

# MAX- MIN MATRICES USING MERSENNE NUMBER

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**Abstract:** Let  $T$  be a finite number of multiple set of real numbers taken as increasing order of numbers.

The purpose of this article is to study the different properties of MIN matrix and MAX matrix of the set  $T$  with

$\min(x_i, x_j)$  and  $\max(x_i, x_j)$  as their  $(i, j)$  entries, respectively. We are going to do this by interpreting these matrices as Mersenne meet and join matrices and applying the determinant formulae and the inverse formulae for Mersenne MIN matrices and Mersenne MAX matrices.

**Keywords:** MIN matrix, MAX matrix, meet matrix, join matrix, Mersenne min matrix and Mersenne max matrix.

## 1 Introduction:

MIN and MAX matrices are simple-structured matrices that appear in many contexts in mathematics and statistics. As is pointed out in the next section, in some cases MIN matrices have an interpretation as covariance matrices of certain stochastic processes. Bhatia [1] shows that the MIN matrix  $[\min(i, j)]$  is infinitely divisible, and in [2] he gives a more comprehensive treatment to this subject. Moyé studies the covariance matrix of Brownian motion, which appears to be a certain MIN matrix. Motivated by Moyé's work, Neudecker, Trenkler and Liu [3] defined a more general matrix

$$A = \begin{bmatrix} a_1 & a_1 & a_1 & a_1 \\ a_1 & a_2 & a_2 & a_2 \\ a_1 & a_2 & a_3 & a_3 \\ & & \vdots & \\ & & \vdots & \\ & & \vdots & \\ a_1 & a_2 & a_3 & a_n \end{bmatrix}$$

( $a_i$  are real numbers for all  $i = 1, \dots, n$ ), and proposed the following problems:

- find a necessary and sufficient condition for  $A$  to be positive definite;
- find the determinant of  $A$ ;
- find the inverse of  $A$  when  $A$  is nonsingular.

Two years later Chu, Puntanen and Styan made use of elementary matrix methods and provided answers to the above questions.

Also in the field of pure mathematics MIN and MAX matrices have appeared in many contexts and by many authors. Probably the first such appearance can be found in the famous book by [4] Pólya and Szegő, where the reader is asked to calculate the determinant of the MIN matrix  $[\min(i, j)]$  and also the determinants of some of its generalizations (in fact, all these exercises can be found already in the original German version of the book published in 1925). Meet matrices were defined by Rajarama Bhat [5] for the first time and in this same article MIN matrices are considered as an example. Da Fonseca studies the eigenvalues of certain MIN and MAX matrices via their matrix inverses, and in bounds for the values of trigonometric functions are found by underestimating the smallest eigenvalue of a MIN matrix. Also the connection between generalized Fibonacci numbers and the characteristic polynomials of MIN and MAX matrices have been studied recently.

As we are going to see, there is a very natural and straight forward way to interpret MIN and MAX matrices as meet and join matrices, whose properties are well studied. On the other hand, because of the simple structure of MIN and MAX matrices it is easy to apply basically any result related to meet and join matrices to MIN and MAX matrices. At the same time we give some thoughts about how difficult it would be to verify these formulas by using only elementary linear algebra. The reader is also very welcome to amuse herself/himself by trying to answer the same question.

## 2 Preliminaries:

We begin by presenting the definition of MIN and MAX matrices.

Let  $T = \{x_1, x_2, x_3, \dots, x_n\}$  be a finite multiple set of real numbers, where  $x_1 \leq x_2 \leq \dots \leq x_n$  (in some cases, however, we need to assume that  $x_1 < x_2 < \dots < x_n$ ). The MIN matrix  $(T)_{\min}$  of the set  $T$  has  $\min(x_i, x_j)$  as its  $(i, j)$  entry, whereas the MAX matrix of the set  $T$  has  $\max(x_i, x_j)$  as its  $(i, j)$  entry and is denoted by  $[T]_{\max}$ . Both matrices are clearly square and symmetric and they may be written explicitly as

$$(T)_{\min} = \begin{bmatrix} x_1 & x_1 & x_1 & x_1 \\ x_1 & x_2 & x_2 & x_2 \\ x_1 & x_2 & x_3 & x_3 \\ \vdots & \vdots & \vdots & \vdots \\ x_1 & x_2 & x_3 & x_n \end{bmatrix} \text{ and } [T]_{\max} = \begin{bmatrix} x_1 & x_2 & x_3 & x_n \\ x_2 & x_2 & x_3 & x_n \\ x_3 & x_3 & x_3 & x_n \\ \vdots & \vdots & \vdots & \vdots \\ x_n & x_n & x_n & x_n \end{bmatrix}$$

**2.1 Remark:**

Here it is convenient to assume that the elements of T are listed in increasing order, since this assumption does not affect most of the basic properties of the matrices  $(T)_{\min}$  and  $[T]_{\max}$ . Rearranging the indexing of the elements of the set T corresponds to multiplying the matrices  $(T)_{\min}$  and  $[T]_{\max}$  from left by a certain permutation matrix Q and from right by the matrix  $Q^T$ . Properties like determinant and positive definiteness remain invariant in this operation.

