CYCLIC SHIFT SEQUENCES WITH PIECEWISE LINEAR COMPANDING TRANSFORM FOR REDUCING PAPR IN OFDM SYSTEMS

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***Abstract* —** Orthogonal frequency division multiplexing (OFDM) signals have an issue of high Peak-to-Average power ratio (PAPR) that is a noteworthy limitation in OFDM communication. High PAPR, which causes distortion when OFDM signal enters into a nonlinear high power amplifier and reduces the system efficiency. We have different techniques for reducing the PAPR, Partial Transmit Sequence (PTS) is a good technique among them. A cyclic shift sequences (CSS) technique is developed using PTS method to enhance the performance in decreasing the PAPR. In CSS scheme we can reduce PAPR by careful selection of shift value (SV) sets. We can reduce PAPR further by the utilization of a new Companding transform known as piece wise linear companding. In this letter, companding technique is used in addition to CSS scheme to accomplish great outcome in reducing PAPR. If the peak amplitude of the signal to be companded and scale of the linear transform are carefully designed the proposed system can have effective reduction in PAPR.

***Keywords*— Orthogonal Frequency Division Multiplexing (OFDM), Peak-to-Average Power Ratio (PAPR), Partial Transmit Sequence (PTS), Cyclic Shift Sequences (CSS), Complementary Cumulative distribution Function (CCDF), Piecewise Linear Companding Transform.**

I.INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is one type of multicarrier modulation method utilizing the orthogonality of subcarriers. Multi carrier modulation is a thought of parallel transmission having several parallel channels between the transmitter and receiver. In OFDM systems the transmitting signals suffers because of high PAPR. The OFDM have applications in WiMAX, DVB/DAB and 4G wireless systems. OFDM has an issue of high PAPR as in many other multicarrier techniques. So it is very expensive to implement. Over the last few decades, various schemes to decrease the PAPR of OFDM signal sequences have been proposed such as clipping, coding, selected mapping (SLM) [2], partial transmit sequence (PTS) [3],[4] and active constellation extension (ACE) [5].

There is a statistical improvement in PAPR and no occurrence of signal distortion in OFDM signals in PTS scheme similar to SLM scheme. PTS scheme undergoes an isolation of input symbol sequences to have various separate input symbol sub-sequences. Later all input symbol sub-sequences are applied with Inverse Fast Fourier transform (IFFT), the resultant sub-sequences of OFDM signal are multiplied with a set of rotation factors and combined together. Later on the sequence of OFDM signal with least PAPR is transmitted which is acquired by the calculation of PAPR for each resultant sequence. In every perspective the cyclic shift sequence (CSS) technique is better compared to the PTS technique. At first, its performance in decreasing the PAPR is good compared to the PTS scheme [4]. Even though analysis on the Class-III SLM technique was done by authors in [2], the approach can be seen a s a mixing of the PTS and CSS schemes, here the OFDM signal subsequences used are both cyclic shifted and multiplied by a rotation factor as in [7].

The CSS scheme will show a better performance in decreasing PAPR compared to PTS scheme even though the Complexity is same for both in the case of computations (utilizing the sub-blocks in equal number) and using the similar number of sequences of the alternative OFDM signal. In this letter, to raise the performance of CSS technique in decreasing PAPR we examine how to choose good Shift Value (SV) sets [1] and companding of peak amplitude. We present a few criteria for the selection of good SV sets by the consideration of autocorrelation function (ACF) of subsequences of the OFDM signal, and then use the simulations to check its validness.

Here in this paper Section2 gives the explanation for Preliminaries. Section3 gives the explanation regarding how to choose shift value (SV) sets for the CSS technique. Section4 gives description for the proposed scheme. Section5 provides the information regarding the simulation results and finally Section6 explains Conclusion.

