LINEAR ESTIMATION IN GAMMA TYPE MODELS

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Abstract: Having defined Gamma type distributions as the unification of (i) Stacy (ii) General Fisher-Tippertt (iii) Fisher-Tippett (iv) Pearson type III (v) Gamma (vi) Erlang (vii) Scaled Chi-Square (vii) Chi-Square (ix) Exponential (x) Shifted Exponential (xi) Standard Exponential (xii) Eien (xiii) Nakagmi (xiv) Chi (xv) Scaled Chi (xvi) Half Normal (xvii) Rayleigh (xviii) Maxwell (xix) Wilson-Hilferty (xx) Generalized Weibull (xxi) Weibull (xxii) Pseudo-Weibull (xxiii) Stretched exponential and (xxiv) Standard Gamma distributions, derived moments of order statistics of Gamma type distributions to facilitate linear estimation of location and scale parameters for specified values of the shape parameter from doubly censored samples using Lloyd's (1952) method. MATLAB programs have been developed for the evaluation of moments of order statistics and coefficients of BLUEs for specified values the parameter from doubly censored samples.

Index Terms: Order Statistics, Gamma Distribution, Generalized Models, Linear Estimation, BLUE's

1. INTRODUCTION

The gamma distribution is a classical distribution which has appeared in the literature since the early 1800s. Amoroso (1925) first discussed the Generalized gamma distribution and used for fitting of income rate. Stacy (1962) proposed the Generalized Gamma distribution with three parameters. Lienhard and Meyer (1967) explained the physical model of generalized gamma distribution. This distribution has many applications in reliability and life testing models and it contains number of distributions stated above as the special cases.

A random variable X has Generalized Gamma distribution with three parameters, $\theta(>0)$ Index parameter; $\sigma(>0)$ scale parameter; and c shape parameter, has probability density function is

$$f(x)=\frac{c}{\sigma\Gamma(\theta)}\left(\frac{x}{\sigma}\right)^{c\theta-1} Exp\left\{-\left(\frac{x}{\sigma}\right)^{c}\right\};\ \theta,\sigma,c>0 \text{ and } 0<x<\infty \qquad (1.1)$$

2. GAMMA TYPE DISTRIBUTIONS

Harter (1967) made four parameter generalized gamma distribution by adding location parameter to Stacy (1962) model.

The probability density function Generalized Gamma Distribution (GGD) with four parameters $\theta(>0)$ index parameter; $c(>0)$ shape parameter; $\sigma(>0)$ scale parameter; and μ location parameter, given by

$$f(x)=\frac{c}{\sigma\,\Gamma(\theta)}\left(\frac{x-\mu}{\sigma}\right)^{c\theta-1} Exp\left\{-\left(\frac{x-\mu}{\sigma}\right)^{c}\right\};x>\mu\ \ \theta,\sigma,c>0 \text{ and } 0<\mu<\infty \qquad (2.1)$$

The reason for calling the above distribution as Gamma type is that many of the distributions related to Gamma distribution can be obtained from this by suitably choosing the values of μ,σ,θ and c . For example:

I.	$\mu=0,\sigma>0,\theta>0$ and $c>0$	Stacy distribution
II.	$\mu>0,\ \sigma=\frac{\sigma}{\theta^{\frac{1}{c}}},\theta$ is an integer and $c>0$	General Fisher-Tippertt distribution
III.	$\mu>0,\ \sigma>0,\theta=1$ and $c>0$	Fisher-Tippett distribution
IV.	$\mu>0,\ \sigma>0,\theta>0$ and $c=1$	Pearson type III distribution
V.	$\mu=0,\ \sigma>0,\theta>0$ and $c=1$	Gamma distribution
VI.	$\mu=0,\ \sigma>0,\theta$ is an integer and $c=1$	Erlang distribution
VII.	$\mu=0,\ \sigma=2\sigma^2,\theta=\left(\frac{k}{2}\right)$ and $c=1$ k is number of independent normal variates	Scaled Chi-Square distribution
VIII.	$\mu=0,\ \sigma=2,\theta=\left(\frac{k}{2}\right)$ and $c=1$ k is number of independent normal variates	Chi-Square distribution

