FUZZY SOFT IDEALS OF A FUZZY SOFT LATTICE

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Abstract ***In this paper, we define the concept of fuzzy soft ideals and filters over a collection of fuzzy soft sets, study their related properties and illustrate them with some examples. We also define the maximum and minimum conditions in fuzzy soft lattice. In addition, we characterized fuzzy soft modularity and fuzzy soft distributivity of fuzzy soft lattices of fuzzy soft ideals.***

Index Terms ***Fuzzy soft ideals and filters, Prime fuzzy soft ideals and filters, Principal fuzzy soft ideal and filter, Modular fuzzy soft lattices, Distributive fuzzy soft lattices.***

I. INTRODUCTION

The theory of soft sets was firstly introduced by Molodtsov[11] in 1999 as a general Mathematical tool for dealing with uncertainty. At present, research works on soft set theory and its application are making progress rapidly. The theory of fuzzy set was introduced by L.A.Zadeh [14] in 1965.Fuzzy set is to used in many areas of daily life such as Engineering,Medicine ,Meteorology. The theory of lattices was introduced by Richard Dedikind. Faruk karaaslam and Naim cagman [7] defined the concept of modular fuzzy soft lattice and distributive fuzzy soft lattice. In this paper we define the concept of fuzzy soft ideal and filter,prime fuzzy soft ideal and filter, principal fuzzy soft ideal and filter. Also, we prove that set of fuzzy soft ideals of a fuzzy soft lattice. Further, we prove fuzzy soft lattice $f_L$is modular if and only if the fuzzy soft ideal lattice $f_I(f_L)$.we also prove that the fuzzy soft lattice f_L is distributive if and if the fuzzy soft ideal lattice $f_I(f_L)$ is distributive.

The readers are asked to refer[10,11] for basic definitions and results of fuzzy soft set theory and [7,12,13] for results on fuzzy soft lattices.

Throughout this work, X refers to the initial universe, P(X) is the power set of X, E is a set of parameters and $A \subseteq E$.F(X) denotes the set of all fuzzy soft sets over X.

II. FUZZY SOFT IDEALS AND FUZZY SOFT FILTERS

In this section we introduce the concept of fuzzy soft ideals and fuzzy soft filters with examples. We prove that every fuzzy soft ideal and fuzzy soft filter of a fuzzy soft lattice $f_L$is a convex fuzzy soft sublattice of f_L and conversely. We also study about prime fuzzy soft ideals and prime fuzzy soft filters. Throughout this work, the fuzzy soft lattice f_L means the fuzzy soft lattice $(f_L,\curlywedge,\curlyvee)$.

2.1 Definition

A non – empty fuzzy soft subset f_I of a fuzzy soft lattice f_L is said to be fuzzy soft ideal if

(f_{I_1}) $f_I(x), f_I(y) \in f_I$ implies $f_I(x) \curlyvee f_I(y) \in f_I$.

(f_{I_2}) $f_I(x) \in f_I$ implies $f_I(x) \curlywedge f_I(a) \in f_I$ for every element $f_L(a)$ of $f_L$or equivalently $f_I(x) \in f_I$ and $f_L(a) \preccurlyeq f_I(x)$ implies $f_I(a) \in f_I$.

2.2 Definition

A non – empty fuzzy soft subset f_F of a fuzzy soft lattice f_L is said to be fuzzy soft filter if

(f_{F_1}) $f_F(x), f_F(y) \in f_F$ implies $f_F(x) \curlywedge f_F(y) \in f_F$.

(f_{F_2}) $f_F(x) \in f_F$ implies $f_F(x) \curlyvee f_F(a) \in f_F$ for every element $f_L(a)$ of $f_L$or equivalently $f_F(x) \in f_F$ and $f_F(x) \preccurlyeq f_L(a)$ implies $f_F(a) \in f_F$.

2.3 Note

Every fuzzy soft ideal of a fuzzy soft lattice of f_L is a fuzzy soft sublattice of f_L.

2.4 Example

Let $X = \{x_1, x_2, x_3 x_4, x_5, x_6, x_7, x_8, x_9\}$, be the universe and $E = \{e_1, e_2, e_3\}$ be the set of parameters, $P = \{e_1\}$, $Q = \{e_2\}$, $R = \{e_3\}$, $S = \{e_1, e_2\}$,$T = \{e_1, e_3\}$, $U = \{e_2, e_3\}$, $V = \{e_1, e_2, e_3\}$ where $P, Q, R, S, T, U, V \subseteq E$ and $f_L = \{f_L(\emptyset), f_L(P), f_L(Q), f_L(R), f_L(S), f_L(T), f_L(U), f_L(V)\} \subseteq F(X)$ with the operations $\tilde{\cup}$ and $\tilde{\cap}$.

Assume that, $f_L(\emptyset) = \emptyset$

$f_L(P) = \{ (e_1, \{x_1\})\}$

$f_L(Q) = \{(e_2, \{x_2\})\}$

$f_L(R) = \{ (e_3, \{x_3\})\}$

$f_L(S) = \{(e_1, \{x_1, x_4\}), (e_2, \{x_2, x_5\})\}$

$f_L(T) = \{(e_1, \{x_1, x_6\}), (e_3, \{x_3, x_7\})\}$

$f_L(U) = \{(e_2, \{x_2, x_8\}), (e_3, \{x_3, x_9\})\}$

$f_L(V) = \{(e_1, \{x_1, x_4, x_6\}), (e_2, \{x_2, x_5, x_8\}), (e_3, \{e_3, e_7, e_9\})\}$

Then $((f_L, \tilde{\cup}, \tilde{\cap})$ is a fuzzy soft lattice. The Hasse diagram of it appears in figure 1.

