PENTAGONAL FUZZY NUMBERS

Supreet Kaur
Mathematics
Patiala, Punjab, India

Abstract: In this paper pentagonal fuzzy numbers in continuation with the other fuzzy numbers are defined. Basic arithmetic operations like addition, subtraction and multiplication of pentagonal fuzzy numbers with examples are also included. In addition, pentagonal fuzzy matrix with its properties is defined

Keywords**: Fuzzy Numbers, fuzzy arithmetic, Fuzzy operations, Fuzzy Matrix, Pentagonal Fuzzy Numbers**

1. INTRODUCTION

L.A.Zadeh introduced fuzzy set theory in 1965. Different types of fuzzy sets are defined in order to clear the vagueness of the existing problems. Membership function of these sets, which have the form A: R → [0, 1] and it has a quantitative meaning and viewed as fuzzy numbers. A fuzzy number is a quantity whose values are imprecise, rather than exact as in the case with single-valued function. So far, fuzzy numbers like triangular fuzzy numbers, trapezoidal fuzzy numbers, and pyramid fuzzy numbers are introduced with its membership functions. These numbers have got many applications like non-linear equations, risk analysis and reliability. Many operations were carried out using fuzzy numbers. In this paper, basics of pentagonal fuzzy numbers with their arithmetic operations along with pentagonal fuzzy matrices with their properties are discussed.

The paper is organized as follows: In section 2, basic definitions of fuzzy number, triangular and trapezoidal fuzzy numbers are discussed. Section 3-6 deals with the concept of Pentagonal fuzzy number (PFN) with their arithmetic properties. In section 7, pentagonal fuzzy matrices with their basic properties are discussed.

2. BASIC DEFINITIONS

I. **Fuzzy Set:** A fuzzy set *A* in *R* (real line) is defined to be a set of ordered pairs, $A= \{(x, \mu_A(x))|x \in X\}$, where $\mu_A(x))$ is called the membership function for the fuzzy set.

II. **α-cut :** The α-cut of α -level set of fuzzy set *A* is a set consisting of those elements of the universe *X* whose membership values exceed the threshold level α .i.e. $A\alpha = \{ x\in X/\mu_A(x) \geq \alpha\}$

III. **Normal fuzzy set**: A fuzzy set *A* is called normal, if there is at least one point $x\in R$ with $\mu_A(x)=1$

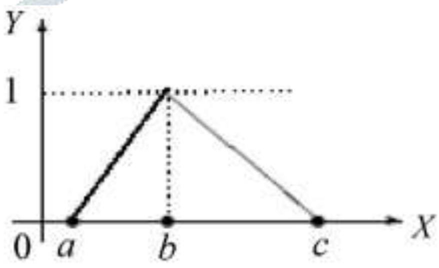
IV. **Convex Fuzzy Set**: A fuzzy set *A* on *R* is convex, if for any $x, y \in R$ and for any $\lambda \in[0,1]$ we have $\mu_A(\lambda x+(1-\lambda)y) \geq \min (\mu_A(x), \mu_A(y))$

V. **Fuzzy Number**: A fuzzy number *A* is a fuzzy set on the real line that satisfies the conditions of normality and convexity.

VI. If a fuzzy set is convex and normalized and its membership function is defined in *R* and piecewise continuous, it is called as fuzzy number. Fuzzy number represents a real number whose boundary is fuzzy.

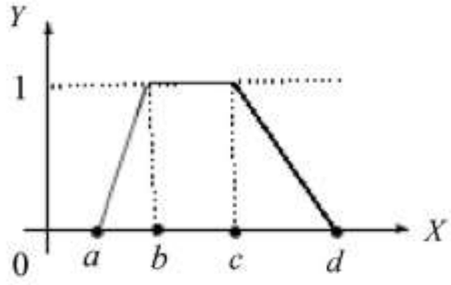
VII. **Triangular fuzzy number:** A triangular fuzzy number is a fuzzy set $A=(a,b,c)$ having following membership function which looks like a triangle graphically

$$\mu_A(x)=\begin{cases} 0, & x \leq a \\ \dfrac{x-a}{b-a}, & a<x\leq b \\ 1, & x=b \\ \dfrac{c-x}{c-b}, & b<x\leq c \\ 0, & x\geq c. \end{cases}$$



