A NEW CLASS OF DELTA FUZZY GENERALIZED β-CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

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Abstract: The aim of this paper is to introduce a new class of sets called fuzzy delta generalized $\boldsymbol{\beta}$-closed sets and a new class of functions called fuzzy delta generalized $\boldsymbol{\beta}$-continuous functions in fuzzy topological spaces. Some of their properties and characterizations are studied.

Indexterms: Fuzzy $\boldsymbol{\beta}$-closed sets, $\boldsymbol{f\delta g\beta}$-closed sets, $\boldsymbol{f\delta g\beta}$-continuous, $\boldsymbol{f\delta g\beta}$-irresolute.

I INTRODUCTION

Among various fuzzy generalized open sets, the notion of fuzzy β-open sets introduced by abd El-Monsef et al. [1] which is equivalent to the notion of semi-preopen sets due to Andrijevic, plays a significant role in General Fuzzy Topology and Real Analysis. Many results have been obtained by using the concept of fuzzy β-closed sets. Dontchev [6] introduced and established the concept of fuzzy generalized semi-preclosed sets as a fuzzy generalization of semi-preclosed sets which is equivalent to the notion of fuzzy generalized β-closed sets due to Tahiliani. In this paper, the concepts of $f\delta g\beta$-closed sets, $f\delta g\beta$-continuous, $f\delta g\beta$-irresolute and fuzzy pre $f\delta g\beta$-continuous functions are introduced and studied their properties and characterizations.

Throughout this paper, (X,τ), (Y,σ) and (Z,η) (or simply X, Y and Z) represent fuzzy topological spaces (or simply spaces) on which no separation axioms are assumed unless explicitly stated.

PRELIMINARIES

Let us recall the following definitions which are useful in the sequel:

1.1 Definition

A fuzzy subset A of fuzzy topological spaces$X$ is called a

i. Fuzzy β-closed sets [2] (or fuzzy semi-preclosed[7]) if $int\left(cl\left(int(A)\right)\right) \le A$.
ii. Fuzzy pre-closed [11] if $cl\left(int(A)\right) \le A$.
iii. Fuzzy b-closed [5] if $cl\left(int(A)\right) \wedge int\left(cl(A)\right) \le A$.
iv. Fuzzy regular-closed [10] if $A = cl\left(int(A)\right)$.
v. Fuzzy α-closed [10] if $cl\left(int\left(cl(A)\right)\right) \le A$.
vi. Fuzzy semi-closed [11] if $int\left(cl(A)\right) \le A$.
vii. Fuzzy δ-closed [10] if $A = cl_\delta(A)$
where$cl_\delta(A) = \{x\epsilon X : int\left(cl(U)\right) \wedge A \ne \phi, U\epsilon\tau \ and \ x\epsilon U\}$

1.2 Definition

A fuzzy subset A of fuzzy topological spaces X is called,

i. Fuzzy generalized β-closed (briefly, $g\beta$-closed) [3] if $\beta cl(A) \le G$ whenever $A \le G$ and G is open in X.
ii. Fuzzy δ generalized b-closed (briefly, $g\beta b$-closed) [7] if $\beta cl(A) \le G$ whenever $A \le G$ and G is open in X.
iii. Fuzzy generalized pre regular closed (briefly, gpr-closed) [9] if $pcl(A) \le G$ whenever $A \le G$ and G is regular open in X.
iv. Fuzzy generalized δ-semiclosed (briefly, $g\delta s$-closed) [7] if $scl(A) \le G$whenever $A \le G$ and $G$is δ-open in X.
v. $fg\delta$-closed [3] if $cl(A) \le U$ whenever $A \le G$ and G is δ-open in X.
vi. $fg\delta^*$-closed [3] if $cl_\delta(A) \le G$ whenever $A \le G$ and G is δ-open in X.
vii. Fuzzy regular generalized b-closed (briefly, rgb-closed) [8] if $bcl(A) \le G$ whenever $A \le G$ and G is regular open in X.
viii. Fuzzy generalized b-closed (briefly, gb-closed) [5] if $bcl(A) \le G$ whenever $A \le G$ and G is open in X.

The complements of the above mentioned closed sets are their respective open sets.

