

FUZZY SEMIOPEN SETS AND FUZZY SEMICLOSED SOFT SETS IN FUZZY TOPOLOGICAL SPACES

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Abstract: This paper introduces fuzzy semi open and fuzzy semi closed soft sets in fuzzy soft topological spaces. A detail study is carried out on properties of fuzzy semi open, fuzzy semi closed fuzzy soft sets, fuzzy semi interior, fuzzy semi closure and fuzzy soft set in a fuzzy soft topological space. Further fuzzy soft semi compactness, fuzzy soft semi connectedness are introduced and studied.

Indexterms: Fuzzy Soft topological space, Fuzzy semi open soft set, Fuzzy soft semi compactness, Fuzzy soft semi continuity, 2000MSC:06D72

I.INTRODUCTION

The notion of fuzzy topological space for fuzzy soft sets was formulated by Shabir et.al[3]. Of late many authors have studied various properties of fuzzy soft topological spaces. This paper aims to introduce and give a detail study of fuzzy semi open soft set, fuzzy semi closed soft set, fuzzy semi continuity, fuzzy semi compactness, fuzzy. Here are some definitions are results in the sequel.

1.1 Definition

[8] A fuzzy set f on X is a mapping $f: X \rightarrow I$. The value $f(x)$ represents the degree of membership of $x \in X$ in the fuzzy set f , for $x \in X$.

Let I^X denotes the family of all fuzzy sets on X . If $f, m \in I^X$ then some basic set operation for fuzzy sets are given by Zadeh [1] as follows:

- (1) $f \leq m \Leftrightarrow f(x) \leq m(x)$, for all $x \in X$.
- (2) $f = m \Leftrightarrow f(x) = m(x)$, for all $x \in X$.
- (3) $n = f \vee m \Leftrightarrow n(x) = f(x) \vee m(x)$, for all $x \in X$.
- (4) $k = f \wedge m \Leftrightarrow k(x) = f(x) \wedge m(x)$, for all $x \in X$.
- (5) $t = f^c \Leftrightarrow t(x) = 1 - f(x)$, for all $x \in X$.

1.2 Definition

A pair (F, A) is called a soft set over X if F is a mapping defined by $F: A \rightarrow 2^X$, where 2^X is the power set of X . In other words, a soft set is a parameterized family of subsets of the set X . Each set $F(e), e \in A$, from this family may be considered as the set of e -elements of the soft set (F, A) .

1.3 Definition

[3]A pair (f, A) is called a fuzzy soft set over X , where $f: A \rightarrow I^X$ is a function. That is, for each $a \in A, f(a) = f_a: X \rightarrow I$ is a fuzzy set on X . A soft set (F, A) can be extended to a soft set type (F, E) , where $F(e) \neq \emptyset$ if $e \in A \subseteq E$ and $F(e) = \emptyset$ if $e \in E - A$.

1.4 Definition

A soft set F_A on the universe X is a mapping from the parameter set E to 2^X , i.e., $F_A: E \rightarrow 2^X$, where $F_A(e) \neq \emptyset$ if $e \in A \subseteq E$ and $F_A(e) = \emptyset$ if $e \notin A$.

The subscript A in the notation F_A indicates where the image of F_A is non-empty. A soft set can be defined by the set of ordered pairs $F_A = \{(e, F_A(e)): e \in E, F_A(e) \in 2^X\}$

The value $F_A(e)$ is a set called the e -element of the soft set for all $e \in E$.

1.5 Definition

A fuzzy soft set f_A on the universe X is a mapping from the parameter set E to I^X , i.e., $f_A: E \rightarrow I^X$, where $f_A(e) \neq 0_X$ if $e \in A \subseteq E$ and $f_A(e) = 0_X$ if $e \notin A$, where 0_X is empty fuzzy set on X .

From now on, we will use $\mathcal{F}(X, E)$ instead of the family of all fuzzy soft sets over X .