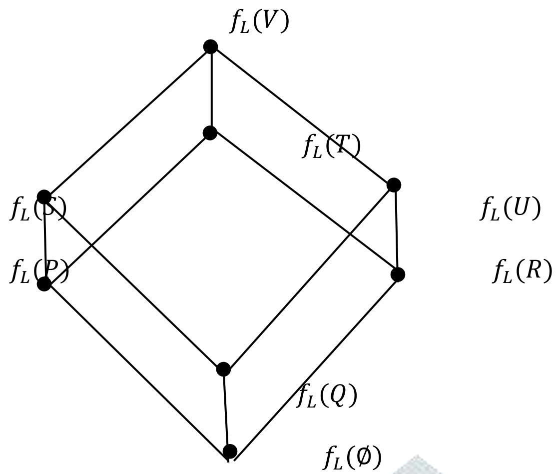

Figure 1

(a) Consider the fuzzy soft set $f_I = \{f_\emptyset, f_P, f_Q, f_S\} \tilde{\subseteq} f_L$ clearly $f_I \neq f_\emptyset$.It also satisfies the properties f_{I_1} and f_{I_2}. Hence f_I is a fuzzy soft ideal of f_L.

(b) Consider the fuzzy soft set $f_I = \{f_P, f_S, f_T, f_V\} \tilde{\subseteq} f_L$ clearly $f_I \neq f_\emptyset$.It also satisfies the properties f_{I_1} but $f_I(T) \in f_I$ and $f_I(R) \preccurlyeq f_I(T)$ implies $f_I(R)$does not belong to f_I. Hence f_I is not a fuzzy soft ideal of f_L.

(c) Consider the fuzzy soft set $f_F = \{f_Q, f_S, f_U, f_V\} \tilde{\subseteq} f_L$ clearly $f_F \neq f_\emptyset$.It also satisfies the properties f_{F_1} and f_{F_2}. Hence f_F is a fuzzy soft filter of f_L.

(d) Consider the fuzzy soft set $f_F = \{f_\emptyset, f_P, f_R, f_T\} \tilde{\subseteq} f_L$ clearly $f_F \neq f_\emptyset$.It also sastisfies the properties f_{F_1} but $f_F(P) \in f_F$ and $f_F(P) \preccurlyeq f_F(S)$ implies $f_F(S)$does not belong to f_F. Hence f_F is not a fuzzy soft filter of f_L.

2.5 Theorem

Every fuzzy soft ideal and fuzzy soft filter of a fuzzy soft lattice f_L is a convex fuzzy soft sublattice of f_L. Conversely, every convex fuzzy soft sublattice of f_L is the fuzzy soft intersection of a fuzzy soft ideal and fuzzy soft filter.

Proof

Let f_I be an fuzzy soft ideal of f_L. Let $f_I(a), f_I(b) \in f_I$.then by (f_{I_1}), $f_I(a) \curlyvee f_I(b) \in f_I, f_I(a) \curlywedge f_I(b) \preccurlyeq f_I(a)$, $f_I(a) \in f_I$ implies $f_I(a) \curlywedge f_I(b) \in f_I$.Therefore f_I is a fuzzy soft sublattice of f_L.Let $f_I(x), f_I(y) \in f_I$ and $f_I(x) \preccurlyeq f_I(y)$.Then $(f_I(y)] = f_L(a) \in f_L / f_L(a) \preccurlyeq f_I(y) \tilde{\subseteq} f_I$. $f_I(x) \preccurlyeq f_I(y)$ implies $(f_I(x), f_I(y) \tilde{\subseteq} (f_I(y)] \tilde{\subseteq} f_I$.therefore $[f_I(x), f_I(y)] \tilde{\subseteq} f_I$.hence f_I is a convex fuzzy soft sublattice of f_L. Similarly, $[f_I(x), f_I(y)] \tilde{\subseteq} [f_I(x) \tilde{\subseteq} f_F$. Hence f_F is a convex fuzzy soft lattice of f_L.Conversely, let f_K be a convex fuzzy soft sublattice of f_L.Let $f_I = \{ f_L(a) \in f_L / f_L(a) \preccurlyeq f_K(v)$ for some $f_K(v) \in f_K\}$. Clearly $f_I(\emptyset) \in f_I$and hence f_I is non-empty. Let $f_I(a), f_I(b) \in f_I$. Then there exist $f_K(v_1), f_K(v_2) \in f_K$ such that $f_I(a) \preccurlyeq f_K(v_1)$ and $f_I(b) \preccurlyeq f_K(v_2)$.since $f_K(v_1), f_K(v_2) \in f_K$,$f_K(v_1) \curlyvee f_K(v_2) \in f_K$. Also since$f_I(a) \curlyvee f_I(b) \preccurlyeq f_K(v_1) \curlyvee f_K(v_2)$,$f_I(a) \curlyvee f_I(b) \in f_I$.Suppose $f_I(a) \in f_I$ and $f_I(b) \preccurlyeq f_I(a)$.then there exist $f_K(v) \in f_K$such that $f_I(a) \preccurlyeq f_K(v)$. Since $f_I(b) \preccurlyeq f_I(a), f_I(b) \preccurlyeq f_K(v)$.therefore $f_I(b) \in f_I$.hencef_I is a fuzzy soft ideal of f_L.

Let $f_F = \{ f_L(a) \in f_L / f_K(w) \preccurlyeq f_L(a)$ for some$f_K(w) \in f_K\}$.Clearly $f_I(\emptyset) \in f_F$and hence f_F is non-empty. Let$f_F(a), f_F(b) \in f_F$. Then there exist $f_K(w_1), f_K(w_2) \in f_K$ such that $f_K(w_1) \preccurlyeq f_F(a)$ and $f_K(w_2) \preccurlyeq f_F(b)$.since$f_K(w_1), f_K(w_2) \in f_K$,$f_K(w_1) \curlywedge f_K(w_2) \in f_K$. Also since$f_K(w_1) \curlywedge f_K(w_2) \preccurlyeq f_F(a) \curlywedge f_F(b)$, $f_F(a) \curlywedge f_F(b) \in f_F$.Suppose $f_F(a) \in f_F$ and $f_F(a) \preccurlyeq f_F(b)$.then there exist $f_K(w) \in f_K$such that $f_K(w) \preccurlyeq f_F(a)$. Since$f_F(a) \preccurlyeq f_F(b), f_K(w) \preccurlyeq f_F(b)$.therefore $f_F(b) \in f_F$.hencef_F is a fuzzy soft filter of f_L.