II. PRELIMINARIES

2.1. OFDM System and PAPR

In an OFDM system, the time domain OFDM signal sequence generated using IFFT is given as

$$x(n) = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} X(K)e^{j\frac{2\pi kn}{N}} \qquad (1)$$

Where N is the number of subcarriers, $X = \{X(0), X(1), \dots\dots, X(N-1)\}$ is a frequency domain sequence of input symbol, and

$x = \{x(0), x(1), \dots\dots, x(N-1)\}$ is a time domain OFDM signal sequence.

The definition for the PAPR for the sequence of OFDM signal x is

$$PAPR = \frac{max_{0\le N}|x(n)|^2}{E[|x(n)|^2]} \quad (2)$$

Where $E[\bullet]$ indicates the expectation.

2.2. Cyclic Shifted Sequences (CSS)

The block diagram for CSS technique to test K alternative OFDM signal sequences is shown in figure 1. In the CSS technique, by using certain partitioning method X is separated into P disjoint sub-blocks, input symbol subsequences $X_1, X_2, \dots \dots \dots X_P$. Then IFFT transforms the P frequency domain sub blocks into P time domain OFDM signal subsequences $x_1, x_2, \dots \dots \dots . x_P$, where

$$x_p = \{x_p(0), x_p(1), \dots \dots \dots x_p(L-1)\}, 1 \le p \le P.$$

Simply, we assumed that L and P are integers of power of two. Next the resultant signals are shifted cyclically and summed together to have a k-th (1≤ k ≤ K) alternative OFDM signal sequence as

$$x^k = \sum_{p=1}^{P} x_p^k \quad (3)$$

Where x_p^k is the signal which gone through a leftward cyclical shift from x_p by an integer τ_p^k $(1 \le p \le P)$. That is,

$$x_p^k = \{x_p(\tau_p^k),\ x_p(\tau_p^k+1) \dots \dots . x_p(L-1), x_p(0) \dots \dots x_p(\tau_p^k - 1)\} \quad (4)$$

As the SLM or PTS methods, the signal with minimum PAPR, is selected by extensive search for transmission with $[log_2 K]$ bits side information. The signal is recovered at the receiver by means of some additional techniques using the side information [3]. Similar to the PTS technique, the CSS technique also uses three partitioning methods, i.e., random, adjacent, and interleaved partitioning methods.

Among them the random partitioning method shows the best performance in reducing the PAPR while the interleaved partitioning method renders poor performance in reducing PAPR but it needs the lowest computational complexity. It is not sensible to use adjacent partitioning method practically for the reason of its much complex computations similar to random partitioning but its Performance in decreasing PAPR is worse compared to random partitioning case. Here in this letter, we examine all the three partitioning methods.

III. DESIRABLE SHIFT VALUE SETS

The main aim of the CSS technique is to decrease the probability of PAPR above some threshold level instead of reducing the PAPR in each sequence of alternative OFDM signal, we have K SV sets which makes the sequences of alternative OFDM signal to have good performance as they are statistically independent.

3.1. Without considering the correlation in the sequence of OFDM signal

In general, we have no mutually independent components of OFDM signal subsequence that will be discussed in 3.2. However, at present simply assume that components in OFDM signal subsequences are mutually independent. That is, given as

$$E[x_{p1}(n1) \bullet \{x_{p2}(n_2)\}^*] = \begin{cases} \sigma^2, & p_1 = p_2 and\ n_1 = n_2 \\ 0, & otherwise \end{cases} \quad (5)$$

Here σ^2 gives the OFDM signal subsequence component power and $\{\bullet\}^*$ indicates a complex conjugate.

Criterion 1: If we have K SV sets; For each (i , j) pair in a total of the K SV sets (i ≠ j), the condition that should be satisfied by the pair is, for all p's the relative distances $\tau_p^i - \tau_p^j$ mod L should be distinct from each other.