Obviously, a classical soft set F_A over a universe X can be seen as a fuzzy soft set by using the characteristic function of the set $F_A(e)$:

$$f_A(e)(a) = \chi_{F_A(e)}(a) = \begin{cases} 1, & \text{if } a \in F_A(e) \\ 0, & \text{otherwise.} \end{cases}$$

1.6 Definition

Let $f_A, m_B \in \mathcal{F}(X, E)$. Then f_A is called a fuzzy soft subset of m_B if $f_A(e) \subseteq m_B(e)$, for each $e \in E$, and we write $f_A \subseteq m_B$. Also f_A is called a fuzzy soft superset of m_B if m_B is a fuzzy soft subset of f_A , and we write $f_A \supseteq m_B$.

1.7 Definition

Let $f_A, m_B \in \mathcal{F}(X, E)$. Then f_A and m_B are said to be equal, denoted by $f_A = m_B$, if $f_A \subseteq m_B$ and $m_B \subseteq f_A$.

1.8 Definition

Let $f_A, m_B \in \mathcal{F}(X, E)$. Then union of f_A and m_B , denoted by $f_A \sqcup m_B$, is the fuzzy soft set $n_{A \cup B}$ defined by $n_{A \cup B}(e) = f_A(e) \vee m_B(e), \forall e \in E$.

That is, $n_{A \cup B} = f_A \sqcup m_B$.

1.9 Definition

Let $f_A, m_B \in \mathcal{F}(X, E)$. Then intersection of f_A and m_B , denoted by $f_A \sqcap m_B$, is the fuzzy soft set $n_{A \cap B}$ defined by $n_{A \cap B}(e) = f_A(e) \wedge m_B(e), \forall e \in E$.

That is, $n_{A \cap B} = f_A \sqcap m_B$.

1.10 Definition

[4] Let $f_A \in \mathcal{F}(X, E)$. Then the complement of f_A , denoted by f_A^c , is the fuzzy soft set defined by f_A^c , is the fuzzy soft set defined by $f_A^c(e) = 1_x - f_A(e), \forall e \in E$.

Let us call f_A^c the fuzzy soft complement function of f_A . Clearly $(f_A^c)^c = f_A$.

Let $f_E \in \mathcal{F}(X, E)$. The fuzzy soft set f_E is called the null fuzzy soft set, denoted by $\tilde{0}_E$, if $f_E(e) = 0_x, \forall e \in E$.

1.11 Definition

[6] A fuzzy soft basis of a fuzzy soft topological space (X_A, τ) is a subcollection \mathcal{B} of fuzzy open soft such that every elements of τ can be expressed as the union of elements of \mathcal{B} .

1.12 Definition

Let (X_A, τ) be a fuzzy soft topological space and $X_B \cong X_A$ then the collection $\tau_{X_B} = \{X_A \cap X_B | X_{A_i} \in \tau, i \in I \cong \mathbb{N}\}$ is called a fuzzy soft subspace topology on X_B .

1.13 Definition

[2] A fuzzy soft set $f_A \in FSS(X)_A$ is called a fuzzy soft point in X_A , denoted by e_F , if for some element $e \in A, f(e) \neq \Phi$ and $f(e') = \Phi$ for all $e' \in A - \{e\}$.

1.14 Definition

A fuzzy soft point e_F in said to be in the fuzzy soft set g_A , denoted by $e_F \in m_A$, if for some element $e \in A$ and $f(e) = m(e)$.

1.15 Definition

A family Ψ of fuzzy soft sets is a cover of a fuzzy soft set m_A if $f_A \cong \tilde{\cup} \{(f_i)_A | (f_i)_A \in \Psi, i \in I$. A sub cover of Ψ is a subfamily of Ψ which is also a cover.

1.16 Definition

A family Ψ of fuzzy soft sets has the finite intersection property (FIP) if the intersection of the members of each finite subfamily of Ψ is not null soft set.

Throughout this study, f_A denotes a fuzzy soft set, (X_A, τ) denotes a fuzzy soft topological space.

II FUZZY SEMIOPEN AND FUZZY SEMICLOSED SOFT SETS

In this section, we introduce fuzzy semi open and fuzzy semi closed soft sets and study various notions related to this structure.