IX.	$\mu=0,\ \sigma>0,\theta=1$ and $c=1$	Exponential distribution
X.	$\mu>0,\ \sigma>0,\theta=1$ and $c=1$	Shifted Exponential distribution
XI.	$\mu=0,\ \sigma=1,\theta=1$ and $c=1$	Standard Exponential distribution
XII.	$\mu=0,\ \sigma>0,\theta=4$ and $c=1$	Eien distribution
XIII.	$\mu>0,\ \sigma>0,\theta=\frac{\theta}{2}$ and $c=2$	Nakagmi distribution
XIV.	$\mu=0,\ \sigma=\sqrt{2},\theta=\frac{k}{2}$ and $c=2$ k is number of standard normal variates	Chi distribution
XV.	$\mu=0,\ \sigma=\sqrt{2\sigma^2},\theta=\frac{k}{2}$ and $c=2$ k is number of standard normal variates	Scaled Chi distribution
XVI.	$\mu=0,\ \sigma=\sqrt{2\sigma^2},\theta=\frac{1}{2}$ and $c=2$	Half Normal distribution
XVII.	$\mu=0,\ \sigma=\sqrt{2\sigma^2},\theta=1$ and $c=2$	Rayleigh distribution
XVIII.	$\mu=0,\ \sigma=\sqrt{2\sigma^2},\theta=\frac{3}{2}$ and $c=2$	Maxwell distribution
XIX.	$\mu=0,\ \sigma>0,\theta>0$ and $c=3$	Wilson-Hilferty distribution
XX.	$\mu>0,\ \sigma=\frac{\sigma}{\theta^{\frac{1}{c}}},\theta>0$ and $c>0$	Generalized Weibull distribution
XXI.	$\mu>0,\ \sigma>0,\theta=1$ and $c>0$	Weibull distribution
XXII.	$\mu=0,\ \sigma>0,\theta=1+\frac{1}{c}$ and $c>0$	Pseudo-Weibull distribution
XXIII.	$\mu=0,\ \sigma>0,\theta=1$ and $c>0$	Stretched exponential distribution
XXIV.	$\mu=0,\ \sigma=0,\theta=1$ and $c=1$	Standard Gamma distribution

Gamma type frequency curves for different values of the parameters θ and c and cdf are shown in Fig.(2.1) through Fig. (2.12)

Figure 2.1: Density function of Generalized Gamma models for various values of Shape Parameter $c(>0)$ with index parameter $\theta=0.5$

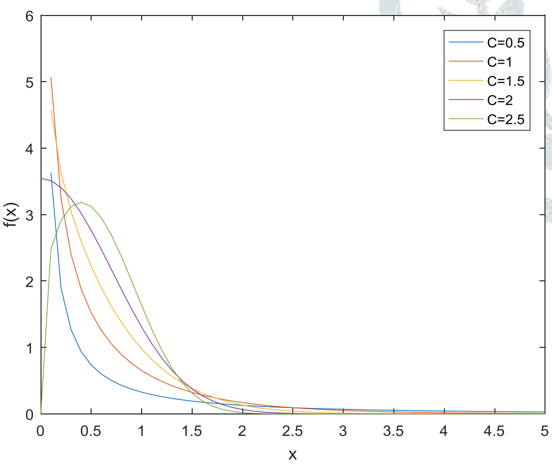


Figure 2.2: Density function of Generalized Gamma models for various values of Shape Parameter $c(>0)$ with index parameter $\theta=1.0$

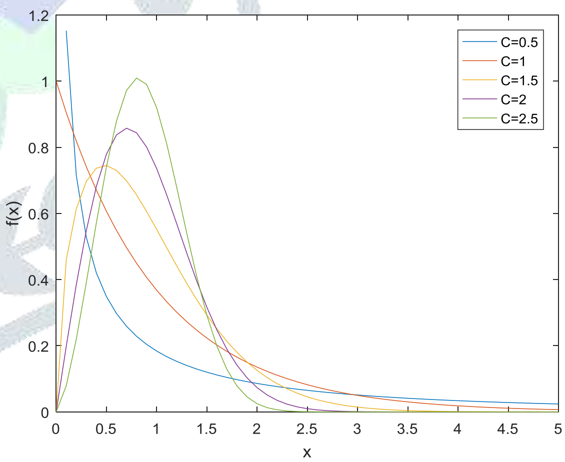


Figure 2.3: Density function of Generalized Gamma models for various values of Shape Parameter $c(>0)$ with index parameter $\theta=1.5$

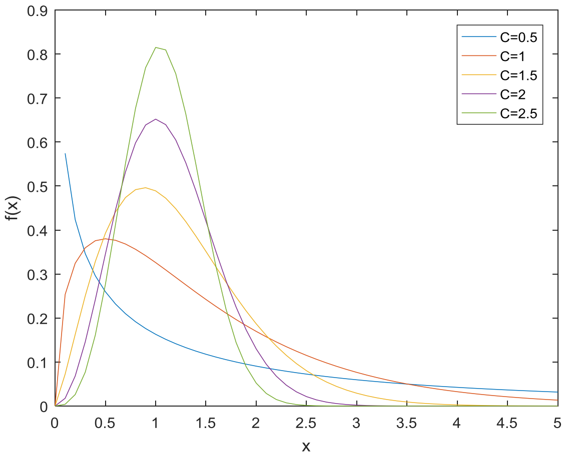


Figure 2.4: Density function of Generalized Gamma models for various values of Shape Parameter $c(>0)$ with index parameter $\theta = 2.0$