An interesting special case of MIN matrices is obtained by setting  $T = \{1, 2, \dots, n\}$ . In this case we have

$$(T)_{\min} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 2 & 3 & n \end{bmatrix} \text{ and } [T]_{\max} = \begin{bmatrix} 1 & 2 & 3 & n \\ 2 & 2 & 3 & n \\ 3 & 3 & 3 & n \\ \vdots & \vdots & \vdots & \vdots \\ n & n & n & n \end{bmatrix}$$

The matrix  $(T)_{\min}$  is, up to a positive scalar, the covariance matrix of a stochastic process with increments which possess the same variance and are uncorrelated. Bhatia provided six alternative proofs for its positive definiteness. This same matrix is also studied in a recent book about matrices in statistics, see [6]. Next we review some basic concepts of lattice theory. A partially ordered set (poset) is a pair  $(P, \leq)$ , where P is a nonempty set and  $\leq$  is a reflexive, antisymmetric and transitive relation. A closed interval  $[x, y]$  in P is the set

$$[x, y] = \{z \in P / x \leq z \leq y\}, \quad x, y \in P.$$

Poset  $(P, \leq)$  is said to be locally finite if the interval  $[x, y]$  is finite for all  $x, y \in P$ . Poset  $(P, \leq)$  is a chain if  $x \leq y$  or  $y \leq x$  for all  $x, y \in P$ . A lattice is a poset, where the infimum  $x \wedge y$  and the supremum  $x \vee y$  exist for all  $x, y \in P$ . It is easy to see that every chain is a lattice with  $x \wedge y = \min(x, y)$  and  $x \vee y = \max(x, y)$ .

For example, the set of real numbers equipped with the usual ordering is a lattice and a chain, but it is not locally finite. The set of positive integers equipped with the divisibility relation is a locally finite lattice with  $x \wedge y = \gcd(x, y)$  and  $x \vee y = \text{lcm}(x, y)$ , but this poset is not a chain.

Next we need to define meet and join matrices. Let  $(P, \leq)$  be a locally finite lattice. Moreover, let  $S = \{x_1, x_2, x_3, \dots, x_n\}$  be a finite subset of P with distinct elements such that  $x_i \leq x_j \Rightarrow i \leq j$  (in other words, the indexing of the elements  $x_i \in S$  is a linear extension[7]). Finally, let f be a function on P to R (or to C). The meet matrix  $(S)_f$  of the set S with respect to the function f is the  $n \times n$  matrix with  $f(x_i \wedge x_j)$  as its (i,j) entry. Similarly, the join matrix  $[S]_f$  of the set S with respect to f is the  $n \times n$  matrix with  $f(x_i \vee x_j)$  as its (i,j) entry.

Like MIN and MAX matrices, meet and join matrices are square and (complex) symmetric as well. A proper way to describe meet and join matrices might be to say that in meet and join matrices the entries are determined partly by the function f and partly by the set S and the underlying lattice structure  $(P, \leq)$ .

**3 Some important results for meet and join matrices:**

In our study of MIN and MAX matrices we are going to make use of a couple of known results for meet and join matrices. The first one is about the structure of  $(S)_f$ . For any two subsets  $S = \{x_1, x_2, \dots, x_n\}$  and  $T = \{y_1, y_2, \dots, y_m\}$  of P, let  $E(S,T) = (e_{ij})$  denote the  $n \times m$  incidence matrix defined as

$$e_{ij} = \begin{cases} 1 & \text{if } y_j \leq x_i \\ 0 & \text{otherwise.} \end{cases}$$