2.1 Definition

In a fuzzy soft topological space (X_A, τ) , a fuzzy soft set

(i) m_A is said to be fuzzy soft sets if \exists an fuzzy open soft set n_A such that $n_A \cong m_A \cong \overline{n_A}$;

(ii) n_A is said to be fuzzy soft sets if \exists an fuzzy open soft set k_A such that $k_A^0 \cong f_A \cong k_A$;

2.2 Example

Consider the fuzzy soft topological spaces (X_A, τ) as defined in Example 3 of [3]. Here $m_A(e_1) = \{h_1, h_2\}$, $n_A(e_2) = \{h_1\}$ is a fuzzy semiopen soft set, as f_{1A} is a fuzzy open soft set such that $f_{1A} \cong m_A \cong \overline{f_{1A}} = f_{1A}$.

$k_A(e_1) = \{h_3\}$, $k_A(e_2) = \{h_3\}$, is a fuzzy semiclosed soft set, as $(f_{1A})^c$ is a fuzzy closed soft set such that $(f_{1A})^c \cong k_A \cong \overline{(f_{1A})^c}$.

2.3 Remark

Every open(closed) fuzzy soft set is a fuzzy semiopen(fuzzy semi closed) fuzzy soft set but not conversely.

2.4 Remark

Φ_A and X_A are always fuzzy semiclosed and fuzzy semiopen.

From now onwards, we shall denote the family of all fuzzy semiopen soft sets(fuzzy semiclosed soft sets) of a soft topological space (X_A, τ) by $FSOSS(X)_A(FSCSS(X)_A)$.

2.5 Theorem

Arbitrary union of fuzzy semiopen soft sets is a fuzzy semiopen soft set.

Proof:

Let $\{(m_A)_\lambda | \lambda \in \Lambda\}$ be a collection of fuzzy semiopen soft sets of a fuzzy soft topological space (X_A, τ) . Then \exists an fuzzy open soft sets $(n_A)_\lambda$ such that $(n_A)_\lambda \cong (m_A)_\lambda \cong \overline{(n_A)_\lambda}$ for each λ ; hence $\tilde{\cup}(n_A)_\lambda \cong \tilde{\cup}(m_A)_\lambda \cong \tilde{\cup}(n_A)_\lambda$ and $\tilde{\cup}(n_A)_\lambda$ is open soft set. So, it is concluded that $\tilde{\cup}(n_A)_\lambda$ is a fuzzy semiopen soft set.

2.6 Theorem

If a fuzzy semiopen soft set m_A is such that $m_A \cong k_A \cong \overline{m_A}$, then k_A is also fuzzy semiopen.

Proof:

As m_A is fuzzy semiopen soft set \exists an fuzzy open soft set n_A such that $n_A \cong m_A \cong \overline{n_A}$; then by hypothesis $n_A \cong k_A$ and $\overline{m_A} \cong \overline{n_A} \Rightarrow k_A \cong \overline{m_A} \cong \overline{n_A}$ i.e., $n_A \cong k_A \cong \overline{n_A}$, hence k_A is a fuzzy semiopen set.

2.7 Theorem

A fuzzy soft set $m_A \in FSOSS(U)_A \Leftrightarrow$ for every fuzzy soft point $e_G \in m_A, \exists$ a fuzzy soft set $n_A \in FSOSS(U)_A$ such that $e_G \in n_A \cong m_A$.

Proof:

(\Rightarrow) Take $n_A = m_A$

$$(\Rightarrow) m_A = \bigcup_{e_G \in m_A} (e_m) \cong \bigcup_{e_G \in m_A} n_A \cong m_A.$$

2.8 Theorem

If m_A is any fuzzy soft set in a fuzzy soft topological space (X_A, τ) then following are equivalent :

- (i) m_A is fuzzy semiclosed soft set;
- (ii) $(\overline{m_A})^0 \cong m_A$
- (iii) $(m_A^c)^0 \cong m_A^c$.
- (iv) m_A^c is fuzzy semiopen soft set;

Proof:

(i) \Rightarrow (ii)