VIII. **Trapezoidal fuzzy number.** A fuzzy number A = (a,b,c,d) is called a trapezoidal fuzzy number if it possesses the following membership function. Graphically, the trapezoidal fuzzy number has a trapezoidal shape with four vertices (a,b,c,d).

$$\mu_A(x)=\begin{cases}0, & x\le a\\ \dfrac{x-a}{b-a}, & a<x\le b\\ 1, & b\le x\le c\\ \dfrac{c-x}{d-c}, & c<x\le d\\ 0, & x\ge d\end{cases}$$



However, real-life problems are sometimes concerned with more than four parameters. To resolve those problems, we propose another concept of the fuzzy number, called pentagonal fuzzy number (PFN) which is discussed in next section.

3. **PENTAGONAL FUZZY NUMBERS:**

I. A fuzzy number $A = (a_1,a_2,a_3,a_4,a_5)$ is called a pentagonal fuzzy number, if its membership function is given by

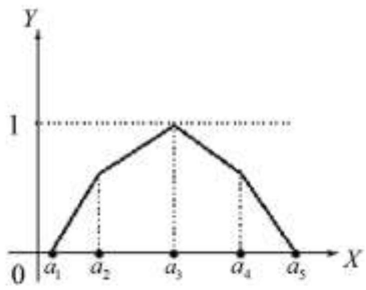
$$\mu_A(x)=\begin{cases}0, & for\quad x<a_1,a_5\le x\\ \dfrac{x-a_1}{a_2-a_1}; & for\quad a_1\le x\le a_2\\ \dfrac{x-a_2}{a_3-a_2}; & for\quad a_2\le x\le a_3\\ 1, & for\quad x=a_3\\ \dfrac{a_4-x}{a_4-a_3}; & for\quad a_3\le x\le a_4\\ \dfrac{a_5-x}{a_5-a_4}; & for\quad a_5\le x\le a_4\end{cases}$$

For the PFN $A= (a_1,a_2,a_3,a_4,a_5)$, a_3 is the middle point and (a_1,a_2) and (a_4,a_5) are the left and right side points of a_3 respectively. The middle point a_3 has the grade of membership 1 and w_1, w_2 are the grades of points a_2, a_4.

Pentagonal fuzzy number (PFN) can be looked upon in a generalized way, so that, two special fuzzy numbers, namely triangular fuzzy number and trapezoidal fuzzy number can be visualized as follows

II. A fuzzy number $A= (a_1,a_2,a_3,a_4,a_5)$, is a pentagonal fuzzy number (PFN) having membership function

$$\mu_A(x;w_1,w_2)=\begin{cases}w_1\dfrac{x-a_1}{a_2-a_1}, & a_1\le x\le a_2\\ 1-(1-w_1)\dfrac{x-a_2}{a_3-a_2}, & a_2\le x\le a_3\\ 1, & x=a_3\\ 1-(1-w_2)\dfrac{x-a_3}{a_4-a_3}, & a_3\le x\le a_4\\ w_2\dfrac{x-a_5}{a_4-a_5}, & a_4\le x\le a_5\\ 0, & x>a_5\end{cases}$$



Case (i) When $w_1 = w_2 = 0$ then the pentagonal fuzzy number is reduced to a triangular fuzzy number. That is A= $(a_1,a_2,a_3,a_4,a_5) \approx (a_2,a_3,a_4)$ where membership function is given by

$$\mu_A(x) = \begin{cases} 0; & for \quad x \le a_2 \\ 1-\left(\dfrac{x-a_2}{a_3-a_2}\right); & for \quad a_2 \le x \le a_3 \\ 1, & for \quad x = a_3 \\ 1-\left(\dfrac{x-a_3}{a_4-a_3}\right); & for \quad a_3 < x \le a_4 \\ 0; & for \quad x \ge a_4 \end{cases}$$

