CONSTRUCTION OF ROBUST CONTROL CHART USING PROCESS CAPABILITY FOR MEAN

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Abstract: The control limits derived for the Median Absolute Deviation (MAD) based mean (S) control chart proposed by Abu-Shawiesh (2008) was for monitoring quality characteristics when a standard value of sigma (σ) is known or given by the management and engineers. When sigma (σ) is unknown and we are interested in monitoring non-normal data, then there is the need to modify the simple robust control limits. In this paper, the control limits for the Shewhart (1931) mean ($\bar{X}$) control chart based on median absolute deviation were modified using the concept of three sigma (3σ) limits and the resultant of modified control limits is more efficient than the existing control limits.

Keywords: Control chart, Control limits interval (CLI), Process capability (Cp) and Robustness.

I. INTRODUCTION

Specifying the control limits is the most important step in designing a control chart. Improper estimation of the process dispersion which results in narrower or wider limits can increase the probability of type I error or the probability of type II error. When the limits are narrow, the risk of a point falling beyond the limits increases, falsely indicating that the process is out of control (Shahriari et al., 2009). When the limits are wider the risk increases the points falling within the limits, this falsely indicates that the process is out of control (Shahriari et al., 2009). The limits on a control chart can either be 0.001 probability limits or 3-sigma limits (Oakland, 2008). The 0.001 probability limit were determined so that, if chance causes alone were at work the probability of a point falling above the upper limit would be one out of a thousand and the probability of points falling below the lower limit would be one out of a thousand. Therefore, 3- sigma limits are the practical equivalent of the 0.001 probability limits. In this paper, the 3-sigma limits approach will be adopted.

The robust methods are one of the most commonly used statistical methods when the underlying normality assumption is violated. These methods offer useful and viable alternative to the traditional statistical methods and can provide more accurate results, often yielding greater statistical power and increased sensitivity and yet still be efficient if the normal assumption is correct. By a robust estimator, we mean an estimator which is insensitive to changes in the underlying distribution and also resistant against the presence of outliers. The robust estimator is considered to be good if it has high efficiency, high breakdown point which is the maximum fraction of outliers an estimator can cape, redescending influence function which measures how an estimator reacts to a small fraction of outliers and has low gross-error sensitivity which measures the worst influence a small amount of contamination of fixed size can have on the value of the estimator. In this paper, we will restrict attention to estimator that has an explicit formula and being easily computable and needs little computation time.

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Abu-Shawiesh (2008) derived the control limits for the standard deviation control chart using the median absolute deviation. However, the derived control limits was for situation when a standard value of sigma is known or specified by the management. Using such control limits to monitor a process where the standard value of sigma is unknown will lead to misplacement of control limits and hence false alarm of out of control. In this paper, we consider situation when the standard value of sigma is unknown and we are interested in monitoring past data. Thus, in this article we modify the control limits Adekeye et al. (2012) for the mean ($\bar{X}$) control chart with an example.

2. CONCEPTS AND TERMINOLOGIES

2.1 Upper Specification Limit (USL)

It is the greatest amount in which a process or product is within the acceptable performance limits.

2.2 Lower Specification Limit (LSL)

It is the smallest amount in which a process or product is within the acceptable performance limits.

2.3 Tolerance Level (TL)

It is the difference between USL and LSL, TL = USL-LSL

2.4 Process Capability (Cp)

This is the ratio of tolerance level to six times standard deviation of the process.

2.5 Subgroup Size (n)

In order to make control chart analysis effective, it is essential to pay due regard to the rational selection of the subgroups. It is the choice of the sample size n and the frequency of sampling. It is also the number of observed values in any given sample or subgroup.

2.6 Number of Samples (N)

It is the number of samples selected for constructing Control Charts.

2.7 Standard deviation (σ)

For many purposes standard deviation is the most useful measure of dispersion of a set of numbers. It is the root mean square value.