**Proposition 3.1.**

[8], Let  $T = \{y_1, y_2, \dots, y_m\}$  be a meet closed subset of P containing  $S = \{x_1, x_2, \dots, x_n\}$  ( $m \geq n$ ). Then

$$(S)_f = E \wedge E^T = A A^T,$$

where  $E = E(S,T)$ ,  $\wedge = \text{diag}(\psi_{T,f}(y_1), \dots, \psi_{T,f}(y_m))$ ,  $A = E \wedge^{\frac{1}{2}}$  and  $\psi_{T,f}$  is defined recursively as

$$\psi_{T,f}(y_j) = f(y_j) - \sum_{y_i < y_j} \psi_{T,f}(y_i).$$

The main idea of this factorization can be generalized for join matrices and even for meet and join matrices on two sets. Furthermore, the other things, to find the following determinant and inverse formulas for meet and join matrices. In Propositions 3.3 and 3.5 the function  $\Phi_{S,f}$  is again the Möbius inversion of  $f$ , but in this case the inversion is executed from above. In other words,

$$\Phi_{S,f}(x_k) = f(x_k) - \sum_{x_k < x_v} \Phi_{S,f}(x_v).$$

**Proposition 3.2.**

If  $S$  is meet closed [9], then

$$\det(S)_f = \prod_{v=1}^n \psi_{S,f}(x_v) = \prod_{v=1}^n \sum_{\substack{z \leq x_v \\ z \leq x_t \\ t < v}} \sum_{w \leq z} f(w) \mu_P(w, z).$$

**Proposition 3.3.**

If  $S$  is join closed [10], then

$$\det[S]_f = \prod_{v=1}^n \sum_{x_v < x_t} f(x_t) \mu_S(x_v, x_t) = \prod_{v=1}^n \sum_{\substack{x_v \leq z \\ x_t \leq z \\ v < t}} \sum_{z \leq w} f(w) \mu_P(z, w).$$

**Proposition 3.4.**

Suppose that  $S$  is meet closed [9]. If  $(S)_f$  is invertible, then the inverse of  $(S)_f$  is the  $n \times n$  matrix  $B = (b_{ij})$ , where

$$b_{ij} = \sum_{k=1}^n \frac{(-1)^{i+j}}{\psi_{S,f}(x_v)} \det E(S_i^k) \det E(S_j^k)$$

Where  $E(S_i^k)$  is the  $(n-1) \times (n-1)$  sub matrix of  $E(S)$  obtained by deleting the  $i^{\text{th}}$  row and the  $k^{\text{th}}$  column of  $E(S)$ , or

$$b_{ij} = \sum_{x_i \vee x_j \leq x_k} \frac{\mu_S(x_i, x_j) \mu_S(x_j, x_k)}{\psi_{S,f}(x_k)},$$

where  $\mu_S$  is the Möbius function of the poset  $(S, \leq)$ .

**Proposition 3.5.**

Suppose that  $S$  is join closed [10]. If  $[S]_f$  is invertible, then the inverse of  $[S]_f$  is the  $n \times n$  matrix  $B = (b_{ij})$ , where

$$b_{ij} = \sum_{k=1}^n \frac{(-1)^{i+j}}{\Phi_{S,f}(x_k)} \det E(S_k^i) \det E(S_k^j),$$

Where  $E(S_k^i)$  is the  $(n-1) \times (n-1)$  sub matrix of  $E(S)$  obtained by deleting the  $k^{\text{th}}$  row and the  $i^{\text{th}}$  column of  $E(S)$ , or

$$b_{ij} = \sum_{x_k \leq x_i \wedge x_j} \frac{\mu_S(x_k, x_i) \mu_S(x_k, x_j)}{\Phi_{S,f}(x_k)},$$

where  $\mu_S$  is the Möbius function of the poset  $(S, \leq)$ .

**4 MIN and MAX matrices as meet and join matrices**

The most straight forward attempt to interpret MIN and MAX matrices as meet and join matrices would be to set  $(P, \leq) = (R, \leq)$ . This, however, cannot be done since the set of real numbers is not locally finite (meet and join matrices are usually studied via Möbius inversion, which requires the local finiteness property). Nevertheless, there is a way around the problem. We set  $P = \{1, 2, \dots, n\}$ ,  $\leq$  is the usual ordering of the integers and  $S = P$ . Since in this case  $(P, \leq)$  is a chain with  $n$  elements, it is trivially a locally finite lattice. Moreover, by defining  $f: P \rightarrow R$  by  $f(i) = z_i$  for all  $i = 1, 2, \dots, n$  we obtain  $(S)_f = (T)_{\min}$  and  $[S]_f = [T]_{\max}$ .