Case (ii) When $w_1 = w_2 = 1$ then the pentagonal fuzzy number is reduced to a trapezoidal fuzzy number. That is A= $(a_1,a_2,a_3,a_4,a_5) \approx (a_1, a_2, a_4\, a_5)$ where membership function is given by

$$\mu_A(x) = \begin{cases} 0, & for \quad x \le a_1 \\ \left(\dfrac{x-a_1}{a_2-a_1}\right); & for \quad a_1 \le x \le a_2 \\ 1, & for \quad a_2 \le x \le a_5 \\ \left(\dfrac{a_4-x}{a_5-a_4}\right); & for \quad a_4 \le x \le a_5 \\ 0, & for \quad x > a_5 \end{cases}$$

4. CONDITIONS ON PENTAGONAL FUZZY NUMBER

A Pentagonal Fuzzy Number A should satisfy the following conditions:

(i) $\mu_A(x)$ is a continuous function in the interval [0,1]

(ii) $\mu_A(x)$ is strictly increasing and continuous function on $[a_1,a_2]$ and $[a_2, a_3]$

(iii) $\mu_A(x)$ is strictly decreasing and continuous function on $[a_3, a_4]$ and $[a_4, a_5]$

5. The pentagonal fuzzy number is represented by the five parameters a_1,a_2,a_3,a_4,a_5 where a_1,a_2 denote the smallest possible values, a_3 the most promising value and a_4,a_5 the largest possible value. The pentagonal fuzzy number can be generated by using the following formula.

$A = (a-2, a-1, a, a+1, a+2)$ for all $a = 3,4,5,6,7$. Since the fuzzy number scale is defined from 1 to 9

6. ARITHMETIC OPERATIONS ON PENTAGONAL FUZZY NUMBERS (PFN):

Let A= (a_1,a_2,a_3,a_4,a_5) and B= (b_1,b_2,b_3,b_4,b_5) be two PFNs then

I. **Addition** of two PFNs is given by A+B = $(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4,a_5+b_5)$
For Example: if A =(1,2,3,4,5) and B=(2,3,4,5,6) then A+B=(3,5,7,9,11)

II. **Subtraction** of two PFNs is given by A-B = $(a_1-b_1,a_2-b_2,a_3-b_3,a_4-b_4,a_5-b_5)$
For example; If A= (1,2,4,5,7) and B= (2,4,6,8,2); then A-B= (-1, -2,- 2, -3,5)

III. **Multiplication** of two PFNs is given by AB = $(a_1b_1,a_2b_2,a_3b_3,a_4b_4,a_5b_5)$
For example; If A= (1,2,4,5,7) and B= (2,4,6,8,2); then AB=(2,8,24,40,14)

IV. **Scalar Multiplication** of a PFN A by k, where k € R be any scalar,
If $k \ge 0$, kA = $(ka_1,ka_2,ka_3,ka_4,ka_5)$ and if $k \le 0$, kA= $(ka_5,ka_4,ka_3,ka_2,ka_1)$
For Example: If A=(2,4,6,8,2) then 2A=(4,8,12,16,4) and -2A=(-4,-8,-12,-16,-4)

V. The **inverse** of a PFN is defined when all its components are non-zero.
If A = (a_1,a_2,a_3,a_4,a_5) then $A^{-1} \approx 1/A \approx (1/a_5, 1/a_4, 1/a_3, 1/a_2, 1/a_1)$
For Example: if A =(1,2,3,4,5) then $A^{-1} \approx (1/5,1/4,1/3,1/2,1)$
If one of the components of a PFN becomes zero, then we cannot find its inverse

VI. **Division** of two PFN is given by $A/B \approx AB^{-1} \approx (a_1/b_5,a_2/b_4,a_3/b_3,a_4/b_2,a_5/b_1)$
Note that a PFN A is divisible by B only when B is having non-zero components
For Example: If A=(2,4,6,8,2) and B =(1,2,3,4,5) then $AB^{-1} \approx (2/5,4/4,6/3,8/2,2/1)$
i.e. $AB^{-1} \approx (2/5,1,2,4,2)$

VII. A PFN A is said to be **positive**, if all its entries are positive. Similarly, A is **negative**, if all of its entries are negative.