Let $f_K(a) \in f_K$.then $f_K(a) \preccurlyeq f_K(a)$ for some $f_K(a) \in f_K$ $f_I(a) \in f_I$and $f_K(a) \preccurlyeq f_K(a)$forsome$f_K(a) \in f_K$,$f_F(a) \in f_F$.Therefore $f_{K\tilde{\cap}F}(a) \in f_I \tilde{\cap} f_F$.Hence $f_K \tilde{\subseteq} f_I \tilde{\cap} f_F$.Let $f_{I\tilde{\cap}F}(a) \in f_I \tilde{\cap} f_F$.then $f_I(a) \in f_I$and $f_F(a) \in f_F$.therefore there exists $f_K(v) \in f_K$, $f_K(w) \in f_K$ such that $f_I(a) \preccurlyeq f_K(v)$ and $f_K(w) \preccurlyeq f_F(a)$. Therefore $f_K(w) \preccurlyeq f_{I \cap F}(a) \preccurlyeq f_K(v)$ for some $f_K(v), f_K(w) \in f_K$. Since $f_K$is a convex fuzzy soft sublattice, $[f_K(w), f_K(v)] \tilde{\subseteq} f_K$implies $f_K(a) \in f_K$.therefore $f_I \tilde{\cap} f_F \tilde{\subseteq} f_K$ hence$f_K = f_I \tilde{\cap} f_F$.

2.6 Definition

A fuzzy soft ideal $f_I$of the fuzzy soft lattice f_L is said to be a prime fuzzy soft ideal if and only if atleast one of an arbitrary pair of elements whose meet is in f_L is contained in f_I.

ie., $f_I(a) \curlywedge f_I(b) \in f_I$ implies $f_I(a) \in f_I$ or $f_I(b) \in f_I$.

2.7 Definition

A fuzzy soft filter $f_F$of the fuzzy soft lattice f_L is said to be a prime fuzzy soft filter, if $f_F(a) \curlyvee f_F(b) \in f_F$ implies $f_F(a) \in f_F$ or $f_F(b) \in f_F$.

2.8 Definition

Let f_L be a fuzzy soft lattice. Let $f_L(x) \in f_L$. Then $\{f_L(a) \in f_L / f_L(a) \preccurlyeq f_L(x)\}$ is a fuzzy soft ideal and is called the principal fuzzy soft ideal generated by $f_L(x)$ and is denoted by $(f_L(x)]$.

2.9 Definition

Let f_L be a fuzzy soft lattice. Let $f_L(x) \in f_L$. Then $\{f_L(a) \in f_L / f_L(x) \preccurlyeq f_L(a)\}$ is a fuzzy soft filter and is called the principal fuzzy soft filter generated by $f_L(x)$ and is denoted by $(f_L(x)]$.

2.10 Example

Let $X = \{x_1, x_2, x_3 x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$, be the universe and $E = \{e_1, e_2, e_3\}$ be the set of parameters, $A = \{e_1\}$, $B = \{e_1, e_2\}$, $C = \{e_1, e_3\}$, $D = \{e_1, e_2, e_3\}$. where $A, B, C, D \subseteq E$ and $f_L = \{f_L(\emptyset), f_L(A), f_L(B), f_L(C), f_L(D)\} \subseteq F(X)$ with the operations $\tilde{\cup}$ and $\tilde{\cap}$.

Assume that, $f_L(\emptyset) = \emptyset$

$f_L(A) = \{ (e_1, \{x_1, x_2\})\}$

$f_L(B) = \{(e_1, \{x_1, x_2, x_3\}), (e_2, \{x_5, x_6\})\}$

$f_L(C) = \{(e_1, \{x_1, x_2, x_4\}), (e_3, \{x_7, x_8\})\}$

$f_L(D) = \{(e_1, \{x_1, x_2, x_3, x_4\}), (e_2, \{x_5, x_6, x_9\}), (e_3, \{e_7, e_8, e_{10}\})\}$

Then $((f_L, \tilde{\cup}, \tilde{\cap})$ is a fuzzy soft lattice. The Hasse diagram of it appears in figure 2.

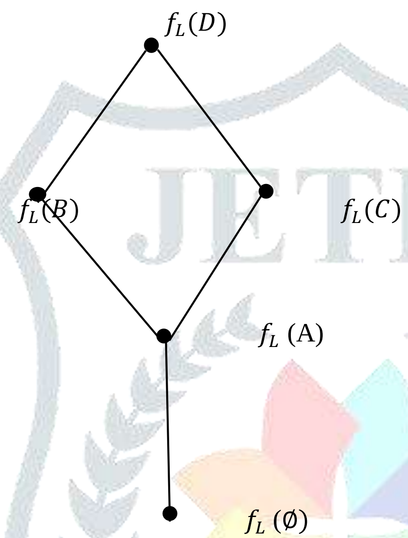

Figure 2

(a) Consider the fuzzy soft ideal $f_I = \{f_\emptyset, f_A, f_B\}$.now, $f_I(B) \wedge f_I(C) = f_I(A) \in f_I$ implies $f_I(B) \in f_I$.Hence f_I is a prime fuzzy soft ideal of f_L.

(b) $(f_I(B)] = \{f_I(\emptyset), f_I(A), f_I(B)\}$ is a principal fuzzy soft ideal generated by $f_I(B)$.

(c) consider the fuzzy soft ideal $f_I = \{f_I(\emptyset), f_I(A)\}$. Let $f_I(B), f_I(C) \in f_L$. Then $f_I(B) \wedge f_I(C) = f_I(A) \in f_I$ implies $f_I(B) \in f_I$.Hence f_I is a prime fuzzy soft ideal of f_L.