1.3 Definition

A function $f : X \to Y$ from a topological space Y is called a

i. Fuzzy β-continuous [1] (resp, β-irresolute, δ-continuous andδ-open) if for every β- $g\beta$-continuous) if $f^{-1}(G)$ is β-closed (resp, β-closed, β-closed, $g\beta$-closed and $g\beta$-closed) set G of Y.
ii. Fuzzy pre β-closed (resp, pre β-open, δ-closed andδ-open) if for every δ-closed (resp, β-open, δ-closed and δ-open) subset A of X, $f(A)$ is β-closed (resp, β-open, δ-closed and δ-open) in Y.

1.4 Definition

A fuzzy topological space X is said to be a

i. Extremely disconnected if the closure of every open set of X is open in X.
ii. Submaximal if every dence set of X is open in X.

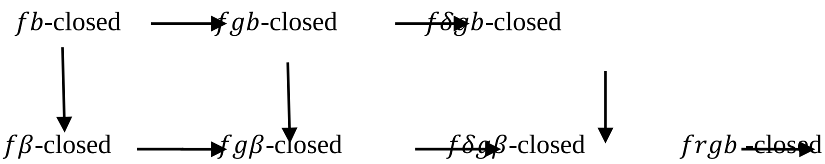
II DELTA FUZZY GENERALIZED β-CLOSED SETS

2.1 Definition

A fuzzy subset A of a space X is called a fuzzy delta fuzzy generalized β-closed (briefly, $f\delta g\beta$-closed) set $\beta cl(A) \leq G$ ehenever $A \leq G$ and $G$is δ-open in X.

The complement of a $f\delta g\beta$-closed set is called $f\delta g\beta$-open.

From the above definition and known results, we have the following diagram of implications and none of its implications is reversible.



2.2 Example

Let $X = \{a,b,c,d\}$and $\tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$. The fuzzy subset $\{a,b,c\}$ is $f\delta g\beta$-closed but neither

2.3 Theorem

Let X be a fuzzy semi regular space. Then $A \leq X$ is $fg\beta$-closed if and only if A is $f\delta g\beta$-closed $f\beta$-closed nor $fg\beta$-closed.

Proof

In a fuzzy semi-regular space X, $\delta O(X) = O(X)$ and hence proof follows.

2.4 Lemma

Let X be a space. Then $fbcl(A) = fscl(A) = f\beta cl(A)$ for every fuzzy semi open set Ain X.

2.5 Theorem

The following are equivalent for any fuzzy semi-open set $A \leq X$:

i. A is $f\delta gs$-closed
ii. A is $f\delta gb$-closed
iii. A is $f\delta g\beta$-closed

2.6 Lemma

If X is extremely disconnected and submaximal space then $fcl(A) = fcl(A)$ for every fuzzy subset Ain X.

2.7 Theorem

The followings are equivalent for any fuzzy subset A of extremely disconnected and submaximal space:

i. A is $f\delta g$-closed
ii. A is $f\delta gb$-closed
iii. A is $fg\delta s$-closed
iv. A is $f\delta g\beta$-closed

2.8 Theorem

A fuzzy subset A of a space X is $f\delta g\beta$-open if and only if $V \leq \beta int(A)$ whenever V is β-closed and $V \leq A$.

Proof

Let V be a δ-closed of $X$and $V \leq A$. Then $(X - A) \leq (X - V)$. Since $(X - A)$ is $f\delta g\beta$-closed, then $\beta cl(X - A) \leq (X - M)$ which implies $M \leq \beta int(A)$.

Conversely, let U be an δ-open set of $X$and $(X - A) \leq U$. Since $(X - U)$ is a δ-closed set contained in A, by hypothesis $(X - U) \leq \beta int(A)$. That is, $X - f\beta int(A) = f\beta cl(X - A) \leq U$. Hence $X - A$ is $f\delta g\beta$-closed and so A is $f\delta g\beta$-open.

2.9 Theorem

Let $B \leq X$ be $f\delta g\beta$-closed then $f\beta cl(B) - B$ contains no non empty $f\delta$-closed set.

Proof

Suppose there exists a non emptyδ-closed set G of X such that $G \leq f\beta cl(B) - B$, then $G \leq f\beta cl(B)$ and $G \leq X - B$ implies $B \leq X - G$ as B is $f\delta g\beta$-closed.