VIII. A PFN *A* is called a **null PFN,** if all of its entries are zero. That is, *A* = (0,0,0,0,0).

IX. A PFN is said to be **null equivalent**, if its middle entry is at the point 0, that is, of the form $(\psi_1,\alpha_1,0,\psi_2,\alpha_2)$ where $\psi_1.\alpha_1,\neq 0$, $\psi_2.\alpha_2\neq 0$

X. A PFN A is said to be a **unit equivalent** PFN when its middle entry is at 1, i.e., of the form $(\psi_1,\alpha_1,1,\psi_2,\alpha_2)$ where $\psi_1.\alpha_1,\neq 0$, $\psi_2.\alpha_2\neq 0$

XI. The subtraction of two PFNs with a common middle entry gives a null equivalent PFN, while their division yields unit equivalent PFN

XII. Addition and multiplication of PFNs are both commutative and associative, while multiplication is also distributive over addition

7. PENTAGONAL FUZZY MATRIX(PFM)

A matrix (a_{ij}) of order m × n is called a pentagonal fuzzy matrix if the entries of the matrix are pentagonal fuzzy numbers i.e. for each (i,j), a_{ij} is a pentagonal fuzzy number.

Pure null PFM: A PFM is said to be a pure null PFM if all its entries are null PFNs, i.e., all the elements are (0,0,0,0,0). It is denoted by O.

Null equivalent PFM. A PFM A $=(a_{ij})$ is said to be a null equivalent PFM if all its elements are of the form $a_{ij=}(\psi_1,\alpha_1,0,\psi_2,\alpha_2)$ where $\psi_1.\alpha_1,\neq 0$, $\psi_2.\alpha_2\neq 0$

Pure unit PFM. A square PFM A $=(a_{ij})$ is said to be a pure unit PFM if a_{ii} = (0,0,1,0,0) and a_{ij} =0, i≠j for all i,j= 1,2,...,n. It is denoted by I.

Unit equivalent PFM. A square PFM A $=(a_{ij})$ is said to be a unit equivalent PFM if all its elements are of the form $a_{ij=}(\psi_1,\alpha_1,1,\psi_2,\alpha_2)$ where $\psi_1.\alpha_1,\neq 0$, $\psi_2.\alpha_2\neq 0$

Fuzzy triangular PFM. A square PFM A = (a_{ij}) is called a fuzzy triangular PFM if either $a_{ij=}(\psi_1,\alpha_1,0,\psi_2,\alpha_2)$ for i>j or $a_{ij=}(\psi_1,\alpha_1,0,\psi_2,\alpha_2)$ for i<j where $\psi_1.\alpha_1,\neq 0$, $\psi_2.\alpha_2\neq 0$

Strictly fuzzy triangular PFM. A square PFM A $=(a_{ij})$ is called a strictly fuzzy triangular PFM if either $a_{ij} =\tilde{0}$ for i ≥j or $a_{ij} = \tilde{0}$ for i≤j; $\tilde{0}$ being the null equivalent PFN

Fuzzy skew symmetric PFM. A square PFM A= (a_{ij}) is called a fuzzy skew symmetric PFM if $A=A^T$ i.e. $a_{ii=}(\psi_1,\alpha_1,0,\psi_2,\alpha_2)$ where $\psi_1.\alpha_1,\neq 0$, $\psi_2.\alpha_2\neq 0$ and $a_{ij}= -a_{ji}$

8. SOME PROPERTIES OF PENTAGONAL FUZZY MATRICES

Let A, B, C be any three matrices of same order and s,t be any scalars then:

- A+B=B+A
- A+(B+C)=(A+B)+C
- A+A=2A
- A-A=$\tilde{0}$; $\tilde{0}$ being the null equivalent PFN
- s(tA)= (st)A.
- s(A+B) =sA+sB
- (s+t)A=sA+tA
- $(A^T)^T = A$
- $(A+B)^T=A^T+B^T$
- $(AB)^T=B^TA^T$
- $(sA)^T=sA^T$
- $(sA+sB)^T=sA^T+sB^T$
- AA^T and A^TA both are symmetric
- $A+A^T$ is a fuzzy symmetric PFM
- $A-A^T$ is a fuzzy skew symmetric PFM
- tr(A+B)=tr(A) + tr(B)
- $tr(A)=tr(A^T)$
- tr(AB)=tr(BA)

9. CONCLUSION:

In this paper, special attention to the pentagonal fuzzy number (PFN) along with their operations and properties is paid. Also their corresponding fuzzy matrices with their properties are discussed.

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