

On Inverse of Heptagonal Fuzzy Number Matrices

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Abstract

Linear system have important applications to many branches of social science, physics and engineering. This paper discuss about fuzzy linear systems with Heptagonal Fuzzy numbers. A new procedure namely matrix inversion method is proposed for solving fuzzy linear system (FLS) of equations. Finally, the method is illustrated by solving a numerical examples.

Keywords

Fuzzy number, Heptagonal fuzzy number(HPFN), Heptagonal fuzzy matrix(HPFM), Fuzzy Linear System (FLS), Inverse of Heptagonal fuzzy matrix.

I. INTRODUCTION

System of Linear equations have many branches for studying and solved a large proportion of this problems in many topics of applied mathematics. System of simultaneous linear equations play of major role in different areas such as matrices, operational research, statistics, computer science engineering and social science. Usually in many applications, some of parameters in the problems are represented by imprecise number instead of crisp number and hence it is important to develop the mathematical models and Numerical procedures to handle the general fuzzy linear system and solve them.

The concept of fuzzy numbers and arithmetic operations are first introduced by Zadeh [13] and Dubois and prade [9]. When the inverse of a square and non singular matrix whose entries are real numbers unconcerned in one of the computing methods of linear equation system composing of the product $n \times n$ matrices.

In [7] Friedman et al proposed a general model for solving a $n \times n$ fuzzy linear system, whose coefficient matrix is crisp and right hand side column is an arbitrary fuzzy number vectors. To find the solution of the original $n \times n$ fuzzy linear system (FLS) is replaced with $2n \times 2n$ crisp linear system. In [10] Buckley, Y. Qu [11] extended several methods to obtain

linear system of interval equation. The exact solution FLS can be found by solving these interval systems proposed fuzzy linear system, whose all parameters are triangular fuzzy numbers.

In [2] Abbasbandy and Jofarin proposed an approximation of the unique solution in fuzzy system of linear equation, applied by the steepest descent method. Abbasbandy.et.al [1] proposed LU decomposition method for solving fuzzy system of linear equation. Muruganndam and Abdul Razak [9] studied matrix inversion method for solving fully fuzzy linear systems with triangular fuzzy numbers, Basaran [8] proposed calculating fuzzy inverse matrix using fuzzy linear equation system. Dinagar and Latha [10] proposed invertible on Type-2 Triangular fuzzy matrices. Dinagar and Harinarayanan [11] proposed

arithmetic operations on hexagonal fuzzy number using α -cut method. In [12] D. Stephen Dinagar and U. Harinarayan, On inverse of Hexagonal fuzzy number matrices.

This paper is organized as follows: In section 2, We give some basic definitions on fuzzy number. In section 3, we recall our proposed definitions of Heptagonal Fuzzy Number (HPFN) and its arithmetic operation. Section 4, we have reviewed the definition of Heptagonal Fuzzy Matrix (HPFM) and its operations are presented with the aid of Heptagonal Fuzzy Matrices (HPFMs). In section 5, we define of fuzzy linear system of equations by matrix inversion method. In section 6, Numerical examples is given for computing the solution of FLS is proposed. Section 7, the conclusion is also included.

II. PRELIMINARIES

Definition 2.1: (Fuzzy Set)

A Fuzzy Set is characterized by a membership function mapping the elements of a domain space or universe of discourse X to the unit interval $[0, 1]$. A fuzzy set A in a universe of discourse X is defined as the following set of pairs

$$A = \{(x, \mu_A(x)) ; x \in X\}$$

Here $\mu_A : X \rightarrow [0,1]$ is mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ the fuzzy set. These membership grades are often represented by real ranging from $[0,1]$.

Definition 2.2: (Convex fuzzy set)

A fuzzy set $A = \{(x, \mu_A(x))\} \subseteq X$ is called convex set in all A_α are convex set (i.e) for every element $x_1 \in A_\alpha$ and $x_2 \in A_\alpha$ for every $\alpha \in [0,1]$. $\lambda x_1 + (1-\lambda)x_2 \in A_\alpha$ for all $\lambda \in [0,1]$. Otherwise the fuzzy set is called the non-convex fuzzy set.