Hence $G \leq f\beta cl(B) \wedge (X - f\beta cl(B)) = \phi$. This shows $G = \phi$.

2.10 Theorem

Let $A \leq X$ be a $f\delta g\beta$-closed set. Then A is β-closed if and only if $f\beta cl(A) - A$ is δ-closed.

Proof

Let A be β-closed, then $f\beta cl(A) = A$ and so $f\beta cl(A) = \phi$ which is δ-closed.

Conversely, let A be $f\delta g\beta$-closed subset of X and $f\beta cl(A) - A$ is a δ-closed. Then by theorem 3.12, $f\beta cl(A) - A = \phi$ and hence A is β-closed.

2.11 Theorem

If $A \leq X$ is both δ-open and $f\delta g\beta$-closed then A is β-closed in X.

Proof

Let A be δ-open $f\delta g\beta$-closed set of Xthen $f\beta cl(A) \leq A$. but always $A \leq f\beta cl(A)$. Therefore $f\beta cl(A) = A$ and hence A is β-closed.

2.12 Theorem

If A is $f\delta g\beta$-closed and $A \leq B \leq f\beta cl(A)$. Then

i. $B$is$f\delta g\beta$-closed.
ii. $f\beta cl(B) - B$contains no non empty δ-closed set.

Proof

i. Let G be a δ-open set in X such that $B \leq G$and $A \leq G$. Since A is $f\delta g\beta$-closed, then $f\beta cl(A) \leq G$. Now, $f\beta cl(B) \leq f\beta cl(f\beta cl(A)) = f\beta cl(A) \leq G$. Therefore $f\beta cl(B) \leq G$.

ii. Follows from theorem 2.10.

2.13 Theorem

If A is $f\delta g\beta$-open and B is any set in X such that $\beta int(A) \leq B \leq A$ then B is $f\delta g\beta$-open in X.

2.14 Remark

The intersection of two $f\delta g\beta$-closed sets need not be $f\delta g\beta$-closed in general as seed from the following example.

2.15 Example

In example 3.2, the fuzzy subsets $\{a, b, c\}$ and $\{a, b, d\}$ are $f\delta g\beta$-closed but their intersection $\{a, b, c\} \wedge \{a, b, d\} = \{a, b\}$ is not $f\delta g\beta$-closed.

2.16 Remark

The union of two $f\delta g\beta$-closed sets need not be $f\delta g\beta$-closed set in general as seen from the following example.

2.17 Example

In example 3.2, the fuzzy subsets $\{a\}$ and $\{b\}$ are $f\delta g\beta$-closed but their union $\{a\} \vee \{b\} = \{a, b\}$ is not $f\delta g\beta$-closed.

2.18 Theorem

If A and B are $f\delta g\beta$-closed sets in extremely disconnected and submaximal space X then $A \vee B$ is $f\delta g\beta$-closed in X.

Proof

Let $A \vee B \leq U$ is δ-open in X, then $A \leq U$ and $B \leq U$. Then $f\beta cl(A) \leq U$ and $f\beta cl(B) \leq U$ since A and B are $f\delta g\beta$-closed sets. By lemma 3.9, $\beta cl(A) = cl(A)$and $\beta cl(B) = cl(B)$. Now, $f\beta cl(A \vee B) \leq fcl(A \vee B) = fcl(A) \vee cl(B) = f\beta cl(A) \vee f\beta cl(B) \leq U \vee U = U$. Thus $\beta cl(A \vee B) \leq U$ whenever $A \vee B \leq U$ and U is δ-open in X and hence $A \vee B$ is $f\delta g\beta$-closed.

2.19 Theorem

Let A and B be two subsets of X with A is semi-closed then $f\beta cl(A \vee B) = f\beta cl(A) \vee f\beta cl(B)$.

2.20 Theorem

If A and B are $f\delta g\beta$-closed sets with A is semi-closed, then $A \vee B$ is $f\delta g\beta$-closed in X.

Proof

Follows from definition 2.1 and theorem 2.19.

2.21 Theorem

The intersection of a $f\delta g\beta$-closed sets and a $f\delta$-closed set of X is always $f\delta g\beta$-closed.