Here the point to note down is, the Criterion 1 is true only when the components of all alternative OFDM signal sub-sequences are mutually independent. But there is no mutual independency for the subsequence components of the OFDM signal due to the fact that the corresponding input symbol sub-sequences in frequency domain have L − L/P zeros and it is well known that if the components pass through the IFFT, the output components are independent if the input components are independent. However, L − L/P zeros says that the input components are correlated, which makes the output components dependent.

3.2. ACF for the sub-sequences of the OFDM signal

Let p-th sub-sequence OFDM signal x_p have the discrete power spectrum as S_p, namely,

$$S_p = \{p(0), p(1), \dots \dots . . p(L-1)\} \quad (6)$$

Where $p(k) = E[|X_p(k)|^2]$, and the value for p(k) is zero or one. This is because of the assumption that all subcarriers will have equal modulation order and normalization is done to make the average power as one. For example, if the interleaved partition is used, S1 = {10101010} and S2 = {01010101} when L = 8 and P = 2.

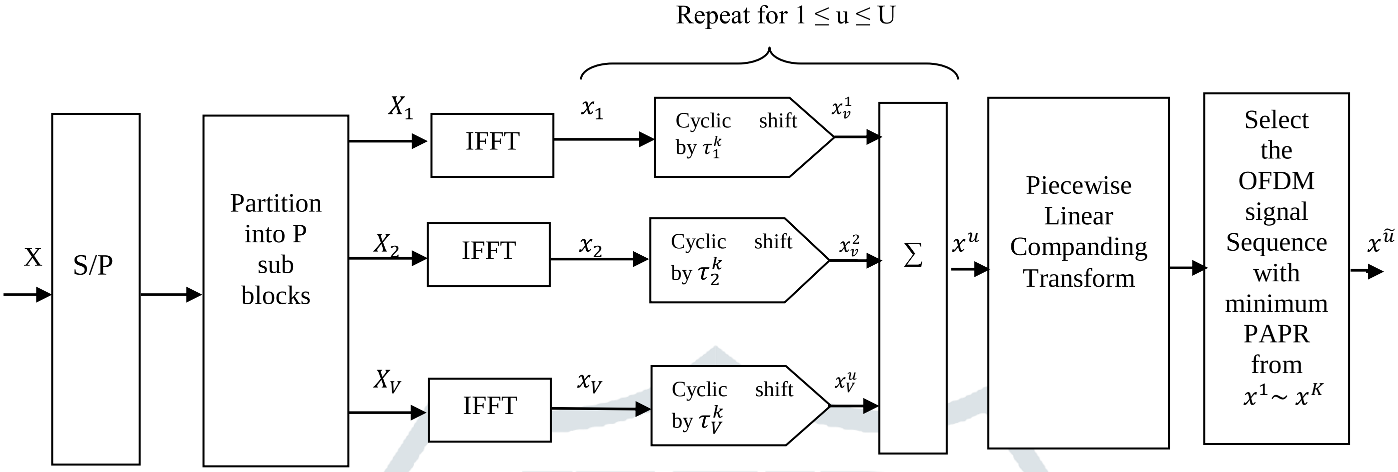

Figure1. Block diagram of the companding transform on Cyclic Shift sequence scheme.

1) Interleaved Partition case: In this circumstance, S_p is given by P interval impulse train. Now, the ACF also transforms into the impulse train.

$$\left|R_{x_p}(m)\right| = \begin{cases} \frac{\sqrt{L}}{P} & if\ m = 0\ mod\ \frac{L}{P} \\ 0 & otherwise \end{cases} \quad (7)$$

2) Adjacent Partition case: In this circumstance, S_p is given as L/P width rectangular function. Then the ACF becomes the function as

$$\left|R_{x_p}(m)\right| = \begin{cases} \frac{\sqrt{L}}{P} & if\ m = 0 \\ \frac{\sin(m\pi/P)}{\sqrt{L}\sin(m\pi/L)} & if\ m \neq 0 \end{cases} \quad (8)$$

3)Random Partition case: In this circumstance, S_p can be considered as a binary pseudo random sequence. Now the shape of ACF is identical to the delta function, here the components are near to zero except at m = 0.