Definition 2.3: (Fuzzy number)

A fuzzy set \tilde{A} , defined on the set of real numbers R is said to be fuzzy number if its membership function has the following characteristics

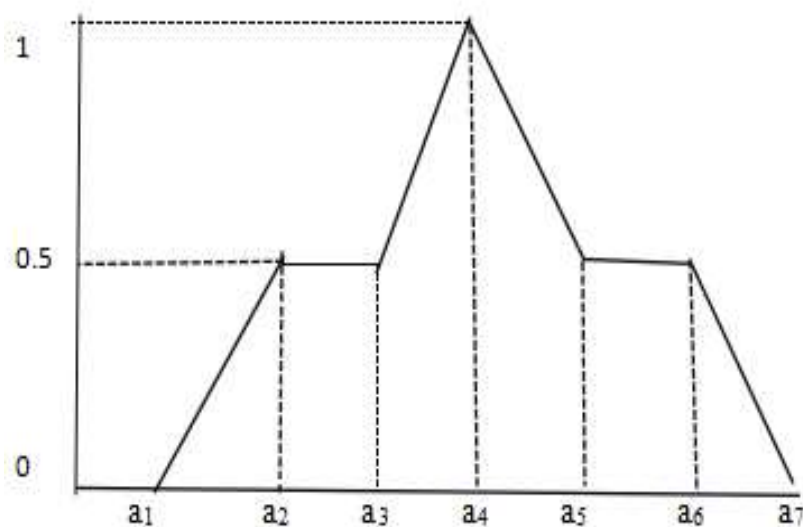
1. \tilde{A} is normal.
2. \tilde{A} is convex set.
3. The support of \tilde{A} is closed and bounded then \tilde{A} is called fuzzy number.

III. HEPTAGONAL FUZZY NUMBER (HPFN)

Definition 3.1: (Heptagonal Fuzzy Number)

A Fuzzy number with seven points as follows $\tilde{A}_{hp} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ is called Heptagonal fuzzy number, where $(a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6 \leq a_7)$. It is denoted by \tilde{A}_{hp} , which are real number satisfying $a_2 - a_1 \leq a_4 - a_3$ and $a_5 - a_4 \leq a_7 - a_6$ and its membership function $\mu_{\tilde{A}_{hp}}(x)$ is given

$$\mu_{\tilde{A}_{hp}}(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \frac{1}{2} \left(\frac{x-a_1}{a_2-a_1} \right) & \text{for } a_1 \leq x \leq a_2 \\ \frac{1}{2} & \text{for } a_2 \leq x \leq a_3 \\ \frac{1}{2} + \frac{1}{2} \left(\frac{x-a_3}{a_4-a_3} \right) & \text{for } a_3 \leq x \leq a_4 \\ \frac{1}{2} + \frac{1}{2} \left(\frac{a_5-x}{a_5-a_4} \right) & \text{for } a_4 \leq x \leq a_5 \\ \frac{1}{2} & \text{for } a_5 \leq x \leq a_6 \\ \frac{1}{2} \left(\frac{a_7-x}{a_7-a_6} \right) & \text{for } a_6 \leq x \leq a_7 \\ 0 & \text{for } x \geq a_7 \end{cases}$$



Heptagonal Fuzzy Number



3.2 Arithmetic Operations on Heptagonal Fuzzy Numbers (HPFNs)

The arithmetic operations between heptagonal fuzzy numbers (HPFNs) are proposed given below. Let $\tilde{A}_{hp} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ and $\tilde{B}_{hp} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7)$ be two Heptagonal fuzzy numbers then,

(i) Addition :

$$\tilde{A}_{hp}(+) \tilde{B}_{hp} = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5, a_6+b_6, a_7+b_7)$$

