Design of Type 2- Interval Fuzzy with LQR Controller for Continuous Stirred Tank Reactor

[1]Tejas Dilip Gangurde, [2]Sudarshan Ukirade, [3]Azhar Inamdar
Department of Electrical Engineering,
Walchand College of Engineering, Sangli, Maharashtra, India

Abstract: This paper proposes Type 2- Interval Fuzzy with LQR (T2IFLQR) controller for a non-linear system. Type 2- Interval fuzzy logic controller (T2IFLC) is self-possessed in such a way that it is an autonomous process. To decipher the influence, the impression of uncertainty on the controller execution to two different types of curves are outlined i.e. aggressive control curve and smoother control curve. A comparative of T2IFLQR and Type 2- Interval Fuzzy PID (T2IFPID) controller are shown in this paper. Popov-Lyapunov approach is used to define the stability of the framework. It is explained how T2IFLC provides extra degrees of freedom which helps to enhance the control performance of linear as well as nonlinear systems.

***IndexTerms* – Type 2- Interval Fuzzy, PID, LQR, CSTR.**

I. INTRODUCTION

In industries, as a whole, it is realized that the most normally utilized controllers are regular proportional–integral–derivative controller since they are basic in structure and low in cost [1]. In [2]-[3], different proportional–integral–derivative controller tuning strategies are testified i.e. Ziegler and Nichols, Cohen and Coon, Internal Model Control etc. Pole Placement Design Strategies are a portion of the outline systems. The use of proportional, integral and derivative controllers in controlling direct framework may be a successful approach to accomplish wanted execution, yet prop-integ-deriv (i.e. PID) controller would not give a palatable execution when the procedures have a questionable model or if the procedure is non-linear. In [4], LQR based compensator and the use of integral feedback to eliminate steady state error is described.

In [5] it is determined that Type 1- Interval Fuzzy Logic Controllers(T1IFLC) can be actualized with single or multiple inputs. Despite the fact that the significant research chip away at fuzzy prop–integ–deriv controller fixates is on the customary, either of the two inputs i.e. PI or PD. This sort of controller proposed by Mamdani, in various works. It demonstrates the solitary information that T1IFLC offers additional noteworthy adaptability and better useful properties. In three different sorts, fuzzy controllers are grouped: the gain scheduling, direct action (DA) sort, and a mix of DA and gain scheduling sorts. The DA sort generally utilized as a part of fuzzy prop–integ–deriv controller application; here in the criticism control circle fuzzy prop–integ–deriv controller is added, and the prop–integ–deriv controller activities computed utilizing fuzzy derivation. In gain scheduling sort controllers, singular prop–integ–deriv controller pick-ups are figured through fuzzy deduction.

As of late, the fundamental research concentrates on T2IFLC. For the most part, T2IFLC finish prevalent exhibitions as it gives an extra level of the opportunity given by the impression of vulnerability in their Type-2 Interval Fuzzy Sets (T2IFS). Key contrasts between T2IFLC and T1IFLC are adaptive-ness, implying that the installed Type 1- Interval Fuzzy Sets (T1IFSs) used to register the limits of the type deduced interval change as input varies. The upper-side membership functions (UMF) and lower-side membership functions (LMF) of a similar T2IFS might be utilized at the same time in processing each bound of the type deduced interval. T1IFLC lack these properties, because of this a T1IFLC is unable to execute the intricate control surface of a T2IFLC utilizing a similar rule base. Interior structure of the T2IFLC is similar to T1IFLC. Principle distinction is desired fuzzy membership function (FM), yet advanced algorithms are developed for designing T2IFLC for generating the control surface. The primary weakness of this approach is that it does not clarify how the footprint of uncertainty parameters influences the execution and robustness of the T2IFLC. In this way, determining the scientific structure of a T2IFLC in the system of the nonlinear control may be an effective approach to look at it. However, the orderly outline and strength examination of the T2IFLC are yet difficult issues because of its generally more intricate structure [6]-[7].

Conventional PID and LQR controllers are not able to give general solution to all control problems because the processes involved are in general complex and with delays, nonlinear, time-variant, non-stationary, and often with poorly defined dynamic. To overcome these difficulties, various types of modified fuzzy controllers are developed [8]-[10]. In [11]-[13] Stability analysis of Fuzzy controllers, Popov Lyapunov method for stability analysis of T2FPID controller is presented which gives better performance as compare to traditional prop-integ-deriv controller. Referring T2IFPID controller, a new controller T2IFLQR controller is designed that gives better performance as compare to prop-integ-deriv controller, LQR controller and T2IFPID controller.