If m_A is fuzzy semiclosed soft set, then \exists fuzzy closed soft set n_A such that $n_A^0 \cong m_A \cong n_A \Rightarrow n_A^0 \cong m_A \cong \overline{m_A} \cong n_A$. By the property of interior we then have $(\overline{m_A})^0 \cong n_A^0 \cong m_A$;

(ii) \Rightarrow (iii)

$$(\overline{m_A})^0 \cong m_A \Rightarrow m_A^c \cong ((\overline{m_A})^0)^c = (\overline{m_A})^0 \cong m_A^c. \quad \text{(iii) } \Rightarrow \text{(iv)}$$

$n_A = (m_A^c)^0$ is an fuzzy open soft set such that $(m_A^c)^0 \cong m_A^c \cong (\overline{m_A^c})^0$, hence m_A^c is fuzzy semiopen.

(iv) \Rightarrow (i)

As m_A^c is fuzzy semiopen \exists an fuzzy open soft set n_A such that $n_A \cong m_A^c \cong \overline{n_A} \Rightarrow n_A^c$ is a fuzzy closed soft set such that $m_A \cong n_A^c$ and $m_A^c \cong \overline{n_A} \Rightarrow (n_A^c)^0 \cong m_A$, hence m_A is fuzzy semiclosed soft set.

III FUZZY SOFT SEMI CONTINUOUS FUZZY SOFT SEMIOPEN, AND FUZZY SOFT SEMICLOSED FUNCTIONS

Here we introduce fuzzy soft functions in fuzzy soft topological spaces.

3.1 Definition

Let (X_A, τ) and (X_B, δ) be two fuzzy soft topological spaces. A fuzzy soft function $f: X_A \rightarrow X_B$ is said to be

- (i) Fuzzy soft semicontinuous if for each fuzzy soft open set m_B of X_B , the inverse image $f^{-1}(m_B)$ is fuzzy soft semiopen set of X_A ;
- (ii) Fuzzy soft semiopen fuction if for each fuzzy open soft set m_A of X_A , the image $f(m_A)$ is fuzzy semiopen soft set of X_B ;
- (iii) Fuzzy soft semiclosed fuction if for each fuzzy closed soft set f_A of X_A , the image $f(f_A)$ is fuzzy semiclosed soft set of X_B ;

3.2 Theorem

A fuzzy soft function $f: X_A \rightarrow X_B$ is fuzzy soft semicontinuous iff $f(sscl f_A) \cong \overline{f(f_A)}$ for every fuzzy soft set f_A of X_A .

Proof:

Let $f: X_A \rightarrow X_B$ is fuzzy soft semicontinuous. Now $\overline{f(f_A)}$ is a fuzzy soft closed set of U_B , so by fuzzy soft semicontinuity of f , $f^{-1}(\overline{f(f_A)})$ is fuzzy soft semiclosed and $f_A \cong f^{-1}(\overline{f(f_A)})$. But $fsscl f_A$ is the smallest fuzzy semiclosed set containing f_A , hence $sscl f_A \cong f^{-1}(\overline{f(f_A)}) \Rightarrow f(sscl f_A) \cong \overline{f(f_A)}$.

Conversely, let F_B be any fuzzy soft closed set of $X_B \Rightarrow f^{-1}(F_B) \in X_A \Rightarrow f(sscl(f^{-1}(F_B))) \cong \overline{f(f^{-1}(F_B))} \Rightarrow f(sscl(f^{-1}(F_B))) \cong \overline{f^{-1}(F_B)} = F_B \Rightarrow sscl f^{-1}(F_B) = f^{-1}(F_B)$, hence is fuzzy semiclosed.

3.3 Theorem

A fuzzy soft function $f: X_A \rightarrow X_B$ is fuzzy soft semicontinuous iff $f^{-1}(h_B)^0 \cong fssint(f^{-1}(h_B))$ for every fuzzy soft set h_B of X_B .