Proof

Let A be $f\delta g\beta$-closed and let F be $f\delta$-closed. If G is a $f\delta$ –open set with $A \wedge F \leq G$. Then $A \leq G \vee F^c$ and $G \vee F^c$ is $f\delta$-open. Since A is $f\delta g\beta$-closed, then $f\beta cl(A) \leq G \vee F^c$ which implies $f\beta cl(A) \leq F \leq G$. Now $f\beta cl(A \wedge F) \leq f\beta cl(A) \wedge f\beta cl(F) \leq f\beta cl(A) \wedge f\delta cl(F) \leq f\beta cl(A) \wedge F \leq G$. Hence $A \wedge F$ is $f\delta g\beta$-closed.

2.22 Theorem

Let $A \leq X$ be $f\delta$-open $f\delta g\beta$-closed and $M \leq X$ is $f\delta$-closed then $A \wedge M$ is $f\delta g\beta$-closed.

Proof

Let $A \leq X$ be $f\delta$-open $f\delta g\beta$-closed. Then by theorem 3.14, A is $f\delta$-closed. Hence $A \wedge M$ is $f\delta$-closed which implies that $A \wedge M$ is $f\delta g\beta$-closed.

2.23 Theorem

Let $A \leq Y \leq X$ and Y be $f\alpha$-open in X, then $f\beta cl_Y(A) = f\beta cl_Y(A) \wedge Y$.

2.24 Theorem

Let Y be a $f\alpha$-open subspace of a space $X$and $A \leq Y$. If A is $f\delta g\beta$-closed in X then is $f\delta g\beta$-closed in Y.

Proof

Let U be a $f\delta$-open set of Y such that $A \leq U$. Then $U = Y \wedge H$ for some $f\delta$-open set $H$of X. Since A is $f\delta g\beta$-closed in X, we have $f\beta cl(A) \leq H$ and $f\beta cl_Y(A) = Y \wedge f\beta cl(A) \leq Y \wedge H = U$. Hence A is $f\delta g\beta$-closed in Y.

2.25 Theorem

Let $A \leq Y \leq X$ and Y be $f\alpha$-open and $f\beta$-closed. If A is $f\delta g\beta$-closed in Y then A is $f\delta g\beta$-closed in X.

Proof

Let U be a $f\delta$-open set of X such that $A \leq U$. Then $A = Y \wedge A \leq Y \wedge U$ where $Y \wedge U$ is δ-open in Y. Since A is $f\delta g\beta$-closed in Y, we have $f\beta cl_Y(A) \leq Y \wedge U$ and by Theorem 3.27, $f\beta cl_Y(A) \leq Y \wedge U \leq U$.

2.26 Theorem

For a space X, the following statements are equivalent:

i. Every $f\delta g\beta$-closed set in $f\delta gb$-closed and
ii. Every $f\beta$-closed set is $f\delta gb$-closed.

Proof

Clearly (i) $\longrightarrow$ (ii).

(ii) $\longrightarrow$ (i): Let A be a $f\delta g\beta$-closed set in X such that $A \leq G$ where G is $f\delta$-open in X, then $f\beta cl(A) \leq G$. As $f\beta cl(A)$ is $f\beta$-closed, by (ii), $f\beta cl(A)$ is $f\delta gb$-closed, $fbcl(A) \leq fbcl\big(f\beta cl(A)\big) \leq G$.

2.27 Theorem

If $f\beta O(X) = f\beta C(X)$, then $f\delta G\beta C(X) = P(X)$.

Proof

Let $A < V$ where V is $f\delta$-open in X, then V is $f\beta$-open.By hypothesis V is $f\beta$-closed. Hence $f\beta cl(A) \leq V$ and so A is $f\delta g\beta$-closed.

2.28 Theorem

For any $x\epsilon X$, the set $X - \{x\}$ is $f\delta g\beta$-closed or $f\delta$-open.

Proof

Suppose $X-\{x\}$ is not $f\delta$-open, then X is the only $f\delta$-open set containing $X-\{x\}$. This implies $f\beta cl(X-\{x\}) \leq X$. Hence $X-\{x\}$ is $f\delta g\beta$-closed.

2.29 Theorem

If A is $f\delta g\beta$-closed, then $fcl_{\delta}\{x\} \wedge A \neq \phi$, for every $x \epsilon f\beta cl(A)$.

