot x^{(-\rho-\nu)} \tag{10}$$

Proof: In order to prove the assertion (10), by the taking Saigo fractional integral operator (4) of (1), we obtain

$$\left(j_{x,\infty}^{\lambda,\nu,\eta} \right) t^{-\rho} {}_pS_q \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) \\ (a_1, A_1) \dots (a_p, A_p); (\delta - \alpha, -\alpha); -\frac{1}{t} \end{matrix} \right] (x)$$

$$= \frac{1}{\Gamma(\lambda)} \int_x^\infty (t-x)^{\lambda-1} t^{-\lambda-\nu-2} F^1\left(\lambda+\nu, -\eta; \lambda; 1-\frac{x}{t}\right) t^{-\rho} \cdot {}_pS_q \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) \\ (a_1, A_1) \dots (a_p, A_p); (\delta - \alpha, -\alpha); -\frac{1}{t} \end{matrix} \right] dt$$

$$= \frac{1}{\Gamma(\lambda)} \sum_{n=0}^{\infty} \left[\frac{\Gamma(b_1+nB_1) \dots \Gamma(b_q+nB_q)}{\Gamma(a_1+nA_1) \dots \Gamma(a_p+nA_p)} \right] \frac{(-1)^n}{\Gamma[-\alpha(n+1)+\delta]}$$

$$\begin{aligned}
 & \frac{1}{n!} \cdot \left(\frac{1}{t}\right)^n \int_x^\infty (t-x)^{\lambda-1} t^{-\rho-\lambda-\nu} {}_2F_1\left(\lambda+\nu, -\eta; \lambda; 1-\frac{x}{t}\right) dt \\
 &= \sum_{n=0}^\infty \left[\frac{\Gamma(b_1+nB_1) \dots \Gamma(b_q+nB_q)}{\Gamma(a_1+nA_1) \dots \Gamma(a_p+nA_p)} \right] \frac{(-1)^n}{\Gamma[-\alpha(n+1)+\delta]} \frac{1}{n!} \left(\int_{x,\infty}^{\lambda,\nu,\eta} \right) t^{-\rho-n} \\
 &= \sum_{n=0}^\infty \left[\frac{\Gamma(b_1+nB_1) \dots \Gamma(b_q+nB_q)}{\Gamma(a_1+nA_1) \dots \Gamma(a_p+nA_p)} \right] \frac{(-1)^n}{\Gamma[-\alpha(n+1)+\delta]} \frac{1}{n!} \frac{\Gamma(\rho+n+\nu)\Gamma(\rho+n+\eta)}{\Gamma(\rho+n)\Gamma(\rho+n+\eta+\lambda+\nu)} \cdot x^{(-\rho-n-\nu)} \\
 &= \sum_{n=0}^\infty \left[\frac{\Gamma(b_1+nB_1) \dots \Gamma(b_q+nB_q)\Gamma(\rho+n+\nu)\Gamma(\rho+n+\eta)}{\Gamma(a_1+nA_1) \dots \Gamma(a_p+nA_p)\Gamma(\rho+n)\Gamma(\rho+n+\eta+\lambda+\nu)} \right] \frac{(-1)^n}{\Gamma[-\alpha(n+1)+\delta]} \frac{1}{n!} \cdot x^{(-\rho-\nu)} \cdot x^{-n} \\
 &= {}_{p+2}S_{q+2} \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) (\rho+\nu, 1) (\rho+\eta, 1) \\ (a_1, A_1) \dots (a_p, A_p) (\rho, 1) (\rho+\eta+\nu+\lambda, 1); (\delta-\alpha, -\alpha); -\frac{1}{x} \end{matrix} \right] \cdot x^{(-\rho-\nu)}.
 \end{aligned}$$

Finally, with the help of (1), we get the desired result (10).