Proof:

Let $f: X_A \rightarrow X_B$ is fuzzy soft semicontinuous. Now $(f(m_A))^0$ is a fuzzy soft open set of X_B , so by fuzzy soft semicontinuity of f , $f^{-1}((f(m_A))^0)$ is fuzzy soft semiopen and $f^{-1}((f(m_A))^0) \cong m$. As $fssint m_A$ is the largest fuzzy soft semiopen set contained in m_A , $f^{-1}((f(m_A))^0) \cong fssint m_A$.

Conversely, take a fuzzy soft open set $m_B \Rightarrow f^{-1}(m_B)^0 \cong fssint(f^{-1}(m_B)) \Rightarrow f^{-1}(m_B) \cong fssint(f^{-1}(m_B)) \Rightarrow f^{-1}(m_B)$ is fuzzy soft semiopen.

3.4 Theorem

A fuzzy soft function $f: X_A \rightarrow X_B$ is fuzzy soft semi open iff $f((f_A)^0) \cong ssint(f(m_A))$ for every fuzzy soft set f_A of X_A .

Proof:

If $f: X_A \rightarrow X_B$ is fuzzy soft semiopen, then $f((f_A)^0) = ssint(f(f_A)^0) \cong ssint(f(f_A))$.

On the other hand, take a fuzzy soft open set m_B of X_B . Then by hypothesis, $f(f_A) = f((m_A)^0) \cong ssint(f(m_A)) \Rightarrow (m_A)$ is fuzzy soft semiopen in X_B .

3.5 Theorem

Let $f: X_A \rightarrow X_B$ be fuzzy soft semiopen. If k_B is a fuzzy soft set and f_A is fuzzy closed soft set containing $f^{-1}(k_B)$ then \exists a fuzzy semiclosed soft set n_B such that $k_B \cong n_B$ and $f^{-1}(n_B) \cong f_A$.

Proof:

Take $h_B = (f(f_A^c))^c$. Now $f^{-1}(k_B) \cong f_A \Rightarrow f(f_A^c) \cong f_A^c$. Then f_A^c open $\Rightarrow f(f_A^c)$ is semiopen, so h_B is fuzzy semiclosed and $k_B \cong h_B$ and $f^{-1}(h_B) \cong f_A$.

IV FUZZY SEMICOMPACT SOFT TOPOLOGICAL SPACES

This section is devoted to introduce fuzzy semicompactness in fuzzy soft topological spaces along with characterization of fuzzy semicompact soft topological spaces.

4.1 Definition

A cover of a fuzzy soft set is said to be a fuzzy semiopen soft cover if every member of the cover is a fuzzy semiopen soft set.

4.2 Definition

A fuzzy soft topological space (X_A, τ) is said to be fuzzy semicompact if each fuzzy semiopen soft cover of X_A has a finite subcover.

4.3 Remark

Every compact soft space is also is also fuzzy semicompact.

4.4 Theorem

A fuzzy soft topological space (X_A, τ) is fuzzy semicompact \Leftrightarrow each family of fuzzy semiclosed soft sets with the FIP has a nonempty intersection.

Proof:

Let $\{(f_A)_\lambda | \lambda \in \Lambda\}$ be a collection of fuzzy semiclosed soft sets with the FIP. If possible, assume

$$\bigcap_{\lambda \in \Lambda} (f_A)_\lambda = \Phi_A \Rightarrow \bigcup_{\lambda \in \Delta} ((f_A)_\lambda)^c = X_A.$$

So, the collection $\{(f_A)_\lambda)^c | \lambda \in \Lambda\}$ forms a fuzzy soft semiopen cover of X_A , which is fuzzy semicompact. So, \exists a finite subcollection Δ of Λ which also covers X_A , i.e.

$$\bigcup_{\lambda \in \Delta} ((f_A)_\lambda)^c = X_A \Rightarrow \bigcap_{\lambda \in \Lambda} (f_A)_\lambda = \Phi_A,$$

a contradiction.