2.11 Example

Let $X = \{x_1, x_2, x_3 x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$, be the universe and $E = \{e_1, e_2, e_3\}$ be the set of parameters, $A = \{e_1\}$, $B = \{e_2\}$, $C = \{e_1, e_2\}$, $D = \{e_1, e_2, e_3\}$ where $A, B, C, D \subseteq E$ and $f_L = \{f_L(\emptyset), f_L(A), f_L(B), f_L(C), f_L(D)\} \subseteq F(X)$ with the operations $\tilde{\cup}$ and $\tilde{\cap}$.

Assume that, $f_L(\emptyset) = \emptyset$

$f_L(A) = \{ (e_1, \{x_1, x_2\})\}$

$f_L(B) = \{(e_1, \{x_3, x_4\})$

$f_L(C) = \{(e_1, \{x_1, x_2, x_5\}), (e_3, \{x_3, x_4, x_6\})\}$

$f_L(D) = \{(e_1, \{x_1, x_2, x_5, x_7\}), (e_2, \{x_3, x_4, x_6, x_8\}), (e_3, \{e_9, e_{10}\})\}$

Then $((f_L, \tilde{\cup}, \tilde{\cap})$ is a fuzzy soft lattice. The Hasse diagram of it appears in figure .

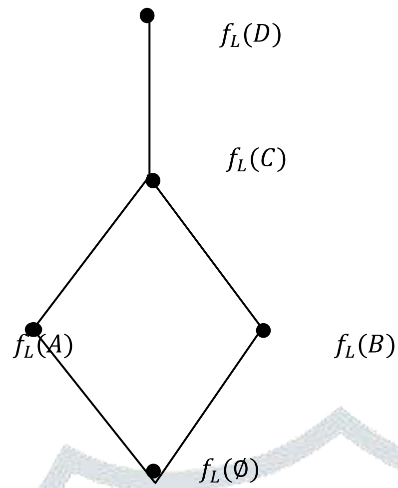

Figure 3.

(a) Consider the fuzzy soft filter $f_F = \{f_A, f_C, f_D\}$.now, $f_I(A) \curlyvee f_I(B) = f_F(C) \in f_F$ implies $f_F(A) \in f_F$.Hence f_F is a prime fuzzy soft filter of f_L.

(b) consider the fuzzy soft filter $f_F = \{f_F(C), f_F(D)\}$. Let $f_F(B), f_F(C) \in f_L$. Then $f_F(B) \curlyvee f_F(A) = f_F(C) \in f_F$ implies $f_F(B) \in f_F$.Hence f_F is a prime fuzzy soft filter of f_L.

(c) $(f_F(A)] = \{f_F(A), f_F(C), f_F(D)\}$ is a principal fuzzy soft filter generated by $f_F(A)$.

2.12 Theorem

Every fuzzy soft lattice has almost one minimal and one maximal element. These elements are at the same time the least and greatest element of that fuzzy soft lattice.

Proof

If possible, let there be two minimal elements $f_L(m), f_L(n) \in f_L$, then $f_L(m) \curlywedge f_L(n) \preccurlyeq f_L(m)$. Since $f_L(m)$ is a minimal element, $f_L(m) \curlywedge f_L(n) \curlywedge f_L(m)$ is impossible. Therefore $f_L(m) \curlywedge f_L(n) = f_L(m)$ and hence $f_L(m) \preccurlyeq f_L(n)$.

Similarly we take$f_L(m) \curlywedge f_L(n) \preccurlyeq f_L(n)$, then $f_L(n) \preccurlyeq f_L(m)$. Therefore $f_L(m) = f_L(n)$. Hence the fuzzy soft lattice f_L has atmost one minimal element and it is the least element of the lattice. By the principle of duality, every fuzzy soft lattice has atmost one maximal element and it is the greatest element of that fuzzy soft lattice.

2.13 Definition

An element $f_L(x)$ of a fuzzy soft lattice f_L is called a greatest element of the fuzzy soft lattice $f_L$if $f_L(a) \preccurlyeq f_L(x)$ for all $f_L(a) \in f_L$.similarly an element $f_L(a)$ of a fuzzy soft lattice $f_L$is called a least element of $f_L$if $f_L(x) \preccurlyeq f_L(a)$ for all $f_L(a) \in f_L$.

III. THE MAXIMUM AND MINIMUM CONDITIONS

In this section, we define the maximum and minimum conditions in fuzzy soft lattice. We also obtain a necessary and sufficient conditions for a fuzzy soft lattice to satisfy the maximum condition. we also define $\curlywedge$ and $\curlyvee$ of two fuzzy soft ideals and we prove that the set of all fuzzy soft ideals of a fuzzy soft lattice.

3.1 Definition

Let f_{C_0}be any element of a poset f_P in the fuzzy soft lattice. Let us form the subchain of $f_C$in the following way: let the greatest element of the subchain be f_{C_0} Let f_{C_K} $(f_K \succcurlyeq 1)$ be an element of f_P such that $f_{C_K} \prec f_{C_{K-1}}$. If each of the chains so formed, commencing at any f_{C_K} is finite, then $f_P$is said to satisfy the mamimum condition.

3.2 Definition

Let the least element of the subchain be f_{C_0} Let f_{C_K} $(f_K \succcurlyeq 1)$ be an element of f_P such that $f_{C_{K-1}} \prec f_{C_K}$. If each of the chains so formed, commencing at any f_{C_K} is finite, then f_P is said to satisfy the minimum condition.

3.3 Result

If a poset $f_P$in a fuzzy soft lattice satisfies the minimum condition then for any $f_P(a) \in f_P$,there exist atleast one minimal element $f_P(m)$ of f_P such that $f_L(m) \preccurlyeq f_L(a)$.