Proof

Let $x \epsilon f\beta cl(A)$. Suppose $fcl_{\delta}\{x\} \wedge A = \phi$, then $A < X - fcl_{\delta}\{x\}$ and $X - fcl_{\delta}\{x\}$ is $f\delta$-open. Since A is $f\delta g\beta$-closed, then $f\beta cl(A) \leq X - fcl_{\delta}\{x\}$ so $x \notin f\beta cl(A)$which is a contradiction. Therefore $fcl_{\delta}\{x\} \wedge A \neq \phi$.

2.30 Definition

The intersection of all $f\delta$-open subsets of X containing A is called the $f\delta$ kernel of A and it is denoted by $fker_{\delta}(A)$.

2.31 Theorem

A subset A of X is $f\delta g\beta$-closed if and only if $f\beta cl(A) \leq fker_{\delta}(A)$.

Proof

Suppose A is $f\delta g\beta$-closed in X such that $x \epsilon f\beta cl(A)$. if possible, let $x \notin fker_{\delta}(A)$, then there exists a $f\delta$-open set G in X such that $A \leq G$ and $x \notin G$. Since A is $\delta g\beta$-closed in X, $f\beta cl(A) \leq G$ implies $x \notin f\beta cl(A)$ which is a contradiction.

Conversely, let $f\beta cl(A) \leq fker_{\delta}(A)$ be true and G is a δ-open set containing A, then $ker_{\delta}(A) \leq G$ which implies $f\beta cl(A) \leq G$. Hence A is $f\delta g\beta$-closed.

2.22 Lemma

For any set $A \leq X$, $f\beta int\big(f\beta cl(A) - (A)\big) = \phi$.

2.23 Theorem

Let $A \leq X$ be $f\delta g\beta$-closed, then $f\beta cl(A) - A$ is $f\delta g\beta$-open.

Proof

Suppose that A is $f\delta g\beta$-closed and M is $f\delta$-closed swet in X such that $M \leq f\beta cl(A) - A$. Then by Theorem 3.12, $M = \phi$ and hence $M \leq f\beta int(f\beta cl(A) - A)$. Therefore by theorem 3.11, $f\beta cl(A) - A$ is $f\delta g\beta$-open.

2.24 Definition

For a subset A of a space X, $f\delta g\beta cl(A) = \wedge \{F: A < F,\ F\ is\ f\delta g\beta - \text{closed in X}\}$.

2.25 Theorem

Let A and B be subsets of a topological space X. Then

i. $f\delta g\beta cl(X) = X$and $f\delta g\beta cl(\Phi) = \Phi$.
ii. If $A \leq B$, then $f\delta g\beta cl(A) \leq f\delta g\beta cl(B)$.
iii. $f\delta g\beta cl(A) \vee f\delta g\beta cl(B) \leq f\delta g\beta cl(A \vee B)$.
iv. $f\delta g\beta cl(A \wedge B) \leq f\delta g\beta cl(A) \wedge f\delta g\beta cl(B)$.
v. If A is $f\delta g\beta$ closed, then $f\delta g\beta cl(A) = A$.
vi. $A \leq f\delta g\beta cl(A) < fg\beta cl(A) < f\beta cl(A)$.

2.26 Theorem

If $f\delta G\beta C(X)$ is closed under finite unions, then $f\delta g\beta cl(A \vee B) = f\delta g\beta cl(A) \vee f\delta g\beta cl(B)$ for all $A, B \in f\delta G\beta C(X)$.

Proof

Let A and B be $f\delta g\beta$ closed sets in X. Then by hypothesis, $A \vee B f\delta g\beta$-closed.

Thus $f\delta g\beta cl(A \vee B) = A \vee B = f\delta g\beta cl(A) \vee f\delta g\beta cl(B)$.

2.27 Theorem

Let A be a subset of a space X. Then $x \in f\delta g\beta cl(A)$ if and only if $G \wedge A \neq \phi$ for every $f\delta g\beta$-open set Gcontaining X.

Proof

Let $x \in f\delta g\beta cl(A)$. suppose that there exists a $\delta g\beta$-open set Gcontaining X such that $G \wedge A = \phi$ then $A \leq X - G$ and $X - G$ is $f\delta g\beta$-closed. Therefore $f\delta g\beta cl(A) \leq X - G$ which implies $x \notin \delta g\beta cl(A)$, a contradiction.