(ii) Subtraction:

$$\tilde{A}_{hp}(-) \tilde{B}_{hp} = (a_1-b_7, a_2-b_6, a_3-b_5, a_4-b_4, a_5-b_3, a_6-b_2, a_7-b_1)$$

(iii) Multiplication:

$$\tilde{A}_{hp}(\times)\tilde{B}_{hp} = \left(\frac{a_1}{7}\sigma_b, \frac{a_2}{7}\sigma_b, \frac{a_3}{7}\sigma_b, \frac{a_4}{7}\sigma_b, \frac{a_5}{7}\sigma_b, \frac{a_6}{7}\sigma_b, \frac{a_7}{7}\sigma_b\right)$$

Where $\sigma_b = (b_1+b_2+b_3+b_4+b_5+b_6+b_7)$

(iv) Division:

$$\tilde{A}_{hp}(\div)\tilde{B}_{hp} = \left(\frac{7a_1}{\sigma_b}, \frac{7a_2}{\sigma_b}, \frac{7a_3}{\sigma_b}, \frac{7a_4}{\sigma_b}, \frac{7a_5}{\sigma_b}, \frac{7a_6}{\sigma_b}, \frac{7a_7}{\sigma_b}\right)$$

Where $\sigma_b = (b_1+b_2+b_3+b_4+b_5+b_6+b_7)$

(v) Scalar Multiplication:

If $k \neq 0$ is scalar k A_{hp} is defined as

$$k\tilde{A}_{hp} = \begin{cases} (ka_1, ka_2, ka_3, ka_4, ka_5, ka_6, ka_7) & \text{if } k \geq 0 \\ (ka_7, ka_6, ka_5, ka_4, ka_3, ka_2, ka_1) & \text{if } k < 0 \end{cases}$$

Definition 3.2.1:(Ranking Function)

We define a ranking function $\check{R}: F(R) \rightarrow R$ which maps each fuzzy numbers to real line $F(R)$ represented the set of all heptagonal fuzzy number. If R be any linear ranking functions

$$\check{R}(\tilde{A}_{hp}) = \left[\frac{a_1+a_2+a_3+a_4+a_5+a_6+a_7}{7} \right]$$

Also we define the order on $F(R)$ by

$$\check{R}(\tilde{A}_{hp}) \geq \check{R}(\tilde{B}_{hp}) \text{ if and only if } \tilde{A}_{hp} \geq \tilde{B}_{hp}$$

$$\check{R}(\tilde{A}_{hp}) \leq \check{R}(\tilde{B}_{hp}) \text{ if and only if } \tilde{A}_{hp} \leq \tilde{B}_{hp}$$

$$\check{R}(\tilde{A}_{hp}) = \check{R}(\tilde{B}_{hp}) \text{ if and only if } \tilde{A}_{hp} = \tilde{B}_{hp}$$

Definition 3.2.2:(Zero Heptagonal Fuzzy Number)

If $\tilde{A}_{hp} = (0, 0, 0, 0, 0, 0, 0)$ then \tilde{A}_{hp} is said to be zero heptagonal fuzzy number. It is denoted by 0 .

Definition 3.2.3:(Zero-Equivalent Heptagonal Fuzzy Number)

A Heptagonal fuzzy number \tilde{A}_{hp} is said to be zero-equivalent heptagonal fuzzy number if $\check{R}(\tilde{A}_{hp}) = 0$. It is denoted by $\tilde{0}$.

Definition 3.2.4: (Unit Heptagonal Fuzzy Number)

If $\tilde{A}_{hp} = (1, 1, 1, 1, 1, 1, 1)$ then \tilde{A}_{hp} is said to be unit heptagonal fuzzy number. It is denoted by 1 .

Definition 3.2.5:(Unit- equivalent Heptagonal fuzzy number)

A Heptagonal fuzzy number \tilde{A}_{hp} said to be unit-equivalent heptagonal fuzzy number if $\check{R}(\tilde{A}_{hp}) = 1$. It is denoted by $\tilde{1}$.