3.4 Result

If a poset $f_P$in a fuzzy soft lattice satisfies the maximum condition then for any $f_P(a) \in f_P$,there exist atleast one maximal element $f_P(m)$ of f_P such that $f_L(a) \preccurlyeq f_L(m)$.

3.5 Corollary

Every fuzzy soft lattice satisfying minimum (maximum) conditions has a least (greatest) element.

3.6 Note

By a fuzzy soft ideal chain of a fuzzy soft lattice f_L,we shall mean a set of fuzzy soft ideals in f_L in which one of every pair of fuzzy soft ideals includes the other.

3.7 Lemma

The fuzzy soft union of any fuzzy soft ideal chain of a fuzzy soft lattice $f_L$is itself a fuzzy ideal in f_L.

Proof

Let f_C be a chain of fuzzy soft ideals of f_L.let $f_I$denote the fuzzy soft union of all fuzzy soft ideals of $f_L$in f_C. Let $f_I(x), f_I(y) \in f_I$ Then there exists fuzzy soft ideals f_{I_1} and f_{I_2} in f_C such that $, f_I(x) \in f_I$ and $f_I(y) \in f_I$.Since either$f_{I_1} \mathrel{\widetilde{\subseteq}} f_{I_2}orf_2 \mathrel{\widetilde{\subseteq}} f_{I_1}$.Let$f_{I_1} \mathrel{\widetilde{\subseteq}} f_{I_2}$.then$f_I(x) f_{I_1} \mathrel{\widetilde{\subseteq}} f_{I_2}$.therefore$f_{I_2}(x) \in f_{I_2}$. since $f_{sI_2}(x), f_{I_2}(y) \in f_{I_2}, f_{I_2}(x) \curlyvee f_{I_2}(y) \in f_{I_2}$.
Hence $f_I(x) \curlyvee f_I(y) \in f_I$. let $f_L(a) \in f_L$.then $f_{I_1}(x) \curlywedge f_{I_1}(a) \in f_{I_1} \mathrel{\widetilde{\subseteq}} f_I$.therefore $f_I(x) \curlywedge f_I(a) \in f_I$.

Hence f_I is a fuzzy soft ideal.

3.8 Theorem

A necessary and sufficient condition for a fuzzy soft ideal f_I in a fuzzy soft lattice f_L to be a principal fuzzy soft ideal is that the fuzzy soft lattice f_L satisfies the maximum condition.

Proof

Suppose the fuzzy soft lattice f_L satisfies the maximum condition.then it is also satisfied in every fuzzy soft ideals f_I of f_L. By corollary the fuzzy soft ideal $f_I$includes a greatest element $f_I(x)$. Then $f_I(f_I(x)]$. Hence every fuzzy soft ideal of $f_L$is a principal fuzzy soft ideal. Conversely, suppose that every fuzzy soft ideal is a principal fuzzy soft ideal.we have to prove fuzzy soft lattice satisfies the maximum condition.

Suppose not, then we can find an infinite subchain of the form $f_C = f_{C_0} \prec f_{C_1} \prec \cdots$ .The set $f_I = \widetilde{\cup}_{n=0}^{\infty} (f_{C_n}]$ being the fuzzy soft union of the elements of the fuzzy soft ideal chain is itself a fuzzy soft ideal by lemma. Hence f_I cannot be a principal fuzzy soft ideal since every one of its elements is less than the other of its elements.

Therefore f_I has no greatest element which is a contradiction.

3.9 Theorem

Let f_I and f_J be fuzzy soft ideals of a fuzzy soft lattice f_L.Define $f_I \curlywedge f_J = \{f_L(a) \in f_L / f_{I\widetilde{\cap}J}(a) \in f_I \widetilde{\cap} f_J\}$ and $f_I \curlyvee f_J = \{ f_L(a) \in f_L / f_L(a) \preccurlyeq f_I(x) \curlyvee f_J(y), f_I(x) \in f_I, f_F(y) \in f_F\}$ then the set of all fuzzy soft ideals $f_I(f_L)$ is a fuzzy soft lattice.

Proof

Clearly $f_I \curlywedge f_J \neq \emptyset$ for $f_I(\emptyset) \in f_I$ and $f_J(\emptyset) \in f_J$. Let $f_{I\curlywedge J}(x), \ f_{I\curlywedge J}(y) \in \ f_I \curlywedge f_J$.Then, $f_{I\widetilde{\cap}J}(x) \in \ f_I \widetilde{\cap} f_J, f_{I\widetilde{\cap}J}(y) \in \ f_I \widetilde{\cap} f_J$. That is $f_I(x) \in f_I$ and $f_J(y) \in f_J$ .Also $f_I(y) \in f_I$ and $f_J(x) \in f_J$.Therefore, $f_I(x) \curlyvee f_I(y) \in f_I$ and $f_J(x) \curlyvee f_J(y) \in f_J$.Hence $f_{I\widetilde{\cap}J}(x) \curlyvee f_{I\widetilde{\cap}J}(y) \in f_I \widetilde{\cap} f_J$ and hence $f_{I\curlywedge J}(x) \curlyvee f_{I\curlywedge J}(y) \in f_I \curlywedge f_J$.Let $f_{I\curlywedge J}(x) \in f_I \curlywedge f_J$ and $f_L(a) \preccurlyeq f_I(x)$. Then $f_I(x) \in f_I$ and $f_J(x) \in f_J$ .since $f_L(a) \preccurlyeq f_I(x) \Longrightarrow f_I(a) \in f_I$ and $f_J(a) \in f_J \Longrightarrow f_{I\curlywedge J}(a) \in f_{I\curlywedge J}$. Therefore $I \curlywedge J$ is a fuzzy soft ideal. Next we prove that $I \curlyvee J$ is a fuzzy soft ideal.