Corollary 1. If we put $\nu = 0$ in (9), then Saigo hypergeometric fractional integral operator reduces to Erdelyi fractional integral operator of generalized Mainardi function

$$\begin{aligned}
 & \left(E_{0,x}^{\lambda,\eta}\right) t^{\rho-1} {}_pS_q \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) \\ (a_1, A_1) \dots (a_p, A_p); (\delta-\alpha, -\alpha); -t \end{matrix} \right] (x) \\
 &= {}_{p+1}S_{q+1} \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) (\rho+\eta, 1) \\ (a_1, A_1) \dots (a_p, A_p) (\rho+\eta+\lambda, 1); (\delta-\alpha, -\alpha); -x \end{matrix} \right] \cdot x^{(\rho-1)} \tag{11}
 \end{aligned}$$

Proof :-By using Erdelyi fractional integral operator (5) of (1) then

$$\begin{aligned}
 & \left(E_{0,x}^{\lambda,\eta}\right) t^{\rho-1} {}_pS_q \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) \\ (a_1, A_1) \dots (a_p, A_p); (\delta-\alpha, -\alpha); -t \end{matrix} \right] (x) \\
 &= \frac{x^{-\lambda-\nu}}{\Gamma(\lambda)} \int_0^x (x-t)^{\lambda-1} t^\eta \cdot t^{\rho-1} {}_pS_q \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) \\ (a_1, A_1) \dots (a_p, A_p); (\delta-\alpha, -\alpha); -t \end{matrix} \right] dt.
 \end{aligned}$$

By the major calculation, we obtain the result

$$\begin{aligned}
 & \left(E_{0,x}^{\lambda,\eta}\right) t^{\rho-1} {}_pS_q \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) \\ (a_1, A_1) \dots (a_p, A_p); (\delta-\alpha, -\alpha); -t \end{matrix} \right] (x) \\
 &= \sum_{n=0}^\infty \left[\frac{\Gamma(b_1+nB_1) \dots \Gamma(b_q+nB_q)\Gamma(\rho+n)\Gamma(\rho+n+\eta)}{\Gamma(a_1+nA_1) \dots \Gamma(a_p+nA_p)\Gamma(\rho+n)\Gamma(\rho+n+\eta+\lambda)} \right] \frac{(-1)^n}{\Gamma[-\alpha(n+1)+\delta]} \cdot \frac{x^n}{n!} \cdot x^{(\rho-1)} \\
 &= {}_{p+1}S_{q+1} \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) (\rho+\eta, 1) \\ (a_1, A_1) \dots (a_p, A_p) (\rho+\eta+\lambda, 1); (\delta-\alpha, -\alpha); -x \end{matrix} \right] \cdot x^{(\rho-1)}.
 \end{aligned}$$

Corollary 2. If we put $\nu = 0$ in (11), then Saigo hypergeometric fractional integral operator reduces to Kober fractional integral operator of generalized Mainardi function

$$\begin{aligned}
 & \left(K_{x,\infty}^{\lambda,\eta}\right) t^{-\rho} {}_pS_q \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) \\ (a_1, A_1) \dots (a_p, A_p); (\delta-\alpha, -\alpha); -\frac{1}{t} \end{matrix} \right] (x) \\
 &= {}_{p+1}S_{q+1} \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) (\rho+\eta, 1) \\ (a_1, A_1) \dots (a_p, A_p) (\rho+\eta+\lambda, 1); (\delta-\alpha, -\alpha); -\frac{1}{x} \end{matrix} \right] \cdot x^{(-\rho)} \tag{12}
 \end{aligned}$$

Proof :- By using Kober fractional integral operator (6) of (1) then

$$\begin{aligned} & \left(K_{x,\infty}^{\lambda,\eta} \right) t^{-\rho} {}_p S_q \left[\begin{matrix} (b_1, B_1) \dots \dots (b_q, B_q) \\ (a_1, A_1) \dots \dots (a_p, A_p); (\delta - \alpha, -\alpha); -\frac{1}{t} \end{matrix} \right] (x) \\ &= \frac{1}{\Gamma(\lambda)} \int_x^\infty (t-x)^{\lambda-1} t^{-\lambda-\eta} \cdot t^{-\rho} {}_p S_q \left[\begin{matrix} (b_1, B_1) \dots \dots (b_q, B_q) \\ (a_1, A_1) \dots \dots (a_p, A_p); (\delta - \alpha, -\alpha); -\frac{1}{t} \end{matrix} \right] dt. \end{aligned}$$