Definition 3.2.6:(Inverse Heptagonal Fuzzy Number)

If \tilde{a}_{hp} is heptagonal fuzzy number and $\tilde{a}_{hp} \neq 0$ then we define $\tilde{a}_{hp}^{-1} = \frac{\tilde{1}}{\tilde{a}}$

IV. HEPTAGONAL FUZZY MATRICES (HPFMs)

In this section, we proposed new definition of Heptagonal Fuzzy Matrix and corresponding its matrix operations

Definition 4.1: (Heptagonal Fuzzy Matrix)

A fuzzy matrix $\tilde{A} = (a_{hpj})_{m \times n}$ of order $m \times n$ is called Heptagonal fuzzy matrix if the elements of the matrix are Heptagonal fuzzy numbers, i.e., of the form $(a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4}, a_{ij5}, a_{ij6}, a_{ij7})$.

4.2: Operations on Heptagonal Fuzzy Matrices (HPFMs)

Through classical matrix algebra, we achieve some algebraic operations of HPFM. Let $\hat{A} = (\tilde{a}_{hpj})_{m \times n}$ and $\hat{B} = (\tilde{b}_{hpj})_{m \times n}$ be two HPFMs of same order. Then we have the following

1. $\hat{A} + \hat{B} = (\tilde{a}_{hpj} + \tilde{b}_{hpj})$
2. $\hat{A} - \hat{B} = (\tilde{a}_{hpj} - \tilde{b}_{hpj})$
3. For $\hat{A} = (\tilde{a}_{hpj})_{m \times n}$ and $\hat{B} = (\tilde{b}_{hpj})_{n \times k}$ then $\hat{A}\hat{B} = (\tilde{c}_{hpj})_{m \times k}$ where $(\tilde{c}_{hpj})_{m \times k} = \sum_{p=1}^n \tilde{a}_{hpi} \tilde{b}_{ipj}$, $i=1, 2, \dots, m$ and $j=1, 2, \dots, k$.
4. \hat{A}^T or $\hat{A}' = (\tilde{a}_{hpi})$
5. $k\hat{A} = (k\tilde{a}_{hpj})$, where k is scalar.

Definition 4.3: (Zero Heptagonal Fuzzy Matrix)

A Heptagonal Fuzzy Matrix (HPFM) is said to be a zero HPFM if all its entries are 0 and it is denoted by \hat{O} .

Definition 4.5: (Zero Heptagonal Fuzzy Matrix)

The square of HPFM is said to be unit HPFM if the diagonal elements are 1 and the rest of elements are 0. It is denoted by \hat{I}

V. MATRIX INVERSION METHOD FOR FUZZY LINEAR SYSTEM (FLS) USING HEPTAGONAL FUZZY NUMBER

In this section, we define the concept of fuzzy linear system is justify in matrix inversion method with the aid of heptagonal fuzzy numbers and the relevant definitions are recalled in nature.

Consider the system of n fuzzy linear non-homogeneous HPFN equations in unknown HPFN vectors $x_{hp1}, x_{hp2}, \dots, x_{hpn}$.

Where $\tilde{a}_{hpj}, x_{hpi}, b_{hpi}$ heptagonal fuzzy numbers.

$$\begin{matrix} \tilde{a}_{hp11}x_{hp1} \oplus \tilde{a}_{hp12}x_{hp2} \oplus \dots \oplus \tilde{a}_{hp1n}x_{hpn} = b_{hp1} \\ \tilde{a}_{hp21}x_{hp1} \oplus \tilde{a}_{hp22}x_{hp2} \oplus \dots \oplus \tilde{a}_{hp2n}x_{hpn} = b_{hp2} \\ \vdots \\ \tilde{a}_{hpn1}x_{hp1} \oplus \tilde{a}_{hpn2}x_{hp2} \oplus \dots \oplus \tilde{a}_{hpn n}x_{hpn} = b_{hpn} \end{matrix}$$

The above linear system is represented in the form is given by

$$\hat{A}x_{hp} = b_{hp}$$

Where $\hat{A} = (\tilde{a}_{hpj}), 1 \leq i, j \leq n$ is n times heptagonal fuzzy matrix and $\tilde{a}_{hpj} \in F(R)$ and $(x_{hpi}, b_{hpi}) \in F(R)$, for all $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, n$. This system is

called fuzzy linear system.