Clearly $f_I \curlyvee f_J \neq \emptyset$ for $f_I(\emptyset) \in f_I$ and $f_J(\emptyset) \in f_J$. Let $f_{I\curlyvee J}(x), \ f_{I\curlyvee J}(y) \in f_I \curlyvee f_J$.Then, $f_{I\curlyvee J}(a) \preccurlyeq f_{I\curlyvee J}(x_1) \curlyvee f_{I\curlyvee J}(y_1), f_{I\curlyvee J}(b) \preccurlyeq f_{I\curlyvee J}(x_2) \curlyvee f_{I\curlyvee J}(y_2)$.where $f_I(x_1), \ f_I(x_2) \in f_I$ and $f_J(y_1) \ , f_J(y_2) \in f_J$. Therefore$f_{I\curlyvee J}(a) \curlyvee f_{I\curlyvee J}(b) \preccurlyeq \left(f_{I\curlyvee J}(x_1) \curlyvee f_{I\curlyvee J}(y_1)\right) \curlyvee \left(f_{I\curlyvee J}(x_2) \curlyvee f_{I\curlyvee J}(y_2)\right) = \left(f_{I\curlyvee J}(x_1) \curlyvee f_{I\curlyvee J}(x_2)\right) \curlyvee \left(f_{I\curlyvee J}(y_1) \curlyvee f_{I\curlyvee J}(y_2)\right)$.Since $f_I(x_1), \ f_I(x_2) \in f_I \ , f_J(x_1) \curlyvee f_J(x_2) \in f_J$.Since $f_J(y_1), \ f_J(y_2) \in \ f_J, f_J(y_1) \curlyvee f_J(y_2) \in \ f_J$. Hence $f_{I\curlyvee J}(x) \curlyvee \ f_{I\curlyvee J}(y) \in f_I \curlyvee \ f_J$. Suppose $f_{I\curlyvee J}(a) \in f_I \curlyvee \ f_Jandf_{I\curlyvee J}(b) \preccurlyeq f_{I\curlyvee J}(a)$.Then $f_{I\curlyvee J}(a) \preccurlyeq f_{I\curlyvee J}(x) \curlyvee f_{I\curlyvee J}(y)$ where $f_I(x) \in f_I$ and $f_J(y) \in \ f_J$.Since $f_{I\curlyvee J}(b) \preccurlyeq \ f_{I\curlyvee J}(a) \preccurlyeq f_{I\curlyvee J}(x) \curlyvee f_{I\curlyvee J}(y), f_{I\curlyvee J}(b) \preccurlyeq f_{I\curlyvee J}(x) \curlyvee f_{I\curlyvee J}(y)$.thus $f_{I\curlyvee J}(b) \in f_I \curlyvee f_J$. Hence $I \curlyvee J$ is a fuzzy soft ideal.

Therefore $f_I(f_L)$ is a fuzzy soft lattice.

3.10 Theorem

The set of all principal fuzzy soft ideals $f_{I_0}(f_L)$ of a fuzzy soft lattice $f_L$is a fuzzy soft sublattice of $f_I(f_L)$and is fuzzy soft isomorphic to f_L.

Proof

We claim that $(f_L(x)] \curlyvee (f_L(y)] = (f_L(x) \curlyvee (f_L(y)]$ and $(f_L(x)] \curlywedge (f_L(y)] = (f_L(x) \curlywedge (f_L(y)]$ holds for every pair of elements $f_L(x) f_L(y)$ of f_L.

First to prove $(f_L(x)] \curlyvee (f_L(y)] = (f_L(x) \curlyvee (f_L(y)]$.

Let $f_{(f_L(x)] \curlyvee (f_L(y)]}(a) \in f_{(f_L(x)]} \curlyvee f_{(f_L(x)]} = (f_L(x)] \mathbin{\widetilde{\cup}} (f_L(y)] \Longrightarrow f_L(a) \in (f_L(x)]$ or $f_{f_L(x)}(a) \in (f_{f_L(y)}] \Longrightarrow f_L(a) \preccurlyeq f_L(x)$ or $f_L(a) \preccurlyeq f_L(y) \Longrightarrow f_L(a) \preccurlyeq f_L(x) \curlyvee f_L(y) \Longrightarrow f_{f_L(x) \curlyvee f_L(y)}(a) \in (f_{f_L(x)} \curlyvee f_{f_L(y)}]$.

Thus$(f_L(x)] \curlyvee (f_L(y)] \mathrel{\widetilde{\subseteq}} (f_L(x) \curlyvee f_L(y)]$. Let$f_{f_L(x) \curlyvee f_L(y)}(a) \in (f_{f_L(x)} \curlyvee f_{f_L(y)}]$.Then $f_L(a) \preccurlyeq (f_L(x) \curlyvee f_L(y)]$.where $f_{f_L(x)}(x) \in (f_{f_L(x)}] , f_{f_L(y)}(y) \in (f_{f_L(y)}]$.(since $f_L(a) \preccurlyeq f_L(x)$ and $f_{f_L(x)}(x) \in (f_{f_L(x)}] , f_{f_L(y)}(a) \in (f_{f_L(x)}]$ and since $f_L(a) \preccurlyeq f_L(y)$ and $f_{f_L(y)}(y) \in (f_{f_L(y)}] , f_{f_L(y)}(a) \in (f_{f_L(y)}]$). $\Longrightarrow f_{(f_L(x)] \curlyvee (f_L(y)]}(a) \in f_{(f_L(x)]} \curlyvee f_{(f_L(x)]}$. Hence $(f_L(x) \curlyvee f_L(y)] \mathrel{\widetilde{\subseteq}} (f_L(x)] \curlyvee (f_L(y)]$.

Thus $(f_L(x)] \curlyvee (f_L(y)] = (f_L(x) \curlyvee (f_L(y)]$.

Next to prove $(f_L(x)] \curlywedge (f_L(y)] = (f_L(x) \curlywedge (f_L(y)]$.