By the major calculation, we get the result

$$\begin{aligned} & \left(K_{x,\infty}^{\lambda,\eta} \right) t^{-\rho} {}_p S_q \left[\begin{matrix} (b_1, B_1) \dots \dots (b_q, B_q) \\ (a_1, A_1) \dots \dots (a_p, A_p); (\delta - \alpha, -\alpha); -\frac{1}{t} \end{matrix} \right] (x) \\ &= \sum_{n=0}^\infty \left[\frac{\Gamma(b_1+nB_1) \dots \dots \Gamma(b_q+nB_q) \Gamma(\rho+n) \Gamma(\rho+n+\eta)}{\Gamma(a_1+nA_1) \dots \dots \Gamma(a_p+nA_p) \Gamma(\rho+n) \Gamma(\rho+n+\eta+\lambda)} \right] \cdot \frac{(-1)^n}{\Gamma[-\alpha(n+1)+\delta]} \cdot \frac{1}{n!} \cdot x^{(-\rho)} \cdot x^{-n} \\ &= {}_{p+1} S_{q+1} \left[\begin{matrix} (b_1, B_1) \dots \dots (b_q, B_q) (\rho + \eta, 1) \\ (a_1, A_1) \dots \dots (a_p, A_p) (\rho + \eta + \lambda, 1); (\delta - \alpha, -\alpha); -\frac{1}{x} \end{matrix} \right] \cdot x^{(-\rho)}. \end{aligned}$$

Corollary 3. If we put $v = -\lambda$ in (9), then Saigo hypergeometric fractional integral operator reduces to Riemann Liouville fractional integral operator of generalized Mainardi function

$$\begin{aligned} & \left(R_{0,x}^\lambda \right) t^{\rho-1} {}_p S_q \left[\begin{matrix} (b_1, B_1) \dots \dots (b_q, B_q) \\ (a_1, A_1) \dots \dots (a_p, A_p); (\delta - \alpha, -\alpha); -t \end{matrix} \right] (x) \\ &= {}_{p+1} S_{q+1} \left[\begin{matrix} (b_1, B_1) \dots \dots (b_q, B_q) (\rho, 1) \\ (a_1, A_1) \dots \dots (a_p, A_p) (\rho + \lambda, 1); (\delta - \alpha, -\alpha); -x \end{matrix} \right] \cdot x^{(\rho+\lambda-1)} \end{aligned} \tag{13}$$

Proof: By using Riemann Liouville fractional integral operator (7) of (1) then

$$\begin{aligned} & \left(R_{0,x}^\lambda \right) t^{\rho-1} {}_p S_q \left[\begin{matrix} (b_1, B_1) \dots \dots (b_q, B_q) \\ (a_1, A_1) \dots \dots (a_p, A_p); (\delta - \alpha, -\alpha); -t \end{matrix} \right] (x) \\ &= \frac{1}{\Gamma(\lambda)} \int_0^x (x-t)^{\lambda-1} t^{\rho-1} {}_p S_q \left[\begin{matrix} (b_1, B_1) \dots \dots (b_q, B_q) \\ (a_1, A_1) \dots \dots (a_p, A_p); (\delta - \alpha, -\alpha); -t \end{matrix} \right] dt \\ &= \frac{1}{\Gamma(\lambda)} \sum_{n=0}^\infty \left[\frac{\Gamma(b_1+nB_1) \dots \dots \Gamma(b_q+nB_q)}{\Gamma(a_1+nA_1) \dots \dots \Gamma(a_p+nA_p)} \right] \cdot \frac{(-1)^n}{\Gamma[-\alpha(n+1)+\delta]} \cdot \frac{1}{n!} \int_0^x (x-t)^{\lambda-1} t^{n+\rho-1} dt \\ &= \sum_{n=0}^\infty \left[\frac{\Gamma(b_1+nB_1) \dots \dots \Gamma(b_q+nB_q) \Gamma(\rho+n) \Gamma(\rho+n+\eta+\lambda)}{\Gamma(a_1+nA_1) \dots \dots \Gamma(a_p+nA_p) \Gamma(\rho+n+\lambda) \Gamma(\rho+n+\eta+\lambda)} \right] \cdot \frac{(-1)^n}{\Gamma[-\alpha(n+1)+\delta]} \cdot \frac{x^n}{n!} \cdot x^{(\rho+\lambda-1)} \\ &= {}_{p+1} S_{q+1} \left[\begin{matrix} (b_1, B_1) \dots \dots (b_q, B_q) (\rho, 1) \\ (a_1, A_1) \dots \dots (a_p, A_p) (\rho + \lambda, 1); (\delta - \alpha, -\alpha); -x \end{matrix} \right] \cdot x^{(\rho+\lambda-1)}. \end{aligned}$$