σ - the known or estimated true value of universe standard deviation and

3. METHODS AND MATERIALS

a. Shewhart control chart for mean

The Shewhart control chart for mean which is one of the most widely used statistical process control technique developed to monitor the standard deviation of a production process σ in order to control the process variability. It is use as a standard practice to estimate σ the average of the subgroup standard deviation $\frac{\sigma}{\sqrt{n}}$. The fundamental assumption of the mean control chart is that the underlying distribution of the quality characteristic is normal, but unfortunately many processes, occur in practice, do not follow the normal distribution and due to the fact that the sample standard deviation, $\frac{\sigma}{\sqrt{n}}$, is non-robust to slight deviations from normality assumption, the need for alternatives to the Shewhart mean control chart comes to play.

$$UCL_{3\sigma} = \bar{\bar{X}} + 3\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$\text{Central Line } CL_{3\sigma} = \bar{\bar{X}}$$

$$LCL_{3\sigma} = \bar{\bar{X}} - 3\left(\frac{\sigma}{\sqrt{n}}\right)$$

b. Robust control chart based on mad

The MAD for a random sample of size n observations x1, x2,...,xn is defined as follows:

$MAD = 1.4826MD\{|X_i - MD|\}, i = 1,2, \ldots, n$, where MD is the sample median.

The Statistics bnMAD is an unbiased estimator of s if the random samples x1, x2... xn are normally distributed. The correction factor bn is given in Table 3 for different values of n.

$$\bar{\bar{X}} \pm \left(3 \times \frac{b_n \overline{MAD}}{\sqrt{n}}\right)$$

c. Process Capability Indices

The Cp, Cpk, and Cpm statistics assume that the population of data values is normally distributed. Assuming a two-sided specification, if µ and σ are the mean and standard deviation, respectively, of the normal data and USL, LSL, and T are the upper and lower specification limits and the target value, respectively, then the population capability indices are defined as follows:

$$C_p = \frac{USL - LSL}{6\sigma}$$

4. CONSTRUCTION OF PROPOSED CONTROL CHART BASED ON MAD USING PROCESS CAPABILITY FOR MEAN

Fix the tolerance level (TL) and process capability (CP) to determine the process standard deviation (σ_{RPC}). Apply the value of σ_{RPC} in the control limits $\bar{\bar{X}} \pm 3\left(\frac{\sigma_{RPC}}{\sqrt{n}}\right)$, to get the control limits based on MAD for standard deviation chart (Radhakrishnan & Balamurugan, 2012). For a specified TL and CP of the process, the value of σ (termed as σ_{RPC}) is calculated from $C_p = \frac{TL}{6\sigma}$ using a JAVA program and presented in Table 2 for various combinations of TL and Cp. The control limits based on MAD using process capability for standard deviation chart are $\bar{\bar{X}} \pm 3\left(\frac{\sigma_{RPC}}{\sqrt{n}}\right)$

5. EXAMPLE

The example provided by Eugene L. Grant and Richard S. Leavenworth (1952, Page No.68) is considered here. The following data are collected based on temperature at which thermostatic switch operates (temperature units not specified).

5.1 Shewhart Control chart for mean

The 3σ control limits suggested by Shewhart (1931) are $\bar{\bar{X}} \pm \left(3 \times \frac{\sigma}{\sqrt{n}}\right)$

$$UCL_{\bar{\bar{X}}} = \bar{\bar{X}} + \left(3 \times \frac{\sigma}{\sqrt{n}}\right) = 54.32 + \left(3 \times \frac{2.42}{\sqrt{5}}\right) = 54.32 + 3.25 = 57.57$$

$$CL_{\bar{\bar{X}}} = \bar{\bar{X}} = 54.32$$

$$LCL_{\bar{\bar{X}}} = \bar{\bar{X}} - \left(3 \times \frac{\sigma}{\sqrt{n}}\right) = 54.32 - \left(3 \times \frac{2.42}{\sqrt{5}}\right) = 54.32 - 3.25 = 51.07$$

Table1: Data for temperature at which thermostatic switch operates (temperature units not specified)

Date	Sub group number	"On" temperature at which thermostatic switch operates (temperature units not specified)					Mean	MAD
		a	b	c	d	e		
Apr. 25	1	54	56	56	56	55	55.4	0.00
	2	51	52	54	56	49	52.4	2.97
	3	54	52	50	57	55	53.6	2.97
	4	56	55	56	53	50	54.0	1.48
	5	53	54	57	56	52	54.4	2.97
	6	53	47	58	55	54	53.4	1.48
	7	52	55	54	55	56	54.4	1.48
	8	56	53	53	54	55	54.2	1.48
	9	55	52	53	56	55	54.2	1.48
	10	50	54	53	55	55	53.4	1.48
Apr. 26	11	57	54	53	52	53	53.8	1.48
	12	52	52	54	53	55	53.2	1.48
	13	54	53	55	52	52	53.2	1.48
	14	54	55	54	53	55	54.2	1.48
	15	56	53	57	56	54	55.2	1.48
Apr. 27	16	58	57	56	54	54	55.8	2.97
	17	55	55	55	56	53	54.8	0.00
	18	54	57	54	55	54	54.8	0.00
	19	54	53	56	53	55	54.2	1.48
	20	53	53	57	54	53	54.0	0.00
	21	53	55	57	56	55	55.2	1.48
	22	59	54	53	54	55	55.0	1.48
	23	54	55	58	55	54	55.2	1.48
	24	56	53	51	55	59	54.8	2.97
	25	56	55	55	55	55	55.2	0.00
							54.32	1.48