Let $f_{(f_L(x)] \curlywedge (f_L(y)]}(a) \in f_{(f_L(x)]} \curlywedge f_{(f_L(x)]} = (f_L(x)] \mathbin{\widetilde{\cap}} (f_L(y)] \Longrightarrow f_L(a) \in (f_L(x)]$ and $f_{f_L(x)}(a) \in (f_{f_L(y)}] \Longrightarrow f_L(a) \preccurlyeq f_L(x)$ and $f_L(a) \preccurlyeq f_L(y) \Longrightarrow f_L(a) \preccurlyeq f_L(x) \curlywedge f_L(y) \Longrightarrow f_{f_L(x) \curlywedge f_L(y)}(a) \in (f_{f_L(x)} \curlywedge f_{f_L(y)}]$.Thus $(f_L(x)] \curlywedge (f_L(y)] \mathrel{\widetilde{\subseteq}} (f_L(x) \curlywedge f_L(y)]$. Let $f_{f_L(x) \curlywedge f_L(y)}(a) \in (f_{f_L(x)} \curlywedge f_{f_L(y)}]$ Then$f_L(a) \preccurlyeq (f_L(x) \curlywedge f_L(y)]$.where$f_{f_L(x)}(x) \in (f_{f_L(x)}]$,$f_{f_L(y)}(y) \in (f_{f_L(y)}]$.(since $f_L(a) \preccurlyeq f_L(x)$ and $f_{f_L(x)}(x) \in (f_{f_L(x)}] , f_{f_L(y)}(a) \in (f_{f_L(x)}]$ and since $f_L(a) \preccurlyeq f_L(y)$ and $f_{f_L(y)}(y) \in (f_{f_L(y)}] , f_{f_L(y)}(a) \in (f_{f_L(y)})$]) $\Longrightarrow f_{(f_L(x)] \curlywedge (f_L(y)]}(a) \in f_{(f_L(x)]} \curlywedge f_{(f_L(x)]}$. Hence $(f_L(x) \curlywedge f_L(y)] \mathrel{\widetilde{\subseteq}} (f_L(x)] \curlywedge (f_L(y)]$.Thus $(f_L(x)] \curlywedge (f_L(y)] = (f_L(x) \curlywedge (f_L(y)]$.

Let us define $\eta : f_L \to f_{I_0}(f_L)$ by $\eta(f_L(a) = (f_L(a)]$. Suppose $\eta(f_L(a) = \eta(f_L(b)$.then$(f_L(a)] = (f_L(b)]$.Since$f_{(f_L(a)]}(a) \in f_{(f_L(a)]} = f_{(f_L(b)]} \Longrightarrow f_{(f_L(b)]}(a) \in f_{(f_L(b)]} \Longrightarrow f_L(a) \preccurlyeq f_L(b)$ and

Since $f_{(f_L(b))}(b) \in f_{(f_L(b))} = f_{(f_L(a))} \Longrightarrow f_{(f_L(a))}(b) \in f_{(f_L(a))} \Longrightarrow f_L(b) \preccurlyeq f_L(a)$. Therefore $f_L(a) = f_L(b)$ and hence η is one – one.For every $(f_L(a)] \in f_{I_0}(f_L)$, there exist an element $f_L(a) \in f_L$ such that $\eta(f_L(a) = (f_L(a)]$. Therefore η is onto.to prove η is a fuzzy soft lattice homomorphism.

$$\eta(f_L(a) \curlyvee f_L(b)) = (f_L(a) \curlyvee f_L(b)] = (f_L(a)] \curlyvee (f_L(b)] = \eta(f_L(a)) \curlyvee \eta(f_L(a))$$
$$\eta(f_L(a) \curlywedge f_L(b)) = (f_L(a) \curlywedge f_L(b)] = (f_L(a)] \curlywedge (f_L(b)] = \eta(f_L(a)) \curlywedge \eta(f_L(a))$$

Therefore η is a fuzzy soft homomorphism.

Hence the map $\eta : f_L \to f_{I_0}(f_L)$ is a fuzzy soft isomorphism and $f_L \cong f_{I_0}(f_L) \subseteqq f_I(f_L)$.

3.11 Theorem

The fuzzy soft lattice $f_L$is modular if and only if the fuzzy soft ideal lattice $f_I(f_L)$ is modular.

Proof

Suppose $f_I(f_L)$ is a modular fuzzy soft lattice. then the set of all principal fuzzy soft ideals $f_{I_0}(f_L)$ of a fuzzy soft lattice $f_L$is a fuzzy soft sublattice of $f_I(f_L)$and is fuzzy soft isomorphic to f_L.

That is $f_L \cong f_{I_0}(f_L) \subseteqq f_I(f_L)$.

$f_I(f_L)$ is a modular fuzzy soft lattice implies its fuzzy soft sublattice $f_{I_0}(f_L)$ is a modular fuzzy soft lattice. $f_{I_0}(f_L)$is a modular fuzzy soft lattice implies its fuzzy soft isomorphic copy f_L is a modular fuzzy soft lattice. Hence $f_I(f_L)$ is a modular fuzzy soft lattice implies that f_L is a modular fuzzy soft lattice. Conversely, let f_L be a modular fuzzy soft lattice. To prove that $f_I(f_L)$ is a modular fuzzy soft lattice. Let f_I, f_J, f_K be fuzzy soft ideals of f_L such that $f_I \preccurlyeq f_K$. clearly $f_I \curlyvee (f_J \curlywedge f_K) \preccurlyeq (f_I \curlyvee f_J) \curlywedge f_K$. it is enough to prove that $(f_I \curlyvee f_J) \curlywedge f_K \preccurlyeq f_I \curlyvee (f_J \curlywedge f_K)$ .let $f_{(I\curlyvee J)\curlywedge K}(a) \in (f_I \curlyvee f_J) \curlywedge f_K$ .then $f_{I\curlyvee J}(a) \in f_I \curlyvee f_J$and $f_K(a) \in f_K$.since $f_{I\curlyvee J}(a) \in f_I \curlyvee f_J$, $f_{I\curlyvee J}(a) \preccurlyeq f_I(x) \curlyvee f_J(y)$. Where $f_I(x) \in f_I$, $f_F(y) \in f_F$. Since $f_I(x) \in f_I$ and $f_I \preccurlyeq f_K$,$f_K(x) \in f_K$.