If the coefficient Heptagonal fuzzy matrix \hat{A} is non singular then \hat{A}^{-1} we get

$$\hat{A}^{-1}(\hat{A}x_{hp}) = \hat{A}^{-1} b_{hp}$$

$$(\hat{A}^{-1}\hat{A}) x_{hp} = \hat{A}^{-1} b_{hp}$$

$$x_{hp} = \hat{A}^{-1} b_{hp}$$

The solution of FLS will be represented by $x_{hp} = \hat{A}^{-1} b_{hp}$

Definition 5.1: (Determinant of HPFM)

The determinant of $n \times n$ HPFM $\hat{A} = (\tilde{a}_{hpj})$ is denoted by $|\hat{A}|$ or $\det(\hat{A})$ and is defined as follows

$$|\hat{A}| = \sum_{q \in S_n} \text{sgn } q \prod_{i=1}^n \tilde{a}_{hpi}$$

$$= \sum_{q \in S_n} \text{sgn } q \tilde{a}_{hp1q(1)} \tilde{a}_{hp2q(2)} \tilde{a}_{hp3q(3)} \dots \tilde{a}_{hpnq(n)}$$

Where $\tilde{a}_{hpq(i)} = (a_{iq(i)1}, a_{iq(i)2}, a_{iq(i)3}, a_{iq(i)4}, a_{iq(i)5}, a_{iq(i)6}, a_{iq(i)7})$ are heptagonal fuzzy number (HPFN) and s_n denotes the symmetric group of all permutations of the indices $\{1,2,\dots,n\}$ and $\text{sgn } q = 1$ or -1 according as the permutation

$$q = \begin{pmatrix} 1 & 2 & \dots & n \\ q(1) & q(2) & \dots & q(n) \end{pmatrix}$$

is even or odd respectively.

Definition 5.2: (Adjoint of HPFM)

Let $\hat{A} = (\tilde{a}_{hpj})$ be a square HPFM of order n . Find the cofactor \tilde{A}_{hpj} of ever element \tilde{a}_{hpj} in \hat{A} and replace every \tilde{a}_{hpj} by its cofactor \tilde{A}_{hpj} in \hat{A} and let it be \hat{B} . ie., $\hat{B} = (\tilde{A}_{hpj})$. Then the transpose of \hat{B} is called the adjoint or adjugate of \hat{A} and is denoted by $\text{adj}\hat{A}$. I.e., $\hat{B}^T = \tilde{A}_{hpj} = \text{adj}\hat{A}$

Definition 5.3: (Singular HPFM)

Let $\hat{A} = (\tilde{a}_{hpj})$ be as square HPFM of order n , then it is said to be singular HPFM if $|\hat{A}| = \tilde{0}$.

Definition 5.4: (Non-Singular HPFM)

Let $\hat{A} = (\tilde{a}_{hpj})$ be a square HPFM of order n , then it is said to be non-singular HPFM if $|\hat{A}| \neq \tilde{0}$.

Definition: 5.5: (Inverse of HPFM)

A non-singular HPFM $\hat{A} = (\tilde{a}_{hpj})$ of order n is said to be invertible if there exist a HPFM \hat{B} of order n such that $\hat{A}\hat{B} = \hat{I} = \hat{B}\hat{A}$.