Conversely, suppose that $x \notin f\delta g\beta cl(A)$. Then there exists a $f\delta g\beta$-closed set F containing A such that $X \notin F$. Hence F^C is a $f\delta g\beta$-open set containing X. Therefore $F^C \wedge \phi$ which contradicts the hypothesis.

2.28 Definition

For a subset A of a space X, $f\delta g\beta int(A) = \vee \{G: G < A, G\ is\ f\delta g\beta - \text{open in X}\}$.

2.29 Theorem

Let A and B be fuzzy subsets of a space X. Then

i. $f\delta g\beta int(X) = X$and $f\delta g\beta int(\Phi) = \Phi$.
ii. If $A < B$, then $f\delta g\beta int(A) \leq f\delta g\beta cl(B)$.
iii. $f\delta g\beta int(A) \vee f\delta g\beta int(B) \leq f\delta g\beta int(A \vee B)$.
iv. $f\delta g\beta int(A \wedge B) \leq f\delta g\beta int(A) \wedge f\delta g\beta int(B)$.
v. If A is $f\delta g\beta$-open, then $f\delta g\beta int(A) = A$.

III. DELTA FUZZY GENERALIZED FUZZY β-CONTINUOUS FUNCTIONS

In this section, the concepts of $f\delta g\beta$-continuous, pre $f\delta g\beta$-continuous and $f\delta g\beta$-irresolute functions in fuzzy topological spaces are introduced. Some of their properties and characterizations are established.

3.1 Definition

A function $f: X \to Y$ is called a

i. $f\delta g\beta$-continuous if the inverse image of every closed set in Y is $f\delta g\beta$-closed in X.
ii. Fuzzy $f\delta g\beta$-continuous if the inverse image of every $f\beta$-closed set in Y is $f\delta g\beta$-closed in X.

iii. $f\delta g\beta$-irresolute if the inverse image of every $f\delta g\beta$-closed in Y is $f\delta g\beta$-closed in X.

Clearly, (i) f is $f\delta g\beta$-continuous if and only if $f^{-1}(V)$ is $f\delta g\beta$-open in X for each open set $V$of Y.

iv. Fuzzy pre $f\delta g\beta$-continuous if and only if $f^{-1}(V)$ is $f\delta g\beta$-open in X for each $f\beta$ -open set $V$of Y.

v. $f$is$f\delta g\beta$-irresoluteif and only if $f^{-1}(V)$ is $f\delta g\beta$-open in X for each $f\delta g\beta$ -open set V of Y.

From the above definitions, we have the following:

3.2 Theorem

i. Every $f\beta$-continuous function is $f\delta g\beta$- continuous.
ii. Every $f\beta$-irresolute function is pre $f\delta g\beta$- continuous.
iii. Every pre $f\delta g\beta$- continuous function is $f\delta g\beta$- continuous.
iv. Every $f\delta g\beta$- irresolute function is pre $f\delta g\beta$- continuous.
v. Every $fg\beta$-continuous function is $f\delta g\beta$- continuous.
vi. Every $f\beta$-$g\beta$- continuous function is pre $f\delta g\beta$- continuous.

Reverse implications of above theorem need not be true in general:

3.3 Theorem

If $f: X \to Y$ is $f\delta g\beta$- continuous, then for each $x \in X$ and for each open set V in $Y$with $f(x) \in V$, there exists a $f\delta g\beta$-open set U in X such that $f(U) \leq V$.

Proof

Let $x \in X$ and V is an fuzzy open set in $Y$with $f(x) \in V$, then $x \in f^{-1}(V)$. Since f is $f\delta g\beta$- continuous, $f^{-1}(V)$ is $f\delta g\beta$-open in X. Put $U = f^{-1}(V)$, then $x \in U$ and $f(U) = f\left(f^{-1}(V)\right) \leq V$.

3.4 Theorem

If the bijective function $f: X \to Y$ is pre $f\delta g\beta$- continuous and $f\delta$-open, then it is $f\delta g\beta$- irresolute.