Therefore $f_K(x) \curlyvee f_K(a) \in f_K$.

Let $f_K(z) = f_K(x) \curlyvee f_K(a)$then$f_K(z) \in f_K$. Also $f_K(a) \preccurlyeq f_K(x) \curlyvee f_K(a)$.Therefore $f_K(a) \preccurlyeq (f_I(x) \curlyvee f_J(y) \curlywedge \big(f_K(x) \curlyvee f_K(a)\big) \preccurlyeq \big(f_I(x) \curlyvee f_J(y)\big) \curlywedge f_K(z)$.Since f_L is a modular fuzzy soft lattice, $f_K(x) \preccurlyeq f_K(z) \Longrightarrow \big(f_I(x) \curlyvee f_J(y)\big) \curlywedge f_K(z) = f_I(x) \curlyvee (f_J(y) \curlywedge f_K(z))$.Therefore$f_K(x) \preccurlyeq f_I(x) \curlyvee (f_J(y) \curlywedge f_K(z))$. Since $f_J(y) \in f_J$ and $f_K(z) \in f_K$, $f_J(y) \curlywedge f_K(z) \in f_J \curlyvee f_K$ Where $f_I(x) \in f_I$,$f_J(y) \curlywedge f_K(z) \in f_J \curlyvee f_K$.

Thus $f_{I\curlyvee(J\curlywedge K)}(a) \in f_I \curlyvee (f_J \curlywedge f_K)$. Therefore $\big(f_I \curlyvee f_J\big) \curlywedge f_K \preccurlyeq f_I \curlyvee \big(f_J \curlywedge f_K\big)$.

Hence $f_I(f_K)$ is a modular fuzzy soft lattice.

3.12 Theorem

The fuzzy soft lattice f_L is distributive if and only if the fuzzy soft ideal lattice $f_I(f_L)$ is distributive.

Proof

Suppose $f_I(f_L)$ is a distributive fuzzy soft lattice. then the set of all principal fuzzy soft ideals $f_{I_0}(f_L)$ of a fuzzy soft lattice $f_L$is a fuzzy soft sublattice of $f_I(f_L)$and is fuzzy soft isomorphic to f_L.

That is $f_L \cong f_{I_0}(f_L) \subseteqq f_I(f_L)$.

$f_I(f_L)$ is a distributive fuzzy soft lattice implies its fuzzy soft sublattice $f_{I_0}(f_L)$ is a distributive fuzzy soft lattice. $f_{I_0}(f_L)$is a distributive fuzzy soft lattice implies its fuzzy soft isomorphic copy f_L is a distributive fuzzy soft lattice. Hence $f_I(f_L)$ is a distributive fuzzy soft lattice implies that f_L is a distributive fuzzy soft lattice. Conversely, let f_L be a distributive fuzzy soft lattice. To prove that $f_I(f_L)$ is a distributive fuzzy soft lattice. Let f_I, f_J, f_K be s fuzzy soft ideals of f_L such that $f_I \preccurlyeq f_K$. clearly $(f_I \curlywedge f_J) \curlyvee (f_I \curlywedge f_K) \preccurlyeq f_I \curlywedge (f_J \curlyvee f_K)$. It is enough to prove that $f_I \curlywedge (f_J \curlyvee f_K) \preccurlyeq (f_I \curlywedge f_J) \curlyvee (f_I \curlywedge f_K)$. Let $f_{I\curlywedge(J\curlyvee K)}(a) \in f_I \curlywedge (f_J \curlyvee f_K)$. Then $f_I(a) \in f_I$ and $f_{J\curlyvee K}(a) \in f_J \curlyvee f_K \Longrightarrow f_I(a) \in f_I$ and $f_I(a) = f_J(y) \curlyvee f_K(z)$.Where $f_J(y) \in f_J$, $f_K(z) \in f_K$. Now $f_I(a) = f_I(a) \curlywedge f_I(a) = f_I(a) \curlywedge (f_J(y) \curlyvee f_K(z)) = (f_I(a) \curlywedge (f_J(y)) \curlyvee (f_I(a) \curlywedge (f_K(z))$.(since f_L is a distributive fuzzy soft lattice).since $f_I(a) \in f_I$, $f_J(y) \in f_J$, $(f_{I\curlywedge J}(a) \curlywedge f_{I\curlywedge J})(y) \in f_I \curlywedge f_J$.since $f_I(a) \in f_I$, $f_K(y) \in f_K$, $(f_{I\curlywedge K}(a) \curlywedge f_{I\curlywedge K})(z) \in f_I \curlywedge f_K$.$(f_I(a) \curlywedge (f_J(y)) \curlyvee (f_I(a) \curlywedge (f_K(z)) \in (f_I \curlywedge f_J) \curlyvee (f_I \curlywedge f_K)$ and Hence $f_I(a) \in \big(f_I \curlywedge f_J\big) \curlyvee (f_I \curlywedge f_K)$. Thus $f_I \curlywedge \big(f_J \curlyvee f_K\big) \preccurlyeq (f_I \curlywedge f_J) \curlyvee (f_I \curlywedge f_K)$.

Hence $f_I(f_L)$ is distributive fuzzy soft lattice.

IV CONCLUSION

In this study, we have defined fuzzy soft ideals and filters, prime fuzzy soft ideals and filters, principal fuzzy soft ideals and filters, discussed their properties and illustrated them with some examples. we have shown that the set of fuzzy soft ideals of a fuzzy soft lattice. Also, we proved modular fuzzy soft lattice and distributive fuzzy soft lattice. An interesting topic for further study is to be discuss methods of fuzzy soft filters of a fuzzy soft lattice.

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