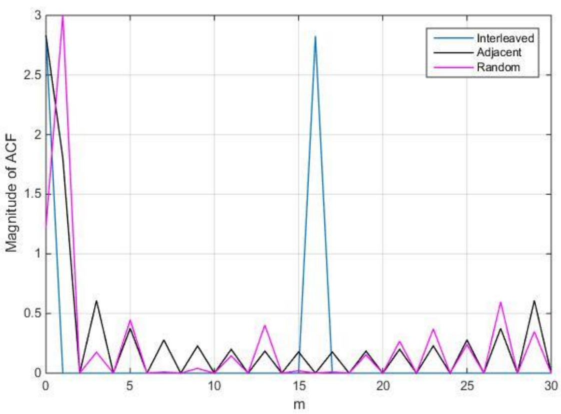

Figure2. Magnitudes of ACFs for different partition cases.

The magnitudes of ACFs related to the given power spectrum were shown in figure2. When L = 32 and P = 2; S_1 = {1010 ⋯⋯ 1010} for an interleaved partition; S_1 = {11 ⋯ 1100 ⋯ 00} for an adjacent partition; S_1= {10010110011111000110111010100000} for a random partition, have a length of 31 and it is a one zero padded m-sequence; clearly, S_2 is a complement of S_1 in each partition case, and the shapes of $|R_{x_p}(m)|$ for p=1 and p=2 are same.

3.3. Desired Shift Value Sets by Considering the ACF for Subsequences of OFDM Signal

By considering the ACF of subsequence of the OFDM signal we examine the desired SV sets for three partitioning cases.

1) For Random Partition: Here criterion1 is valid as the ACF shape is identical to a delta function.

2) For Interleaved Partition: ACF in (7) is an impulse train which says that OFDM signal subsequence components are associated with one another as

$$E\left[x_{p1}(n_1)\cdot\{x_{p2}(n_2)\}^*\right] = \begin{cases} \sigma^2, p_1 = p_2\ and\ n_1 = n_2 = mod\ \frac{L}{P} \\ 0,\ otherwise \end{cases} \quad (9)$$

Therefore, Criterion 1 has to be slightly modified as follows.

Criterion 2: If we have K SV sets; In these K SV sets, in each (i, j) pair (i ≠ j), the condition that should be satisfied by the pair is, for all p's the relative distances $\tau_p^i - \tau_p^j$ mod L/P are distinct from one pair to another pair.

3) For Adjacent Partition: Similar to Criterion 1 and Criterion 2, we can have a derivation for the optimal condition for K SV sets here also. But it is very complex and not as simple as done in the previous cases. So, we give a rough criterion for the adjacent partition case based on the rough interpretation.

Criterion 3: If we have K SV sets; In these K SV sets each (i, j) pair (i ≠ j), the condition that should be satisfied by the pair is, the relative distances $\tau_p^i - \tau_p^j$ mod L are distinct from each other for all p's. Furthermore, the mutual differences of the P relative distances ($\tau_1^i - \tau_1^j, \tau_2^i - \tau_2^j, \ldots \ldots . \tau_p^i - \tau_p^j$ mod L) should be as close to L/2 as possible.

IV. LINEAR COMPANING TRANSFORM

Here we proposed a new Piecewise linear companding transform. Depending on the theoretical analysis the parameters of the transform are designed carefully. Peak amplitude A_c is used for companding the original signal x_n, for signals having amplitudes above A_c the companding scheme proposed is shown in Fig.3 clips the signals to have a reduction in the peak power and to have power compensation it transforms the signals linearly having amplitudes near to A_c. Then, the companding function for the companding scheme proposed here is

$$h(x) = \begin{cases} x & |x| \le A_i \\ mx + (1-m)A_c & A_i < |x| \le A_c \\ sgn(x)A_c & |x| > A_c \end{cases} \quad (10)$$

Here $sgn\ (x)$ is the $signum$ function.