Proof

Let V be a $f\delta g\beta$-closed set in Y and F be a $f\delta$-open in X such that $f^{-1}(V) \leq F$, then $V \leq f(F)$ and $f(F)$ is $f\delta$-open in Y as f is $f\delta$-open. Since V is $f\delta g\beta$-closed in Y, $\beta cl(V) \leq f(F)$. This implies $f^{-1}\left(\beta cl(V)\right) \leq F$.

Since f is pre $f\delta g\beta$- continuous and $\beta cl(V)$ is $f\beta$-closed in Y it follows that $f^{-1}\left(\beta cl(V)\right)$ is $f\delta g\beta$-closed in X. Therefore $\beta cl\left(f^{-1}\left(\beta cl(V)\right)\right) \leq F$. That is, $\beta cl\left(f^{-1}(V)\right) \leq F$ and hence $f^{-1}(V)$ is $f\delta g\beta$-closed set in X. Thus f is $f\delta g\beta$- irresolute.

3.5 Theorem

Let $f: X \to Y$ and $g: Y \to Z$ be any two functions. Then;

i. If f is $f\delta g\beta$-continuous and g is continuous then gof is $f\delta g\beta$-continuous.
ii. If f and g are $f\delta g\beta$- irresolute, then gof is $f\delta g\beta$- irresolute.

Proof

(i) Let $h = gof$ and U be a closed set in Z.Since g is continuous, $g^{-1}(U)$ is closed in Y. Therefore $f^{-1}[g^{-1}(U)] = h^{-1}(U)$ is $f\delta g\beta$-closed in X because f is $f\delta g\beta$-continuous. Hence gof is $f\delta g\beta$-continuous.

(ii) Similar to (i).

3.6 Theorem

If $f: (X, \tau) \to (Y, \sigma)$ is a $f\delta g\beta$-continuous and A is a $f\delta$-open $f\delta g\beta$-closed subset of a space X and assume the class $f\delta G\beta C(X, \tau)$ is closed under finite intersections, then the restriction $f/A: (A, \tau/A) \to (Y, \sigma)$ is $f\delta g\beta$-continuous.

Proof

Let U be a fuzzy closed subset of Y. By hypothesis, $f^{-1}(U) \wedge A = V$ (say) is $f\delta g\beta$-closed in X. Since $(f/A)^{-1}(U) = V$, then it is sufficient to show that V is $f\delta g\beta$-closed in A. Let $V \leq M$ where M is a $f\delta$-oen set in A. then there exists a $f\delta$-open set N in X such that $M = N \wedge A$. Then $V \leq N$ implies $pcl_X(V) \leq N$ and $\beta cl_X(V) \wedge A \leq N \wedge A$ which implies $\beta cl_X(V) \leq M$. Hence V is $f\delta g\beta$-closed in A.

3.7 Definition

A space X is said to be $T_{f\delta g\beta}$-space if every $f\delta g\beta$-closed subset of X is closed.

3.8 Theorem

Every $T_{f\delta g\beta}$-space is $f\delta g\beta T_{\frac{1}{2}}$-space but not conversely.

Proof

Let X be $T_{f\delta g\beta}$-space and A be $f\delta g\beta$-closed, then A is closed. Therefore A is $f\beta$-closed and hence X is $f\delta g\beta T_{\frac{1}{2}}$-space.

3.9 Theorem

If $f: X \to Y$ and $g: Y \to Z$ are $f\delta g\beta$-continuous with Y is $T_{f\delta g\beta}$-space, then gof is $f\delta g\beta$-continuous.

Proof

Let $h = gof$ and V be a fuzzy closed set in Z. Since g is $f\delta g\beta$-continuous, $g^{-1}(V)$ is $f\delta g\beta$-closed in Y. Therefore $g^{-1}(V)$ is fuzzy closed in Y is $T_{f\delta g\beta}$-space. Since f is $f\delta g\beta$-continuous, $f^{-1}[g^{-1}(V)] = h^{-1}(V)$ is $f\delta g\beta$-closed in X and hence gof is $f\delta g\beta$-continuous.

IV CONCLUSION

In this paper, we discussed properties of new class of generalized closed sets in fuzzy topological spaces and also discussed between delta fuzzy generalized β-closed sets, delta generalized β-continuous functionsetc. In this paper through in future we introduced some applications.

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