Executing the Möbius inversion is now easy due to the simple chain-structure of the poset  $(P, \leq)$  (general information about Möbius inversion and Möbius functions on posets can be found). For the Möbius function of the chain  $(P, \leq)$  we have for  $i, j \in P$  that

$$\mu_P(j, i) = \begin{cases} 1 & \text{if } i = j \\ -1 & \text{if } i = j + 1 \\ 0 & \text{otherwise.} \end{cases}$$

The function  $\mu_P$  can then be used to define two other functions  $\psi_P$  and  $\Phi_P$  as

$$\text{and } \begin{aligned} \psi_P(1) &= x_1, & \psi_P(i) &= \sum_{1 \leq j \leq i} \mu_P(i, j)x_j = x_i - x_{i-1} \quad \text{for } 1 < i \leq n \\ \Phi_P(n) &= x_n, & \Phi_P(n) &= \sum_{i \leq j \leq n} \mu_P(i, j)x_j = x_i - x_{i+1} \quad \text{for } 1 < i \leq n. \end{aligned}$$

It turns out that the values of the functions  $\psi_P$  and  $\Phi_P$  characterize many key properties of the matrices  $(T)_{\min}$  and  $[T]_{\max}$  by [11].

**4.1 Remark:**

Similarly defined functions  $\psi_{P,S,f}$  and  $\Phi_{P,S,f}$  are also used in the study of more general meet and join matrices, but here these functions take particularly simple forms due to the simple chain-structure of the set P.

Meet and join matrices and their special cases GCD and LCM matrices have been studied in dozens of research papers and their basic properties are rather well known. In this article we are going to formulate these general results for MIN and MAX matrices. Since most of the results presented in this paper follow directly from some stronger theorem for meet and join matrices, it would not be absolutely necessary to reprove these statements. However, we are going to see that in many cases it is still interesting and useful to find simpler proofs that are also accessible to those who are not so familiar with the methods used in the study of meet and join matrices.

**5 SOME DEFINITIONS:**

**5.1 MINIMUM AND MAXIMUM MATRICES:**

Let us define the matrix operations  $\wedge$  and  $\vee$  by  $(a_{i,j})_{n \times m} \wedge (b_{i,j})_{n \times m} = (\min(a_{i,j}, b_{i,j}))$  and  $(a_{i,j})_{n \times m} \vee (b_{i,j})_{n \times m} = (\max(a_{i,j}, b_{i,j}))$ . Let C denote the  $n \times n$  matrix with  $c_{ij} = x_i$  for all  $1 \leq i, j \leq n$ . Then  $(T)_{\min} = C \wedge C^T$  and  $[T]_{\max} = C \vee C^T$ .

**5.2 MERSENNE MATRICES:**

Let  $S = \{x_1, x_2, \dots, x_n\}$  be a set of distinct positive integers and the  $n \times n$  matrix and  $[M] = (m_{ij})$ , where  $m_{ij} = 2^{(x_i x_j)} - 1$ , call it to be Mersenne matrix on S [12].

**5.3 MERSENNE MINIMUM MATRICES:**

Let  $S = \{x_1, x_2, \dots, x_n\}$  be a set of distinct positive integers and the  $n \times n$  matrix and  $[M] = (m_{ij})$ , where  $m_{ij} = 2^{\min(x_i, x_j)} - 1$ , call it to be Mersenne MIN matrix on S.

**5.4 MERSENNE MAXIMUM MATRICES:**

Let  $S = \{x_1, x_2, \dots, x_n\}$  be a set of distinct positive integers and the  $n \times n$  matrix and  $[M] = (m_{ij})$ , where  $m_{ij} = 2^{\max(x_i, x_j)} - 1$ , call it to be Mersenne MAX matrix on S.

**6 DETERMINANTS OF MERSENNE MIN AND MERSENNE MAX MATRICES:**

**Theorem 6.1:**

We consider the determinants of the matrices

$$\det(T)_{\min} = \psi_P(1)\psi_P(2) \dots \dots \dots \psi_P(n) = x_1(x_2 - x_1)(x_3 - x_2) \dots \dots \dots (x_n - x_{n-1}).$$

$$\det[T]_{\max} = \Phi_P(1)\Phi_P(2) \dots \dots \dots \Phi_P(n) = (x_1 - x_2)(x_2 - x_3) \dots \dots \dots (x_{n-1} - x_n)x_n.$$

Proof: These determinant formulas follow directly from Proposition 3.2 and Proposition 3.3.