Then \hat{B} is called the inverse of \hat{A} and is denoted by \hat{A}^{-1} . Thus $\hat{A}\hat{A}^{-1} = \hat{I} = \hat{A}^{-1}\hat{A}$. Also

$$\hat{A}^{-1} = \frac{1}{R(|\hat{A}|)} \text{adj}\hat{A}$$

VI. NUMERICAL EXAMPLE

In this section two example are given in order to illustrate the proposed method.

Example 6.1

Consider the following fuzzy liner system and solve by matrix inversion method.

$$\begin{aligned} (-3, 0, 1, 2, 3, 5, 6) x_{hp1} \oplus (-3, -1, 1, 0, 1, 2, 3, 5) x_{hp2} &= (-1, 0, 8, 9, 10, 11, 12) \\ (-4, 0, 3, 4, 5, 6, 7) x_{hp1} \oplus (-3, -2, 0, 3, 4, 5, 7) x_{hp2} &= (0, 10, 11, 12, 13, 14, 17) \end{aligned}$$

Solution

The given system may be written as

$$\begin{pmatrix} (-3, 0, 1, 2, 3, 5, 6) & (-3, -1, 1, 0, 1, 2, 3, 5) \\ (-4, 0, 3, 4, 5, 6, 7) & (-3, -2, 0, 3, 4, 5, 7) \end{pmatrix} \begin{pmatrix} x_{hp1} \\ x_{hp2} \end{pmatrix} = \begin{pmatrix} (-1, 0, 8, 9, 10, 11, 12) \\ (0, 10, 11, 12, 13, 14, 17) \end{pmatrix}$$

$$x_{hp} = \hat{A}^{-1} b_{hp}$$

$$\text{Now, } |\hat{A}| = (-21, -9, -4, 1, 6, 13, 21)$$

$$\text{Then } \check{R}(|\hat{A}|) = 1 \neq \tilde{0}$$

Since \hat{A} is non-singular, then \hat{A}^{-1} is exist

$$\hat{A}^{-1} = \frac{1}{\check{R}(|\hat{A}|)} \text{adj}\hat{A}$$

$$\text{adj}\hat{A} = \begin{pmatrix} (-3, -2, 0, 3, 4, 5, 7) & (-5, -3, -2, -1, 0, 1, 3) \\ (-7, -6, -5, -4, -3, 0, 4) & (-3, 0, 1, 2, 3, 5, 6) \end{pmatrix}$$

$$\hat{A}^{-1} = \begin{pmatrix} (-3, -2, 0, 3, 4, 5, 7) & (-5, -3, -2, -1, 0, 1, 3) \\ (-7, -6, -5, -4, -3, 0, 4) & (-3, 0, 1, 2, 3, 5, 6) \end{pmatrix}$$

The solution is $x_{hp} = \hat{A}^{-1} b_{hp}$

$$\begin{pmatrix} x_{hp1} \\ x_{hp2} \end{pmatrix} = \begin{pmatrix} (-76, -47, -22, 10, 28, 46, 82) \\ (-82, -42, -24, 6, 12, 55, 94) \end{pmatrix}$$

The solution is

$$x_{hp1} = (-76, -47, -22, 10, 28, 46, 82)$$

$$x_{hp2} = (-82, -42, -24, 6, 12, 55, 94)$$

Example 6.2

Consider the following fuzzy linear system and solve by matrix inversion method.

$$(-2, 2, 3, 4, 6, 7, 8) x_{hp1} \oplus (-2, 0, 1, 2, 3, 4, 6) x_{hp2} \oplus (-3, -2, 0, 1, 2, 3, 6) x_{hp3} = (0, 1, 3, 5, 7, 9, 10)$$

$$(-2, 0, 2, 3, 5, 6, 7) x_{hp2} \oplus (-2, 0, 1, 2, 3, 4, 6) x_{hp2} \oplus (-3, -2, 0, 1, 2, 3, 6) x_{hp3} = (-2, 0, 2, 3, 5, 6, 7)$$

$$(-3, -2, 0, 1, 2, 3, 6) x_{hp1} \oplus (-3, -2, 0, 1, 2, 3, 6) x_{hp2} \oplus (-3, -2, 0, 1, 2, 3, 6) x_{hp3} = (-2, 2, 1, 3, 7, 8, 9)$$