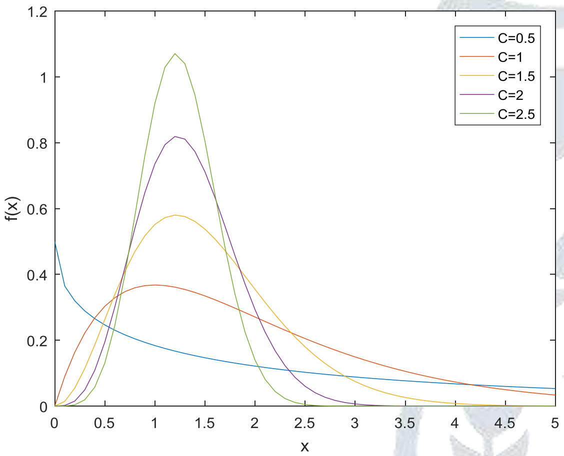


Figure 2.5: Density function of Generalized Gamma models for various values of Shape Parameter $c(>0)$ with index parameter $\theta = 2.5$

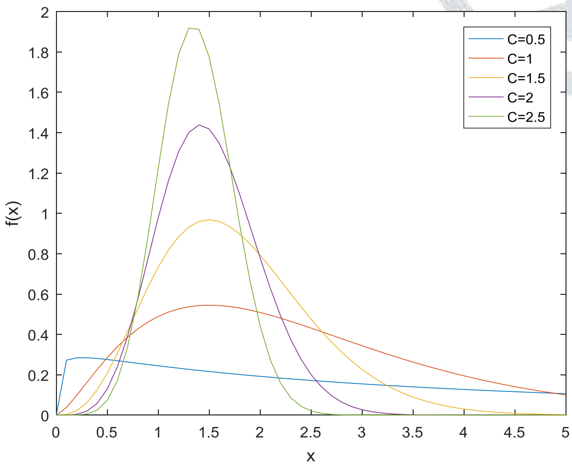


Figure 2.6: Density function of Generalized Gamma models for various values of Shape Parameter $c(>0)$ with index parameter $\theta = 3$

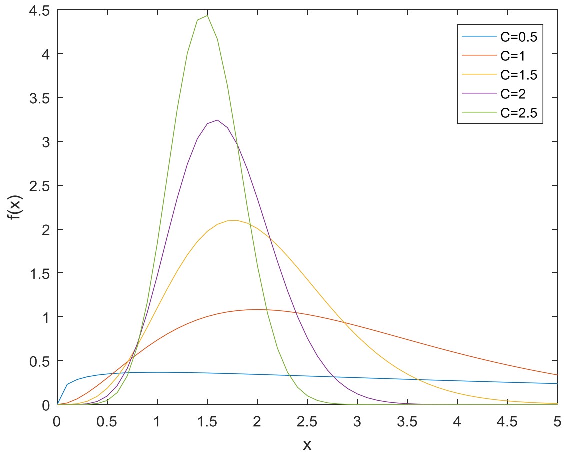


Figure 2.7: Distribution function of Generalized Gamma models for various values of Shape Parameter $c(>0)$ with index parameter $\theta = 0.5$

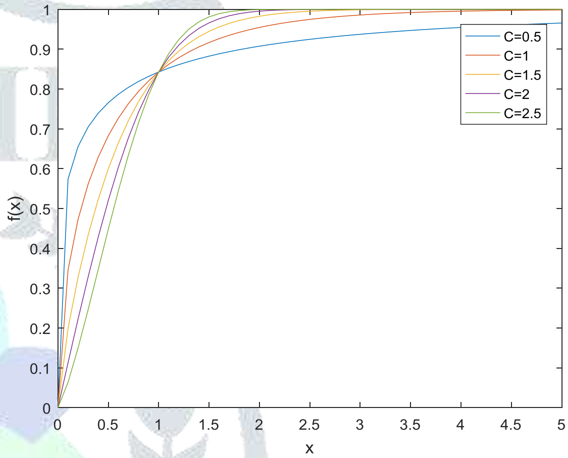


Figure 2.8: Distribution function of Generalized Gamma models for various values of Shape Parameter $c(>0)$ with index parameter $\theta = 1.0$

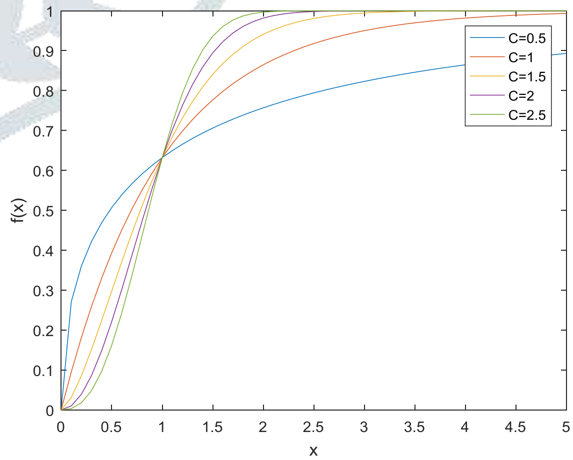


Figure 2.9: Distribution function of Generalized Gamma models for various values of Shape Parameter $c(>0)$ with index parameter $\theta = 1.5$