**Theorem 6.2:**

Next we consider the determinants of the mersenne min and max matrices

$$\text{Det mer } (T)_{\min} = \psi_P(1)\psi_P(2) \dots \dots \dots \psi_P(n) = x_1(x_2 - x_1)(x_3 - x_2) \dots \dots \dots (x_n - x_{n-1}),$$

where  $x_i = 2^{\min(x_i, x_j)} - 1 \quad j = 1, 2, \dots, n$ .

$$\text{Det mer } [T]_{\max} = \Phi_P(1)\Phi_P(2) \dots \dots \dots \Phi_P(n) = (x_1 - x_2)(x_2 - x_3) \dots \dots \dots (x_{n-1} - x_n)x_n,$$

$$\text{where } x_i = 2^{\max(x_i, x_j)} - 1 \quad j = 1, 2, \dots, n.$$

**Example 1**

If  $S = \{2, 3\}$  is a lower closed set. Consider 2x2 Mersenne Min matrix on S is

$$\text{Mer}(S)_{\min} = \begin{bmatrix} 3 & 3 \\ 3 & 7 \end{bmatrix}$$

$$\text{Det Mer}(S)_{\min} = x_1(x_2 - x_1) = 3 \times 4 = 12.$$

Consider 2x2 Mersenne Max matrix on S is

$$\text{Mer}(S)_{\max} = \begin{bmatrix} 3 & 7 \\ 7 & 7 \end{bmatrix}$$

$$\text{Det Mer}(S)_{\max} = (x_1 - x_2)x_1 = -4 \times 7 = -28.$$

**Example 2**

If  $S = \{2, 4, 6\}$  is a lower closed set. Consider 3x3 Mersenne Min matrix on S is

$$\text{Mer}(S)_{\min} = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 15 & 15 \\ 3 & 15 & 63 \end{bmatrix}$$

$$\text{Det Mer}(S)_{\min} = x_1(x_2 - x_1)(x_3 - x_2) = 3 \times 12 \times 48 = 1748.$$

Consider 3x3 Mersenne Max matrix on S is

$$\text{Mer}(S)_{\max} = \begin{bmatrix} 3 & 15 & 63 \\ 15 & 15 & 63 \\ 63 & 63 & 63 \end{bmatrix}$$

$$\text{Det Mer}(S)_{\max} = (x_1 - x_2)(x_2 - x_3)x_3 = (-12) \times (-48) \times 63 = 36288.$$

**Example 3**

If  $S = \{1, 2, 3, 6\}$  is a lower closed set. Consider 4x4 Mersenne Min matrix on S is

$$\text{Mer}(S)_{\min} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 3 & 3 \\ 1 & 3 & 7 & 7 \\ 1 & 3 & 7 & 63 \end{bmatrix}$$

$$\text{Det Mer}(S)_{\min} = x_1(x_2 - x_1)(x_3 - x_2)(x_4 - x_3) = 1 \times 2 \times 4 \times 56 = 448.$$

Consider 4x4 Mersenne Max matrix on S is

$$\text{Mer}(S)_{\max} = \begin{bmatrix} 1 & 3 & 7 & 63 \\ 3 & 3 & 7 & 63 \\ 7 & 7 & 7 & 63 \\ 63 & 63 & 63 & 63 \end{bmatrix}$$

$$\text{Det Mer}(S)_{\max} = (x_1 - x_2)(x_2 - x_3)(x_3 - x_4)x_4 = (-2) \times (-4) \times (-56) \times 63 = -28224.$$

**7 INVERSES OF MERSENNE MIN AND MAX MATRICES:**

Under the assumption that the elements of the set T are distinct the MIN and MAX matrices of the set T are usually invertible. Next we shall find their inverses.



**Theorem 7.1:**

Suppose that the elements of the set T are distinct. If  $x_1 \neq 0$ , then the MIN matrix is invertible and the inverse matrix is the nxn tridiagonal matrix  $B = (b_{ij})$ , where

$$B_{ij} = \begin{cases} 0 & \text{if } |i - j| > 1 \\ \frac{x_2}{x_1(x_2 - x_1)} & \text{if } i = j = 1 \\ \frac{1}{x_i - x_{i-1}} + \frac{1}{x_{i+1} - x_i} & \text{if } 1 < i = j < n \\ \frac{1}{x_n - x_{n-1}} & \text{if } i = j = n \\ \frac{-1}{|x_i - x_j|} & \text{if } |i - j| = 1. \end{cases}$$

where  $x_i = 2^{x_i} - 1 \quad i = 1, 2, \dots, n$ .