Consequently, at the receiver the decompanding signal is given as

$$h^{-1}(x) = \begin{cases} x & |x| \le A_i \\ (x-(1-m)A_c)/m & (1-m)A_c < |x| \le A_c \\ sgn(x)A_c & |x| > A_c \end{cases} \quad (11)$$

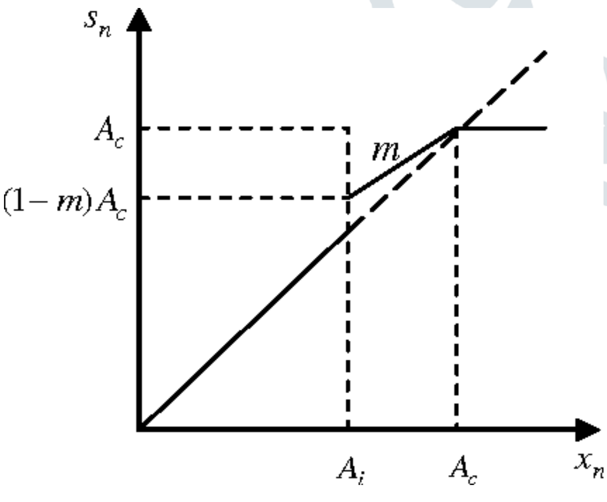

Figure 3. Linear companding transform.

Companding transform which is the proposed method can be specified by the parameters A_c ,A_i and k. The companded signals have highest amplitude as A_c. Average signal power is Kept as constant so, from (2) A_c determines the PAPR value theoretically which is going to be achieved in the proposed technique**.** From the value of PAPR preset theoretically, A_c can be resolved as $A_c = \sigma_x 10^{PAPR_{preset}/20}$. We can obtain parameters A_i and k from A_c, by solving in the paper [8].

V. SIMULATION RESULTS

5.1. Comparison of partition techniques

Prior verifying our proposed criteria for the Piecewise linear companding on CSS scheme. The CSS technique uses the well-designed K SV sets satisfying Criterion 1 and Criterion 2. Now, to verify the above proposed criteria for the CSS technique are valid, we construct the K SV sets in two different ways. That is, the solid lines in Fig.4, show the performance in reduction of PAPR reduction for the case when the K SV sets satisfy the above criteria well. Next, the dotted lines in Figure 4, show the performance in reduction of PAPR for the case that does not obey the criteria. In the results, we consider L = 128, K = 4, and P = 4 in common. The 16-QAM is used for all following simulations.

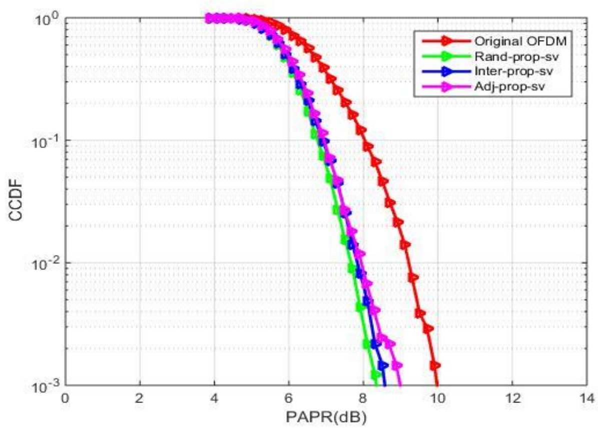

Figure 4. Comparison of the performance of the CSS technique in reducing PAPR for three partition cases that is random, interleaved, and adjacent partition cases for selected SV sets when L = 128, K = 4, and P = 4 according to the used SV sets.