Solution

The given linear system may be written as

$$\begin{pmatrix} (-2, 2, 3, 4, 6, 7, 8) & (-2, 0, 1, 2, 3, 4, 6) & (-3, -2, 0, 1, 2, 3, 6) \\ (-2, 0, 2, 3, 5, 6, 7) & (-2, 0, 1, 2, 3, 4, 6) & (-3, -2, 0, 1, 2, 3, 6) \\ (-3, -2, 0, 1, 2, 3, 6) & (-3, -2, 0, 1, 2, 3, 6) & (-3, -2, 0, 1, 2, 3, 6) \end{pmatrix} \begin{pmatrix} x_{hp1} \\ x_{hp2} \\ x_{hp3} \end{pmatrix} = \begin{pmatrix} (0, 1, 3, 5, 7, 9, 10) \\ (-2, 0, 2, 3, 5, 6, 7) \\ (-2, 2, 1, 3, 7, 8, 9) \end{pmatrix}$$

$$x_{hp} = \hat{A}^{-1}b_{hp}$$

$$\text{Now, } |\hat{A}| = (-17, -8, -3, 1, 6, 10, 18)$$

$$\text{Then } \check{R}(|\hat{A}|) = 1 \neq \tilde{0}$$

Since \hat{A} is non-singular, then \hat{A}^{-1} is exist

$$\hat{A}^{-1} = \frac{1}{\check{R}(|\hat{A}|)} \text{adj}\hat{A}$$

$$\text{adj}\hat{A} = \begin{pmatrix} (-8, -3, -1, 1, 3, 6, 9) & (-9, -6, -3, -1, 1, 3, 8) & (-14, -6, -3, 0, 3, 8, 12) \\ (-10, -8, -5, -2, 0, 3, 8) & (-8, -1, 1, 3, 6, 9, 11) & (-17, -13, -6, -1, 3, 7, 20) \\ (-8, -4, -1, 1, 4, 6, 9) & (-10, -7, -5, -2, 0, 2, 8) & (-22, -8, -3, 2, 9, 14, 22) \end{pmatrix}$$

$$\hat{A}^{-1} = \begin{pmatrix} (-8, -3, -1, 1, 3, 6, 9) & (-9, -6, -3, -1, 1, 3, 8) & (-14, -6, -3, 0, 3, 8, 12) \\ (-10, -8, -5, -2, 0, 3, 8) & (-8, -1, 1, 3, 6, 9, 11) & (-17, -13, -6, -1, 3, 7, 20) \\ (-8, -4, -1, 1, 4, 6, 9) & (-10, -7, -5, -2, 0, 2, 8) & (-22, -8, -3, 2, 9, 14, 22) \end{pmatrix}$$

The solution is, $x_{hp} = \hat{A}^{-1}b_{hp}$

$$\begin{pmatrix} x_{hp1} \\ x_{hp2} \\ x_{hp3} \end{pmatrix} = \begin{pmatrix} (-123, -57, -26, 2, 30, 71, 117) \\ (-142, -95, -46, -5, 30, 70, 153) \\ (-158, -73, -32, 7, 56, 92, 157) \end{pmatrix}$$

The solution is

$$x_{hp1} = (-123, -57, -26, 2, 30, 71, 117)$$

$$x_{hp2} = (-142, -95, -46, -5, 30, 70, 153)$$

$$x_{hp3} = (-158, -73, -32, 7, 56, 92, 157)$$

VII. CONCLUSION

In this paper, we introduced fuzzy linear system obtained by matrix inversion method in the form of heptagonal fuzzy number. The method is illustrated with numerical examples. The notation of fuzzy linear system can be applying in Cramers rule and LU Decomposition method by this proposed method in future.

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