**Theorem 7.2:**

If  $x_n \neq 0$ , then the inverse of the MAX matrix is invertible and the inverse matrix is the nxn tridiagonal matrix  $C = (C_{ij})$ , where

$$C_{ij} = \begin{cases} 0 & \text{if } |i - j| > 1 \\ \frac{1}{(x_1 - x_2)} & \text{if } i = j = 1 \\ \frac{1}{x_{i-1} - x_i} + \frac{1}{x_i - x_{i+1}} & \text{if } 1 < i = j < n \\ \frac{1}{x_{n-1} - x_n} + \frac{1}{x_n} & \text{if } i = j = n \\ \frac{1}{|x_i - x_j|} & \text{if } |i - j| = 1. \end{cases}$$

where  $x_i = 2^{x_i} - 1 \quad i = 1, 2, \dots, n$ .

Proof: The inverse formulas follow straight from Proposition 3.4 and Proposition 3.5. An elementary approach would be to construct the supposed inverse matrices and multiply them with the matrices (T) min and [T] max.

**Example 4**

(S) is a Mersenne Min matrix on lower closed set  $S = \{2, 3\}$ . Then by definition 6.1

$$(S)^{-1} = B = (b_{ij})$$

Therefore since  $(S)^{-1} = B$  is the symmetric we have

$$(S)^{-1} = B = \begin{bmatrix} \frac{3}{2} & -1 \\ -1 & 1 \end{bmatrix}$$

(S) is a Mersenne Max matrix on lower closed set  $S = \{2, 3\}$ . Then by definition 6.2

$$(S)^{-1} = C = (c_{ij})$$

Therefore since  $(S)^{-1} = C$  is the symmetric we have

$$(S)^{-1} = C = \begin{bmatrix} -1 & 1 \\ 1 & \frac{-2}{3} \end{bmatrix}$$

**Example 5**

(S) is a Mersenne Min matrix on lower closed set  $S = \{2, 4, 6\}$ . Then by definition 6.1

$$(S)^{-1} = B = (b_{ij})$$

Therefore since  $(S)^{-1} = B$  is the symmetric tridiagonal we have

$$(S)^{-1} = B = \begin{bmatrix} \frac{5}{12} & \frac{-1}{12} & 0 \\ \frac{-1}{12} & \frac{5}{48} & \frac{-1}{48} \\ 0 & \frac{-1}{48} & \frac{1}{48} \end{bmatrix}$$

(S) is a Mersenne Max matrix on lower closed set  $S = \{2, 4, 6\}$ . Then by definition 6.2

$$(S)^{-1} = C = (c_{ij})$$

Therefore since  $(S)^{-1} = C$  is the symmetric tridiagonal we have

$$(S)^{-1} = C = \begin{bmatrix} \frac{-1}{12} & \frac{1}{12} & 0 \\ \frac{1}{12} & \frac{-5}{48} & \frac{1}{48} \\ 0 & \frac{1}{48} & \frac{-5}{1008} \end{bmatrix}$$

### Example 6

(S) is a Mersenne Min matrix on lower closed set  $S = \{1, 2, 3, 6\}$ . Then by definition 6.1

$$(S)^{-1} = B = (b_{ij})$$

Therefore since  $(S)^{-1} = B$  is the symmetric tridiagonal we have

$$(S)^{-1} = B = \begin{bmatrix} \frac{3}{2} & \frac{-1}{2} & 0 & 0 \\ \frac{-1}{2} & \frac{3}{4} & \frac{-1}{4} & 0 \\ 0 & \frac{-1}{4} & \frac{15}{56} & \frac{-1}{56} \\ 0 & 0 & \frac{-1}{56} & \frac{1}{56} \end{bmatrix}$$

(S) is a Mersenne Max matrix on lower closed set  $S = \{1, 2, 3, 6\}$ . Then by definition 6.2

$$(S)^{-1} = C = (c_{ij})$$

Therefore since  $(S)^{-1} = C$  is the symmetric tridiagonal we have

$$(S)^{-1} = C = \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{-3}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{-15}{56} & \frac{1}{56} \\ 0 & 0 & \frac{1}{56} & \frac{-1}{504} \end{bmatrix}$$

### CONCLUSION:

In this paper, the different properties of MIN and MAX matrices of the set T with  $\min(x_i, x_j)$  and  $\max(x_i, x_j)$  as their  $(i, j)$  entries like determinant value and inverse of MIN and MAX matrices have been studied. The study is carried out by applying known results of meet and joins matrices to Mersenne min and Mersenne max matrices.

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