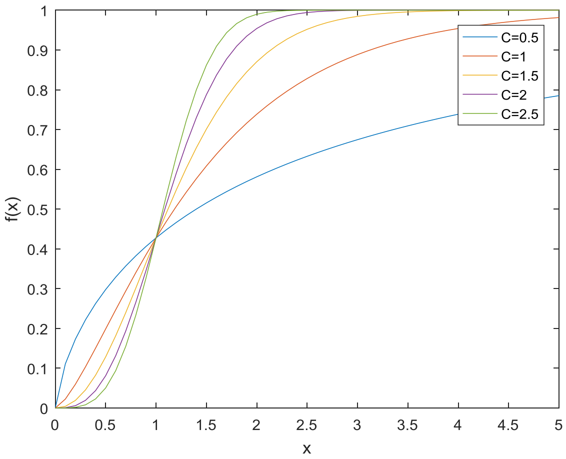


Figure 2.10: Distribution function of Generalized Gamma models for various values of Shape Parameter $c(>0)$ with index parameter $\theta = 2.0$

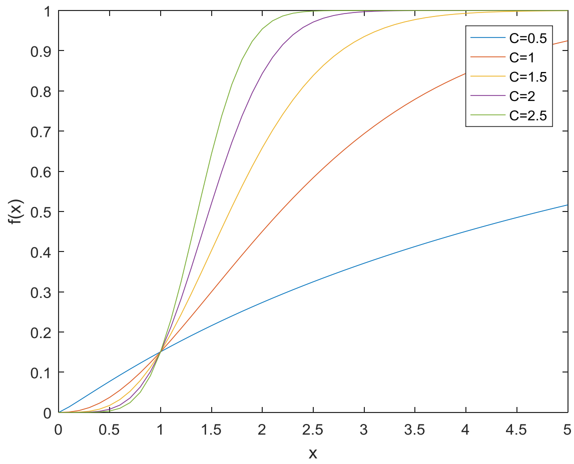
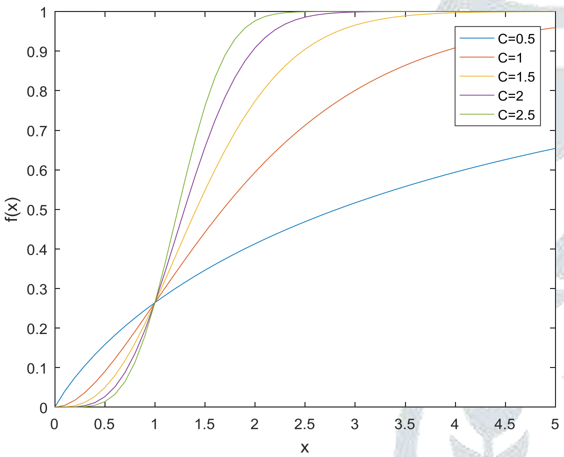


Figure 2.12: Distribution function of Generalized Gamma models for various values of Shape Parameter $c(>0)$ with index parameter $\theta = 3$



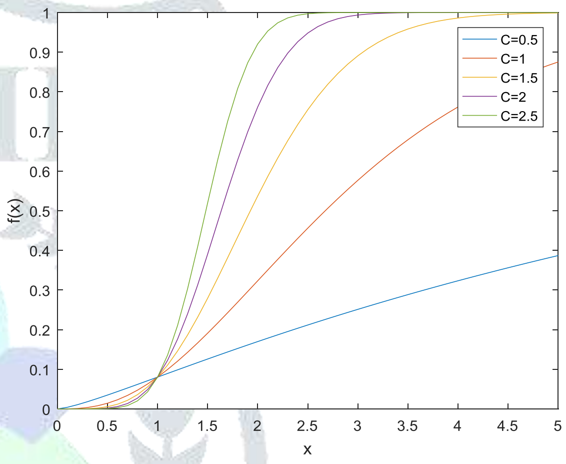


Figure 2.11: Distribution function of Generalized Gamma models for various values of Shape Parameter $c(>0)$ with index parameter $\theta = 2.5$

3. MOMENTS OF ORDER STATISTICS

To study the linear estimations, the required moments of order statistics are derived by us and the details are as under.

Let

$$Y = \left(\frac{X-\mu}{\sigma}\right)$$

then the density function Standard Generalized Gamma distribution becomes

$$g(y) = \frac{c}{\sigma\,\Gamma(\theta)}(y)^{c\theta-1} Exp\left\{-y^{c}\right\} \qquad (3.1)$$

and its cumulative distribution function is $F(y) = \frac{\gamma(y;\theta,c)}{\Gamma(\theta)}\ ;\ 0 \le Y \le y$

$$= 1 - \frac{\Gamma(y;\theta,c)}{\Gamma(\theta)}\ ;\ \ y \le Y \le \infty \qquad (3.2)$$

where $\gamma(y;\theta,c)$ is lower incomplete gamma function and $\Gamma(y;\theta,c)$ is upper incomplete gamma function.