	Grand mean	Mean of MAD

5.2 Robust control chart for mean based on MAD

The 3σ control limits based on MAD suggested by Moustafa Omar Ahmed Abu-Shawiesh (2008) are

$$\bar{\bar{X}} \pm \left(3 \times \frac{b_n \overline{MAD}}{\sqrt{n}}\right)$$

$$UCL_{\bar{\bar{X}}.Rob} = \bar{\bar{X}} + \left(3 \times \frac{b_n \overline{MAD}}{\sqrt{n}}\right) = 54.32 + \left(3 \times \frac{(1.206 \times 1.48}{\sqrt{5}}\right) = 54.32 + 2.4 = 56.72$$

$$CL_{\bar{\bar{X}}.Rob} = \bar{\bar{X}} = 54.32$$

$$LCL_{\bar{\bar{X}}.Rob} = \bar{\bar{X}} - \left(3 \times \frac{b_n \overline{MAD}}{\sqrt{n}}\right) = 54.32 - \left(3 \times \frac{(1.206 \times 1.48}{\sqrt{5}}\right) = 54.32 - 2.4 = 51.92$$

Table 2: The correction factor bn for various sample size

Sample size (n)	b_n	c_4
2	1.196	0.7979
3	1.495	0.8862
4	1.363	0.9213
5	1.206	0.9400
6	1.200	0.9515
7	1.140	0.9594
8	1.129	0.9650
9	1.107	0.9693
10	1.087	0.9727

5.3 Construction of Robust control chart for mean using process capability based on MAD

For a given TL = 0.023 (USL - LSL = 0.0024 – 0.001) & Cp = 2.0, it is found the value of σ_{RCP} is 0.25. The control limits based on MAD using process capability for mean for a specified TL with the control limits

$$\bar{\bar{X}} \pm \left(3 \times \frac{\sigma_{RPC}}{\sqrt{n}}\right)$$

$$UCL_{\bar{\bar{X}}.RPC} = \bar{\bar{X}} + \left(3 \times \frac{\sigma_{RPC}}{\sqrt{n}}\right) = 54.32 + \left(3 \times \frac{0.25}{\sqrt{5}}\right) = 54.32 + 0.34 = 54.66$$

$$CL_{\bar{\bar{X}}.RPC} = \bar{\bar{X}} = 54.32$$

$$LCL_{\bar{\bar{X}}.RPC} = \bar{\bar{X}} - \left(3 \times \frac{\sigma_{RPC}}{\sqrt{n}}\right) = 54.32 - \left(3 \times \frac{0.25}{\sqrt{5}}\right) = 54.32 - 0.34 = 53.98$$

Table 3: The control chart comparison

Control Limits	Shewhart using Mean	Robust using bnMAD	Robust using based on MAD using process capability
LCL	51.07	51.92	53.98
CL	54.32	54.32	54.32
UCL	57.57	56.72	54.66
CLIs	6.49	4.79	0.67

It is found from the results that the many points fall outside the control limits in the robust control chart for mean based on MAD using process capability than the other charts namely Shewhart mean chart and robust using MAD. It is clear that the product/Service is not in good Quality as expected using the robust chart based on MAD using process capability, so a correction is needed in the process/System.

6. CONCLUSION

We have modified the simple robust control limits for monitoring process data when the standard value of sigma is not given and Median Absolute Deviation (MAD) is used to give an estimate. A JAVA programme has also been used for the proposed control limits and the computation of control limit interval. From the results in Table 3, it is clear that the mean chart for the modified control limits using process capability is more efficient than the earlier control limits for all the distributional data sets under consideration. Therefore, the modified control limits proposed by process capability derived in this work for the mean chart can be used when monitoring non-normal data or when there is no standard value of sigma specify by the process engineer/management.

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