Corollary 4. If we put $v = -\lambda$ in (11), then Saigo hypergeometric fractional integral operator reduces to fractional integral operator of generalized Mainardi function

$$\left(W_{x,\infty}^\lambda \right) t^{-\rho} {}_p S_q \left[\begin{matrix} (b_1, B_1) \dots \dots (b_q, B_q) \\ (a_1, A_1) \dots \dots (a_p, A_p); (\delta - \alpha, -\alpha); -\frac{1}{t} \end{matrix} \right] (x)$$

$$= {}_{p+1}S_{q+1} \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) (\rho - \lambda, 1) \\ (a_1, A_1) \dots (a_p, A_p) (\rho, 1); (\delta - \alpha, -\alpha); -\frac{1}{x} \end{matrix} \right] \cdot x^{(-\rho+\lambda)} \quad (14)$$

Proof: By using Weyl fractional integral operator (8) of (1), then

$$\begin{aligned} & (W_{x,\infty}^\lambda) t^{-\rho} {}_pS_q \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) \\ (a_1, A_1) \dots (a_p, A_p); (\delta - \alpha, -\alpha); -\frac{1}{t} \end{matrix} \right] (x) \\ &= \frac{1}{\Gamma(\lambda)} \int_x^\infty (t-x)^{\lambda-1} \cdot t^{-\rho} {}_pS_q \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) \\ (a_1, A_1) \dots (a_p, A_p); (\delta - \alpha, -\alpha); -\frac{1}{t} \end{matrix} \right] dt \\ &= \frac{1}{\Gamma(\lambda)} \sum_{n=0}^\infty \left[\frac{\Gamma(b_1+nB_1) \dots \Gamma(b_q+nB_q)}{\Gamma(a_1+nA_1) \dots \Gamma(a_p+nA_p)} \right] \cdot \frac{(-1)^n}{\Gamma[-\alpha(n+1)+\delta]} \cdot \frac{1}{n!} \cdot \left(\frac{1}{t}\right)^n \int_x^\infty (t-x)^{\lambda-1} t^{-\rho} dt \\ &= \sum_{n=0}^\infty \left[\frac{\Gamma(b_1+nB_1) \dots \Gamma(b_q+nB_q) \Gamma(\rho+n-\lambda) \Gamma(\rho+n+\eta)}{\Gamma(a_1+nA_1) \dots \Gamma(a_p+nA_p) \Gamma(\rho+n) \Gamma(\rho+n+\eta)} \right] \cdot \frac{(-1)^n}{\Gamma[-\alpha(n+1)+\delta]} \cdot \frac{1}{n!} \cdot x^{(-\rho+\lambda)} \cdot x^{-n} \\ &= {}_{p+1}S_{q+1} \left[\begin{matrix} (b_1, B_1) \dots (b_q, B_q) (\rho - \lambda, 1) \\ (a_1, A_1) \dots (a_p, A_p) (\rho, 1); (\delta - \alpha, -\alpha); -\frac{1}{x} \end{matrix} \right] \cdot x^{(-\rho+\lambda)}. \end{aligned}$$

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