Laplacian Minimum Dominating Energy of some special classes of Graphs.

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Abstract:

In this paper ,we defined set energy of a graph .We also study the special case of a set S and the corresponding Laplacian minimum dominating energy of S for some special classes of graphs . We also attained their bounds .

Keywords :

Minimum dominating set ,Laplacian minimum dominating matrix, Laplacian minimum dominating eigen values ,Laplacian minimum dominating energy of a graph.

Subject classification : 05C50,05C69.

1.Introduction :

The concept of energy of a graph was introduced by I.Gutman [8] in 1978.

Let G=(V,E) be a simple, finite, connected undirected graph with order n and size m. A dominating set in G is a subset D of V(G) such that each element of V(G) –D is adjacent to atleast one vertex of D by means of a matrix as follows; in the adjacency matrix A(G) of G replace the a_{ii} by 1 if and only if Vi ϵ S and A= (a_{ij}) be the adjacency matrix of the graph G and the eigen values of the adjacency matrix are $\lambda_1,\lambda_2\ldots\lambda_n$. It is assumed that these eigen values are in the non increasing order.

The Energy E(G) of a graph G is defined to be the sum of the absolute values of the eigen values of G .ie, E(G) $=\sum_{i=1}^{n}|\lambda_i|$. For the details on the mathematical aspects of the theory of graph energy, we can make reviews [11] and the references cited therein.

I.Gutman and B.Zhou [2] defined the Laplacian energy of a graph G in the year 2006.The Laplacian matrix of the graph G denoted by $L=(L_{ij})$ is a square matrix of order n whose elements are defined as

$$L_{ij} = \begin{cases} -1 \text{ if } v_i \text{ and } v_j \text{ are adjacent} \\ 0 \text{ if } v_i \text{ and } v_j \text{ are not adjacent} \\ d_i \text{ if } i = j, \text{where } d_i \text{ is the degree of the vertex} \end{cases}$$

Let $\mu_1,\mu_2,\ldots\ldots\ldots.\mu_n$ be the Laplacian eigen values of G Laplacian energy L[E(G)] of G is defined as $L[E(G)] = \sum_{i=1}^{n}\left|\mu_i - \frac{2m}{n}\right|$. The basic properties including various upper and lower bounds for Laplacian energy of a graph G have been established in [5,6,7,13,14,19,20,21] and it has found remarkable chemical applications, the molecular orbital theory of conjugated molecules [7].

Definition 1.1 :

The minimum Dominating Energy of a graph G:

Let G= (V,E) be a simple graph of order n with vertex set $V=\{v_1,v_2,\dots v_n\}$ and size m. A non-empty subset D of V is called a dominating set of G if every vertex in V-D is adjacent to atleast one vertex in D. Any Dominating set with minimum cardinality is called the minimum dominating set .

Let D be the minimum dominating set of a graph G.The adjacency matrix of the minimum dominating set of G is the (n×n) matrix denoted by $A_D(G)$ and it is defined as $A_D(G) = (a_{ij})$ Where

$$a_{ij} = \begin{cases} 1 \text{ if } v_i, v_j \in D \\ 1 \text{ if } i = j \text{ and } v_i \in D \\ 0 \text{ otherwise}. \end{cases}$$

The characteristic polynomial of $A_D(G)$ is denoted by $f_n(G,\lambda) = \det(\lambda_i - A_D(G))$. The minimum dominating eigen values of the graph G are the eigen values of $A_D(G)$. Since $A_D(G)$ is real and symmetric matrix its eigen values are real numbers and we label them in non-increasing order $\lambda_1 \geq \lambda_2 \geq \dots\dots \geq \lambda_n$.

The minimum dominating energy of G is defined as

$$E_D(G) = \sum_{i=1}^{n} |\lambda_i|$$ and the trace of $A_D(G)$ = Domination number of the graph G.

Definition 1.2:

Laplacian minimum Dominating energy of a graph G:

Let D(G) be the diagonal matrix denoting the vertex degrees of the adjacent vertices of the graph G . The Laplacian minimum dominating matrix of G is denoted by $L_D(G)$ and it is defined as $L_D(G)=D(G)-A_D(G)$.Let $\mu_1,\mu_2,\dots\mu_n$ be the eigen values of $L_D(G)$,arranged in non-increasing order . These eigen values are called Laplacian minimum dominating eigen values of G.