Generalized gamma being location scale family, to find linear estimates for specified values of the shape parameter c and index parameter θ, the moments of order statistics are derived by us as follow

$E\left(y_{i:n}^{k}\right)$ is denoted as $\alpha_{i:n}^{(k)}$ and is given by

$$\alpha_{i:n}^{(k)} = \frac{1}{\beta(i,n-i+1)}\int_{0}^{\infty} y_i^{k}\left[F(y_i)\right]^{i-1}\left[1-F(y_i)\right]^{n-i} g(y_i)\,dy_i$$

$$= \frac{n!}{(n-i)!(i-1)!\Gamma(\theta)}\left[\sum_{m=0}^{i-1}(-1)^{m}\binom{i-1}{m}\sum_{l=0}^{\infty}\left(\frac{1}{\Gamma(\theta-l)}\right)_{l}^{(n-i+m)}\right.$$

$$\left.\frac{\Gamma\left\{(\theta-1)(n-i+m)+\theta+\frac{k}{c}-l\right\}}{(n-i+m+1)^{\left\{(\theta-1)(n-i+m)+\theta+\frac{k}{c}-l\right\}}}\right];\quad n\le\theta \qquad (3.3a)$$

where $\left(\frac{1}{\Gamma(\theta-l)}\right)_l^{(n-i+m)}$ is calculated by using power product of sum of series

$$=\frac{n!}{(n-i)!(i-1)!\Gamma(\theta)}\left[\sum_{m=0}^{n-i}(-1)^m\binom{n-i}{m}\sum_{l=0}^{\infty}\left(\frac{1}{\Gamma(\theta+l+1)}\right)_l^{(m+i-1)}\right.$$

$$\left.\frac{\Gamma\left\{(\theta)(i+m)+\frac{k}{c}+l\right\}}{(i+m)^{\left\{(\theta)(i+m)+\frac{k}{c}+l\right\}}}\right];\quad n>\theta \qquad (3.3b)$$

where $\left(\frac{1}{\Gamma(\theta+l+1)}\right)_l^{(i+m-1)}$ is calculated by using power product of sum of series

$E\left(y_{i:n}^1, y_{j:n}^1\right)$ is denoted as $\alpha_{i,j:n}^{(1,1)}$ and is given by

$$\alpha_{i,j:n}^{(1,1)}=\frac{n!}{(i-1)!(j-i-1)!(n-j)!}\left[\int_0^\infty\int_{y_i}^\infty y_i y_j\left[F(y_i)\right]^{i-1}\left[F(y_j)-F(y_i)\right]^{j-i-1}\right.$$

$$\left.\left[1-F(y_j)\right]^{n-j}g(y_i)g(y_j)dy_i dy_j\right]$$

$$=\frac{n!}{(n-j)!(j-i-i)!(i-1)!\left(\Gamma(\theta)\right)^2}\left[\sum_{a=0}^{i-1}\sum_{b=0}^{j-i-1}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}(-1)^{a+b}\binom{i-1}{a}\binom{j-i-1}{b}\right.$$

$$\left.\left(\frac{1}{\Gamma(\theta-p)}\right)_p^{(\alpha-1)}\left(\frac{1}{\Gamma(\theta-q)}\right)_q^{(\beta-1)}\frac{\Gamma(\omega)}{\beta}\sum_{l=0}^{\infty}\frac{\Gamma(\lambda+\omega-l)}{\Gamma(\omega-l)(\alpha+\beta)^{(\lambda+\beta-l)}\beta^l}\right];\ n\le\theta \qquad (3.4a)$$

where $\left(\frac{1}{\Gamma(\theta-p)}\right)_p^{(\alpha-1)}$ & $\left(\frac{1}{\Gamma(\theta-q)}\right)_q^{(\beta-1)}$ are calculated by using power product of sum of series.

$\alpha=j-i+a-b;\beta=n-j+b+1;\lambda=(\theta-1)(\alpha-1)+\theta-p;\omega=(\theta-1)(\beta-1)+\theta+1-q$

$$\alpha_{i,j:n}^{(1,1)}=\frac{n!}{(i-1)!(j-i-1)!(n-j)!}\left[\int_0^\infty\int_0^{y_j} y_i y_j\left[F(y_i)\right]^{i-1}\left[F(y_j)-F(y_i)\right]^{j-i-1}\right.$$

$$\left.\left[1-F(y_j)\right]^{n-j}g(y_i)g(y_j)dy_i dy_j\right]$$

$$=\frac{n!}{(n-j)!(j-i-i)!(i-1)!\left(\Gamma(\theta)\right)^2}\left[\sum_{a=0}^{i-1}\sum_{b=0}^{j-i-1}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}(-1)^{a+b}\binom{i-1}{a}\binom{j-i-1}{b}\right.$$

$$\left.\left(\frac{1}{\Gamma(\theta-p)}\right)_p^{(\alpha-1)}\left(\frac{1}{\Gamma(\theta-q)}\right)_q^{(\beta-1)}\frac{\Gamma(\omega)}{\beta}\sum_{l=0}^{\infty}\frac{\Gamma(\lambda+\omega-l)}{\Gamma(\omega-l)(\alpha+\beta)^{(\lambda+\beta-l)}\beta^l}\right];\ n>\theta \quad (3.4a)$$

where $\left(\frac{1}{\Gamma(\theta-p)}\right)_p^{(\alpha-1)}$ & $\left(\frac{1}{\Gamma(\theta-q)}\right)_q^{(\beta-1)}$ are calculated by using power product of sum of series.