For the converse, if possible, let (X_A, τ) be not semicompact. Then \exists a fuzzy semiopen cover $\{(m_A)_\lambda | \lambda \in \Lambda\}$ of X_A , Such that for every finite subcollection Δ of Λ we have

$$\bigcup_{\lambda \in \Delta} (m_A)_\lambda \neq X_A \Rightarrow \bigcap_{\lambda \in \Lambda} ((m_A)_\lambda)^c \neq \Phi_A.$$

Hence $\{(m_A)_\lambda)^c | \lambda \in \Lambda\}$ has the FIP.

So, by hypothesis

$$\bigcap_{\lambda \in \Lambda} ((m_A)_\lambda)^c \neq \Phi_A \Rightarrow \bigcup_{\lambda \in \Delta} (m_A)_\lambda \neq X_A,$$

a contradiction.

4.5 Theorem

Fuzzy semi continuous image of a fuzzy soft semi compact space is fuzzy soft compact.

Proof:

Let $f: X_A \rightarrow X_B$ be a fuzzy semicontinuous function from a fuzzy semicompact soft topological space (X_A, τ) to (X_B, δ) . Take a fuzzy soft open cover $\{(m_B)_\lambda | \lambda \in \Lambda\}$ of $X_B \Rightarrow \{f^{-1}((m_B)_\lambda) | \lambda \in \Lambda\}$ forms a fuzzy soft semiopen cover of $X_A \Rightarrow \exists$ a finite subset Δ of Λ such that $\{f^{-1}((m_B)_\lambda) | \lambda \in \Delta\}$ forms a fuzzy semiopen cover of $X_A \Rightarrow (m_B)_\lambda | \lambda \in \Delta\}$ forms a finite soft open cover of X_B .

4.6 Theorem

Fuzzy semiclosed subspace of a fuzzy semicompact soft topological space is fuzzy soft semicompact.

Proof:

Let X_B a fuzzy semiclosed subspace of a fuzzy semicompact soft topological space (X_A, τ) and $\{(m_B)_\lambda | \lambda \in \Lambda\}$ be a fuzzy semiopen cover of $X_B \Rightarrow$ for each $(m_B)_\lambda, \exists$ a fuzzy semiopen soft set m_A of X_B such that $m_B = m_A \tilde{\cap} m_B$. Then the family $\{(m_B)_\lambda | \lambda \in \Lambda\} \tilde{\cup} (X_A - X_B)$ is a fuzzy soft semi open cover of X_A , which has a finite subcover. So $\{(m_B)_\lambda | \lambda \in \Lambda\}$ has a finite subfamily to cover X_B . Hence X_B is fuzzy semicompact.

V CONCLUSION:

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. In the present paper, we have continued to study the properties of fuzzy topological spaces. We introduce fuzzy soft semi-interior, fuzzy semi-closure and fuzzy soft semi open and fuzzy soft semi closed set and have established several interesting properties. In our future work, we will go on studying the properties of fuzzy soft semi connectedness in fuzzy topological spaces. We hope that the findings in this paper will help researcher enhance and promote the further study on soft topology to carry out a general framework for their applications in practical life.

REFERENCES:

- [1] N.Cagman, S.Karatas, S.Enginoglu, (2011) Soft Topology, Comp. Math. Appl. 62,351-358.
- [2] J.Mahanta,P.K. Das, 3 Mar (2012) Results on fuzzy soft topological spaces, arXriv:1203.0634v1[cs.IT],
- [3]Maji, P.K., Biswas, R. (2001) And Roy, A.R. fuzzy soft sets, J. Fuzzy Math. 9(3), 589-602.
- [4]P.K. Maji, R. Biswas and A.R. Roy. (2003) Soft set theory, Computers and Mathematics with Applications 45,555-562.s
- [5]D.Molodstov, (1999) Soft Set Theory-First Results, Comp. Math. Appl. 37, 19-31.
- [6]S. Roy, T.K. Samanta, (2012) A note on fuzzy soft topological spaces, Ann. Fuzzy Math. Inform., 3(2),305-311.
- [7]M. Shabir, M.Naz, (2011) 1786-1799. On Soft topological spaces, Comp. Math. App.61
- [8]Zadeh,L.A. (1965) Fuzzy sets, Information and Control 8,338-353.