The Laplacian minimum dominating energy of the graph G is defined as

$$LE_D(G)=\sum_{i=1}^{n}\left|\mu_i - \frac{2m}{n}\right|$$ where m and n are the order and the size of the graph G and $\frac{2m}{n}$ denotes the average degree of G.

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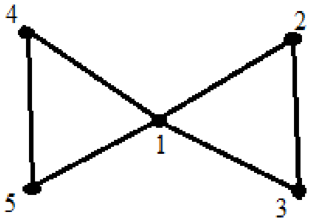
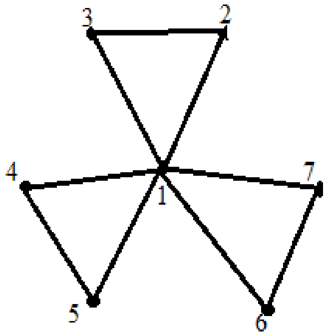
Definition 2.1:

Friendship graph or Dutch Windmill graph :

Friendship graph is a planar undirected graph with order 2n+1 and size 3n. It states that the finite graphs with the property that every two vertices have exactly one neighbour in common .





Theorem 2.1 :

For $n \geq 2$, the Laplacian minimum dominating energy of friendship graph $k_{1,n}$ or n-fan graph is $\frac{n(8n+1)}{2n+1}$ approximately.

Proof:

Consider the friendship graph $k_{1,n}$ with vertex V={1,2,3….n} with centre at 1.Then minimum dominating set is D={1} and hence the domination number $\gamma(G)$ =1 .

Consider the adjacency matrix of the minimum dominating matrix of $k_{1,n}$ is

$$A_D(K_{1,n}) = \begin{pmatrix} 1 & 1 & 1 & \dots\dots & 1 \\ 1 & 1 & 1 & \dots\dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & \dots\dots & 1 \end{pmatrix}_{(2n+1)X(2n+1)}$$

and the diagonal matrix of $K_{1,n}$ is

$$D(K_{1,n}) = \begin{pmatrix} 2n & 0 & 0 & \dots\dots & 0 \\ 0 & 2 & 0 & \dots\dots & 0 \\ 0 & 0 & 2 & \dots\dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots\dots & 2 \end{pmatrix}_{(2n+1)X(2n+1)}$$

The Laplacian minimum dominating matrix of $K_{1,n}$ is given by

$L_D(K_{1,n}) = D(K_{1,n}) - A_D(K_{1,n})$.

$$L_D(K_{1,n}) = \begin{pmatrix} 2n-1 & -1 & -1 & \ldots\ldots & -1 \\ -1 & 1 & -1 & \ldots\ldots & 0 \\ -1 & -1 & 1 & \ldots\ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & 0 & 0 & \ldots\ldots & 1 \end{pmatrix}_{(2n+1)X(2n+1)}$$

The characteristic equation of $L_D(K_{1,n})$ is $\mu(\mu-1)^{n-1}(\mu-3)^n(\mu-2n) = 0$

Average degree of the Friendshipship graph $K_{1,n} = \frac{6n}{2n+1}$

Laplacian minimum dominating energy of

$$LE_D(K_{1,n}) = \left|0-\frac{6n}{2n+1}\right| + \left|1-\frac{6n}{2n+1}\right|(n-1) + \left|3-\frac{6n}{2n+1}\right|n + \left|2n-\frac{6n}{2n+1}\right|$$

$$= \left|\frac{6n}{2n+1}\right| + \left|\frac{2n+1-6n}{2n+1}\right|(n-1) + \left|\frac{3(2n+1)-6n}{2n+1}\right|n + \left|\frac{2n(2n+1)-6n}{2n+1}\right|$$

$$= \frac{n(8n+1)}{2n+1}.$$

Definition 2.2:

Wheel Graph

Wheel graphs are planar graphs. A wheel graph W_n is a graph with order n and size 2(n-1) (n≥4) formed by connecting a single vertex to all the vertices of a (n-1) cycle .

Theorem 2.2 :

For $n \geq 2$, the Laplacian minimum dominating energy of the Wheel graph $W_{1,n-1}$ is $\frac{2(n^2-5n+2)}{n}$ approximately.