$\alpha=i+a;\beta=j-i-a+b;\lambda=(\theta)(\alpha-1)+\theta+p+1;\omega=(\theta)(\beta-1)+\theta+q+1$

Special cases:

Substituting specified values for the parameters in section 2, we can get the single and product moments of (i) Stacy (ii) General Fisher-Tippertt (iii) Fisher-Tippett (iv) Pearson type III (v) Gamma (vi) Erlang (vii) Scaled Chi-Square (vii) Chi-Square (ix) Exponential (x) Shifted Exponential (xi) Standard Exponential (xii) Eien (xiii) Nakagmi (xiv) Chi (xv) Scaled Chi (xvi) Half Normal (xvii) Rayleigh (xviii) Maxwell (xix) Wilson-Hilferty (xx) Generalized Weibull (xxi) Weibull (xxii) Pseudo-Weibull (xxiii) Stretched exponential and (xxiv) Standard Gamma distributions as special cases by proper substitution of μ,σ,θ and c as indicated I through XXIV in section 2 respectively in equations (3.3a and 3.3b) and (3.4a and 3.4b).

4. REVIEW OF EXISTING LITERATURE

Gupta (1960) has derived and evaluated moments of order statistics in case of standard gamma distribution for integer values of index parameter (θ) using Sterling second kind of approximations. Krishnaiah and Rizvi (1967) have derived expressions for the moments of order statistics and claimed that these expressions hold good for real values of index parameter. Breiter and Krishnaiah (1968) have derived and evaluated moments of ordered statistics using Gauss Lagrange quadrature formula and tabulated the same for $\theta = 0.5(1)10.5$ for $n = 1(1)16$. Moments of order statistics for the sample size $n = 1(1)40$ for integer values of index parameter (θ) were evaluated by Harter (1970). Moments of order statistics for the sample size $n = 1(1)20$ for index parameter θ =0.5, 1.5, 2.5 and 3.5 were evaluated by Pearson and Hartley (1972). Prescott (1974) have evaluated variance and covariance using the expressions given by Gupta (1960) for $n = 2(1)10$ with index parameter $\theta = 2(1)5$. The moment method of generalized gamma distribution was discussed by Stacy and Mihram (1965). Vasudeva Rao, Kantam and Narasimham (1991), derived and evaluated the moments and the coefficients of BLUEs of location and scale parameters of censored samples of the generalized gamma distribution with integer values of index parameter $\theta = 2(1)4$, shape parameter values $c = 0.25, 0.5, 2.0$ and 4 for sample size $n = 3(1)10$, also proposed some alternative linear unbiased estimate methods of location and scale parameters and studied their relative efficiencies compared to BLUEs. In this we have derived and evaluated the moments, variances and covariance of the censored samples of four parameter generalized gamma distribution for all values of index parameter θ and shape parameter c irrespective of sample size.

5. RESULTS

Using (3.3a), (3.3b), (3.4a) and (3.) mean vector of i^{th} order statistic and covariance matrix of i^{th} and j^{th} order statistics have been completed for the sample size $n = 2(1)15$ for specified values of index parameter $\theta = 0.5, 1.0, 1.5, 2.0$ and 3.0, and shape parameter $c =$ 0.5, 1, 1.5, 2 and 3 from doubly censored samples (where r_1 and r_2 respectively denoting number of observations censored on the left and right) and are available with the first author and not presented here to save space. However, they are tabulated here for $c = 1.5$ and $\theta = 1.5$ for the sample size up to $n = 5$ in case of Generalized Gamma distribution are presented respectively Table 5.1 and 5.2.

Table 5.1: Mean vector $\alpha_{i:n}$ with $c = 1.5$ for Ordered Generalized Gamma Distribution up to n=5

n	r_1	r_2	i	$\alpha_{i:n}^{(1)}$	n	r_1	r_2	i	$\alpha_{i:n}^{(1)}$	n	r_1	r_2	i	$\alpha_{i:n}^{(1)}$
2	0	0	1	8.487957E-01	4	0	1	2	1.165197E+00	5	3	0	5	1.593782E+00
	0	0	2	1.593782E+00				0	1.808075E+00		0	1	1	5.981405E-01
3	0	0	1	6.905948E-01		0	2	1	8.487957E-01				2	9.679576E-01
			2	1.165197E+00				2	1.593782E+00				3	1.362437E+00
			3	1.808075E+00		1	1	2	8.487957E-01				4	1.956621E+00
	1	0	2	8.487957E-01				3	1.593782E+00		0	2	1	6.905948E-01
			3	1.593782E+00	5	0	0	1	5.357705E-01				2	1.165197E+00
	0	1	1	8.487957E-01				2	8.476204E-01				3	1.808075E+00
			2	1.593782E+00				3	1.148464E+00		0	3	1	8.487957E-01
4	0	0	1	5.981405E-01				4	1.505086E+00				2	1.593782E+00
			2	9.679576E-01				5	2.069504E+00		1	1	2	6.905948E-01
			3	1.362437E+00		1	0	2	5.981405E-01				3	1.165197E+00
			4	1.956621E+00				3	9.679576E-01				4	1.808075E+00
	1	0	2	6.905948E-01				4	1.362437E+00		2	1	3	8.487957E-01
			3	1.165197E+00				5	1.956621E+00				4	1.593782E+00
			4	1.808075E+00		2	0	3	6.905948E-01		1	2	2	8.487957E-01
	2	0	3	8.487957E-01				4	1.165197E+00				3	1.593782E+00
			4	1.593782E+00				5	1.808075E+00					
	0	1	1	6.905948E-01		3	0	4	8.487957E-01					