5.2. Optimality of the criteria

It is hard to match the performance of PAPR reduction in the case using best SV sets to the cases using ALL available SV sets through simulation because there are too many possible SV sets. Instead, in Fig. 5, we drew curve using randomly generated SV sets, where L=64, K=4, and P=4 are used. Also, we drew the curve using best SV sets satisfying Criterion 2.

The SV sets $\bar{\tau}^1$ = {1,1,1,1), $\bar{\tau}^2$ = {1,2,3,4), $\bar{\tau}^3$ = {1,3,5,7), $\bar{\tau}^4$ = (1,4,7,10) are used as the best SV sets. In Fig. 5, we can observe the best results in the reduction of PAPR using these SV sets.

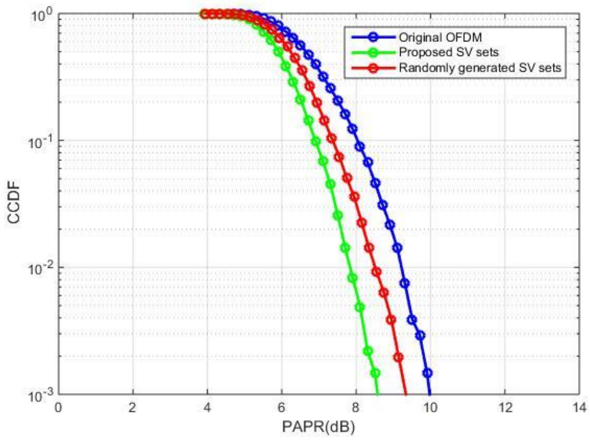

Figure 5. The optimality of the proposed SV sets when L=128, K=4, interleaved partition, and P=4 are used.

5.3. Proposed method

Prior to verifying our proposed method on CSS scheme, the comparison of performance between the CSS scheme without applying piecewise linear companding transform and CSS scheme with piecewise linear companding transform in reducing PAPR is shown in Figure 6, we can examine that the performance in reducing PAPR is improved by 3.61 dB after applying piecewise linear companding transform on the CSS technique.

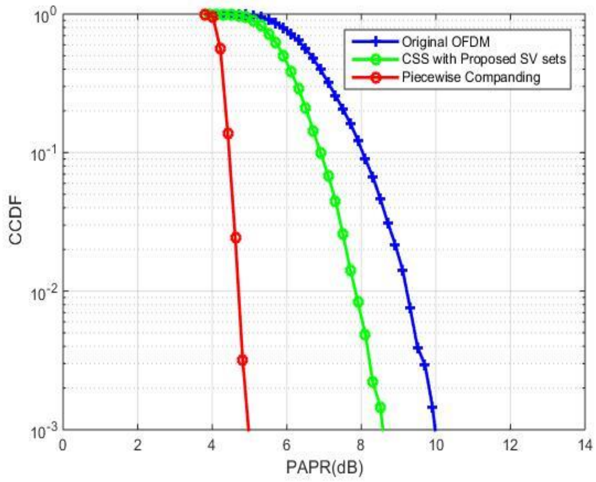

Figure 6. Comparing the performance of CSS with piecewise linear companding and CSS without piecewise linear companding in reducing PAPR for L=128, P=4 and K=4 are used.

VI. CONCLUSION

The CSS scheme is the very popular and promising method in reducing PAPR which is evolved from the PTS scheme. Here in this paper, the criteria to choose good SV sets and a piecewise linear companding technique were proposed, which can guarantee the optimal performance in reducing PAPR in CSS scheme. The criterion are proposed by considering the ACF of the subsequence of OFDM signal for three different partitioning cases, which are random, interleaved, and adjacent partitioning cases. In the simulation results, compared to the SV sets which are not properly designed and having a PAPR of 9.34 dB, the SV sets satisfying the proposed criteria shows good performance with a PAPR of 8.58 dB and by the use of companding in the CSS technique the PAPR is reduced to 4.98 dB with a reduction of 3.6 dB. So the performance is improved with CSS scheme using piecewise linear companding compared to CSS scheme without companding.

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