Proof:

Consider the wheel graph W_n with vertex set $\{v_1,v_2,v_3\ldots.v_n\}$. The minimum dominating set is D = $\{v_1\}$and hence the domination number $\gamma(G)$ =1. Consider the adjacency matrix of the minimum dominating set of $W_{1,n-1}$.

$$\begin{pmatrix} 1 & 1 & 1 & 1 & \ldots\ldots & 1 \end{pmatrix}$$

$$A_D(W_{1,n-1}) = \begin{matrix} 1 & 1 & 1 & 0 & \ldots\ldots & 1 \\ 1 & 1 & 1 & 1 & \ldots\ldots & 0 \\ \ldots & & & & & \\ \ldots & & & & & \\ 1 & 1 & 0 & 0 & \ldots\ldots & 1 \end{matrix} \quad (nXn)$$

The diagonal matrix of $W_{1,n-1}$ is

$$D(W_{1,n-1}) = \begin{pmatrix} n-1 & 0 & 0 & \ldots\ldots & 0 \\ 0 & 3 & 0 & \ldots\ldots & 0 \\ 0 & 0 & 3 & \ldots\ldots & 0 \\ \ldots & & & & \\ \ldots & & & & \\ 0 & 0 & 0 & \ldots\ldots & 3 \end{pmatrix} (nXn)$$

The Laplacian minimum dominating matrix is given by

$L_D(W_{1,n-1}) = D(W_{1,n-1,}) - A(W_{1,n-1})$

$$L_D(W_{1,n-1}) = \begin{pmatrix} n-2 & -1 & -1 & \ldots\ldots & -1 \\ -1 & 2 & -1 & \ldots\ldots & -1 \\ -1 & -1 & 2 & \ldots\ldots & 0 \\ \ldots & & & & \\ \ldots & & & & \\ -1 & -1 & 0 & \ldots\ldots & 2 \end{pmatrix} (nXn)$$

The characteristic equation of $L_D(W_{1,n-1})$ is $\mu(\mu-2)^{\frac{n-3}{2}}(\mu-4)^{\frac{n-3}{2}}(\mu-n+2)(\mu-n+1)=0$

Average degree of the wheel graph $W_{1,n-1} = \frac{4(n-1)}{n}$.

Laplacian minimum dominating energy of $L_D(W_{1,n-1})$ is

$$LE_D(W_{1,n-1}) = \left|\frac{4(n-1)}{n}\right| + \left|2-\frac{4(n-1)}{n}\right|\left(\frac{n-3}{2}\right) + \left|4-\frac{4(n-1)}{n}\right|\left(\frac{n-3}{2}\right) + \left|(n-2)-\frac{4(n-1)}{n}\right| + \left|(n-1)-\frac{4(n-1)}{n}\right|$$

$$= \frac{4(n-1)}{n} + \frac{(2n-4)(n-3)}{2n} + \frac{4(n-3)}{2n} + \frac{n^2-6n+4}{n} + \frac{n^2-5n+4}{n}$$

$$= \frac{2(n^2-5n+2)}{n}.$$

3. Upper bounds of Laplacian minimum dominating energy of Friendship and Wheel graph

Theorem:

Let G be a graph with order n and size m and D is the minimum dominating set of G then

$$LE_D(G) \leq \sqrt{2Mn + 4m(1-m)}$$

Proof :

Cauchy schwarz inequality is

$$(\sum_{i=1}^{n} a_i b_i)^2 \leq (\sum_{i=1}^{n} a_i^2)(\sum_{i=1}^{n} b_i^2)$$

Put $a_i = 1$, $b_i = \left| \mu_i - \frac{2m}{n} \right|$ then

$$(\sum_{i=1}^{n} \left|\mu_i - \frac{2m}{n}\right|)^2 \leq (\sum_{i=1}^{n} 1)(\sum_{i=1}^{n} \left|\mu_i - \frac{2m}{n}\right|)^2$$

ie., $[LE_D(G)]^2 = n\, [\sum_{i=1}^{n} \mu_i^2 + \sum_{i=1}^{n} \frac{4m^2}{n^2} - \frac{4m}{n} \sum_{i=1}^{n} \mu_i]$

$$= n[\, 2M + \frac{4m^2}{n^2} n - \frac{4m}{n}(2m - |D|)]$$

$$= n\, [2M + \frac{4m^2}{n} - \frac{8m^2}{n} + \frac{4m|D|}{n}]$$

$$= 2Mn + 4m\,(|D| - m)$$

$$LE_D(G) \leq \sqrt{2Mn + 4m(|D| - m)}$$

In both Friendship and Wheel graph , the cardinality of minimum dominating set is one.

ie., $|D| = 1$

Hence $LE_D(G) \leq \sqrt{2Mn + 4m(1-m)}$.

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