Table 5.2: Variance Covariance Matrix $\beta_{i,j:n}^{(1,1)}$ with $c = 1.5$ Ordered Generalized Gamma Distribution up to $n = 5$

n	r_1	r_2	i	j	$\beta_{i,j:n}^{(1,1)}$	n	r_1	r_2	i	j	$\beta_{i,j:n}^{(1,1)}$	n	r_1	r_2	i	j	$\beta_{i,j:n}^{(1,1)}$
2	0	0	1	1	2.039833E-01				2	2	2.019191E-01				4	4	2.019191E-01
				2	1.387513E-01					3	1.542136E-01					5	1.542136E-01
			2	2	4.272807E-01				3	3	4.021978E-01				5	5	4.021978E-01
3	0	0	1	1	1.299328E-01		0	2	1	1	2.039833E-01		3	0	4	4	2.039833E-01
				2	9.153236E-02					2	1.387513E-01					5	1.387513E-01
				3	6.880421E-02				2	2	4.272807E-01				5	5	4.272807E-01
			2	2	2.019191E-01		1	1	2	2	2.039833E-01		0	1	1	1	9.507453E-02
				3	1.542136E-01					3	1.387513E-01					2	6.765750E-02
			3	3	4.021978E-01				3	3	4.272807E-01					3	5.316908E-02

	1	0	2	2	2.039833E-01	5	0	0	1	1	7.492960E-02					4	4.196696E-02
				3	1.387513E-01					2	5.352364E-02				2	2	1.319340E-01
			3	3	4.272807E-01					3	4.272963E-02					3	1.047027E-01
	0	1	1	1	2.039833E-01					4	3.530824E-02					4	8.317903E-02
				2	1.387513E-01					5	2.869374E-02				3	3	1.940970E-01
			2	2	4.272807E-01				2	2	9.785398E-02					4	1.558880E-01
4	0	0	1	1	9.507453E-02					3	7.869620E-02				4	4	3.833012E-01
				2	6.765750E-02					4	6.528456E-02		0	2	1	1	1.299328E-01
				3	5.316908E-02					5	5.324262E-02					2	9.153236E-02
				4	4.196696E-02				3	3	1.287501E-01	5	0	2	1	3	6.880421E-02
			2	2	1.319340E-01					4	1.075032E-01				2	2	2.019191E-01
				3	1.047027E-01					5	8.810242E-02					3	1.542136E-01
				4	8.317903E-02				4	4	1.867896E-01				3	3	4.021978E-01
			3	3	1.940970E-01					5	1.543545E-01		0	3	1	1	2.039833E-01
				4	1.558880E-01				5	5	3.687156E-01					2	1.387513E-01
			4	4	3.833012E-01		1	0	2	2	9.507453E-02				2	2	4.272807E-01
	1	0	2	2	1.299328E-01					3	6.765750E-02		1	1	2	2	1.299328E-01
				3	9.153236E-02					4	5.316908E-02					3	9.153236E-02
				4	6.880421E-02					5	4.196696E-02					4	6.880421E-02
			3	3	2.019191E-01				3	3	1.319340E-01	5	1	1	3	3	2.019191E-01
				4	1.542136E-01					4	1.047027E-01					4	1.542136E-01
			4	4	4.021978E-01					5	8.317903E-02				4	4	4.021978E-01
	2	0	3	3	2.039833E-01				4	4	1.940970E-01		2	1	3	3	2.039833E-01
				4	1.387513E-01					5	1.558880E-01					4	1.387513E-01
			4	4	4.272807E-01				5	5	3.833012E-01				4	4	4.272807E-01
	0	1	1	1	1.299328E-01		2	0	3	3	1.299328E-01		1	2	2	2	2.039833E-01
				2	9.153236E-02					4	9.153236E-02					3	1.387513E-01
				3	6.880421E-02					5	6.880421E-02				3	3	4.272807E-01

Using Lloyd's method, the coefficients of BLUEs for specified values of index parameter $\theta = 0.5, 1.0, 1.5, 2.0$ and 3.0, and shape parameter $c =$ 0.5, 1, 1.5, 2 and 3 for $n = 2(1)15$ in case of doubly censored case samples (where r_1 and r_2 respectively denoting number of observations censored on the left and right)are evaluated and are available with the first author. However, the same are presented for $n = 5$ and $\theta = 1.5$ and $c = 1.5$ in case of Generalized Gamma distribution and are presented in Table (5.3)

Table 5.3. Coefficients of the BLUE's of the parameters of Generalized Gamma Distribution from doubly censored samples up to n=5.

n	r_1	r_2	i	$\gamma_{i:n}$	$\delta_{i:n}$	$V(\hat{\mu})$	$V(\hat{\sigma})$	$Cov(\hat{\mu},\hat{\sigma})$
2	0	0	1	2.139343E+00	-1.342306E+00	8.118452E-01	6.374019E-01	-6.285852E-01
	0	0	2	-1.139343E+00	1.342306E+00			
3	0	0	1	1.691151E+00	-1.091078E+00	3.543047E-01	3.025790E-01	-2.679039E-01
			2	-1.271664E-01	3.410570E-01			
			3	-5.639843E-01	7.500209E-01			
	1	0	2	2.139343E+00	-1.342306E+00	8.118452E-01	6.374019E-01	-6.285852E-01
			3	-1.139343E+00	1.342306E+00			
	0	1	1	2.139343E+00	-1.342306E+00	8.118452E-01	6.374019E-01	-6.285852E-01
			2	-1.139343E+00	1.342306E+00			
4	0	0	1	1.465068E+00	-9.623897E-01	2.159046E-01	1.949845E-01	-1.610800E-01
			2	5.917493E-02	1.224108E-01			
			3	-1.550861E-01	3.136479E-01			
			4	-3.691570E-01	5.263310E-01			

	1	0	2	1.691151E+00	-1.091078E+00	3.543047E-01	3.025790E-01	-2.679039E-01
			3	-1.271664E-01	3.410570E-01			
			4	-5.639843E-01	7.500209E-01			
	2	0	3	2.139343E+00	-1.342306E+00	8.118452E-01	6.374019E-01	-6.285852E-01
			4	-1.139343E+00	1.342306E+00			
	0	1	1	1.691151E+00	-1.091078E+00	3.543047E-01	3.025790E-01	-2.679039E-01
			2	-1.271664E-01	3.410570E-01			
			0	-5.639843E-01	7.500209E-01			
	0	2	1	2.139343E+00	-1.342306E+00	8.118452E-01	6.374019E-01	-6.285852E-01
			2	-1.139343E+00	1.342306E+00			
	1	1	2	2.139343E+00	-1.342306E+00	8.118452E-01	6.374019E-01	-6.285852E-01
			3	-1.139343E+00	1.342306E+00			
5	0	0	1	1.323128E+00	-8.797879E-01	1.515983E-01	1.426701E-01	-1.121222E-01
			2	1.230056E-01	3.342729E-02			
			3	-3.295147E-02	1.691601E-01			
			4	-1.413315E-01	2.705669E-01			
			5	-2.718502E-01	4.066336E-01			
	1	0	2	1.465068E+00	-9.623897E-01	2.159046E-01	1.949845E-01	-1.610800E-01
			3	5.917493E-02	1.224108E-01			
			4	-1.550861E-01	3.136479E-01			
			5	-3.691570E-01	5.263310E-01			
	2	0	3	1.691151E+00	-1.091078E+00	3.543047E-01	3.025790E-01	-2.679039E-01
			4	-1.271664E-01	3.410570E-01			
			5	-5.639843E-01	7.500209E-01			
	3	0	4	2.139343E+00	-1.342306E+00	8.118452E-01	6.374019E-01	-6.285852E-01
			5	-1.139343E+00	1.342306E+00			
	0	1	1	1.465068E+00	-9.623897E-01	2.159046E-01	1.949845E-01	-1.610800E-01
			2	5.917493E-02	1.224108E-01			

Table.5.3: (Contd.) Coefficients of the BLUE's of the parameters of Generalized Gamma Distribution from doubly censored samples up to n=5.

n	r_1	r_2	i	$\gamma_{i:n}$	$\delta_{i:n}$	$V(\hat{\mu})$	$V(\hat{\sigma})$	$Cov(\hat{\mu},\hat{\sigma})$
			3	-1.550861E-01	3.136479E-01			
			4	-3.691570E-01	5.263310E-01			
	0	2	1	1.691151E+00	-1.091078E+00	3.543047E-01	3.025790E-01	-2.679039E-01
			2	-1.271664E-01	3.410570E-01			
			3	-5.639843E-01	7.500209E-01			
	0	3	1	2.139343E+00	-1.342306E+00	8.118452E-01	6.374019E-01	-6.285852E-01
			2	-1.139343E+00	1.342306E+00			
	1	1	2	1.691151E+00	-1.091078E+00	3.543047E-01	3.025790E-01	-2.679039E-01
			3	-1.271664E-01	3.410570E-01			
			4	-5.639843E-01	7.500209E-01			
	2	1	3	2.139343E+00	-1.342306E+00	8.118452E-01	6.374019E-01	-6.285852E-01
			4	-1.139343E+00	1.342306E+00			
	1	2	2	2.139343E+00	-1.342306E+00	8.118452E-01	6.374019E-01	-6.285852E-01
			3	-1.139343E+00	1.342306E+00			

Future work: It is planned to extend this work on Burr type models.

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