II. TYPE 2- INTERVAL FUZZY LOGIC CONTROLLER STRUCTURE

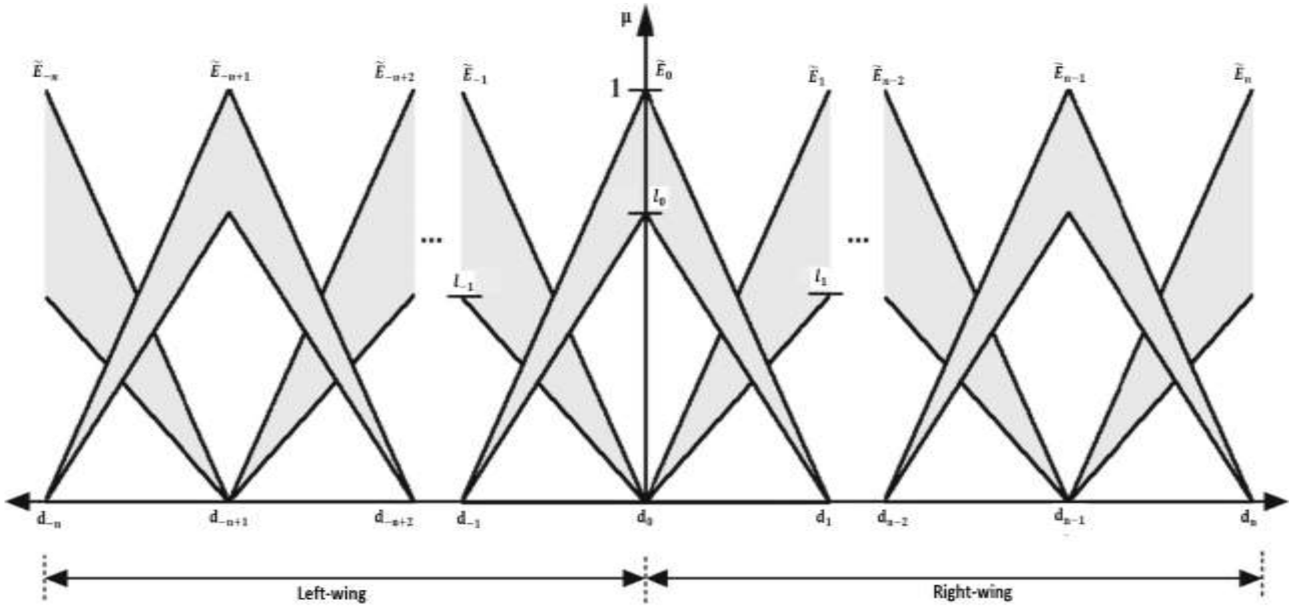

Figure 1: Demonstration of Type-2 Interval triangular membership function for error

In this segment, the proposed T2IFLC structures input-output mapping is determined. For effortlessness, the antecedent part for the T2IFLC FM of the fuzzy rule base is presented by consistently conveyed symmetrical triangular. From the error ($\underline{e}$) i.e. the linguistic value of the input are indicated as $\tilde{E}_i$ where i = {-n, (-n+1) ,(-n+2), . . . , -2, -1, 0, 1, 2, . . . ,(n-2), (n-1), n}. The presented type-2 fuzzy set ($\tilde{E}_i$) is referred as LMF ($\underline{\mu}\tilde{E}_i$) and UMF$(\overline{\mu}\tilde{E}_i)$. As appeared in Fig.1, the height of the LMF is represented by l_n though the center of the $\tilde{E}_i$ is indicated by d_n. The input T2IFSs are symmetrical, subsequently, the information space ($\underline{e}$) is separated in two fundamental districts which are named as the Left wing ($\underline{e} \in [d_{-n} \quad d_0]$) and Right wing ($\underline{e} \in [d_0 \quad d_n]$). In addition, following properties are controlled by an additional type reduction system since T2IFLC utilize and process T2IFSs. A few investigations have been introduced to examine the impact of the footprint of uncertainty and additional type reduction process on wanted fuzzy enrolment (i.e., control surface). The type-2 interval fuzzy mapping, characterized T2IFLC membership functions-

(i) $\overline{\mu}\tilde{E}_i(\underline{e}) + \overline{\mu}\tilde{E}_{i+1}(\underline{e}) = 1$, i = -n, . ,-2,-1, 0, 1,2, . .,n
(ii) $\underline{\mu}\tilde{E}_i(\underline{e}) = l_i * \overline{\mu}\tilde{E}_i(\underline{e})$, -n, . ,-2,-1, 0, 1,2, . .,n
(iii) $l_{-i} = l_i$, i = 1,2,3, . . . ,n

The proposed T2IFLC run development is described as:

$$r_i\text{: IF } (\underline{e}) \text{ is } \tilde{E}_i \text{ THEN } (\underline{e}'_{fuz}) \text{ is } B_i,\ i = 1,2,\ldots,N \quad (1)$$

The aggregate total sum of rule numbers is given as N = 2n + 1, while the resultant part deciphers the crisp singleton esteems (B_i) which are consistently disseminated in the scope of [−1, 1]. Liang and Mendel revealed that the defuzzified output of a T2IFLC can be computed as:

$$\underline{e}_{fuz} = \frac{\underline{e}^{l'}{}_{fuz} + \underline{e}^{r'}{}_{fuz}}{2} \quad (2)$$

where ($\underline{e}^{r'}{}_{fuz}$) and ($\underline{e}^{l'}{}_{fuz}$) projects the endpoints of the type deduced set.
Moreover ($\underline{e}^{r'}{}_{fuz}$), ($\underline{e}^{l'}{}_{fuz}$) are calculated as below:

$$\underline{e}^{r'}{}_{fuz} = \frac{\sum_{j=1}^{R} \underline{\mu}\tilde{E}_i(\underline{e}) * B_i + \sum_{j=R+1}^{N} \overline{\mu}\tilde{E}_i(\underline{e}) * B_i}{\sum_{j=1}^{R} \underline{\mu}\tilde{E}_i(\underline{e}) + \sum_{j=R+1}^{N} \overline{\mu}\tilde{E}_i(\underline{e})} \quad (3)$$

$$\underline{e}^{l'}{}_{fuz} = \frac{\sum_{j=1}^{R} \overline{\mu}\tilde{E}_i(\underline{e}) * B_i + \sum_{j=R+1}^{N} \underline{\mu}\tilde{E}_i(\underline{e}) * B_i}{\sum_{j=1}^{R} \overline{\mu}\tilde{E}_i(\underline{e}) + \sum_{j=R+1}^{N} \underline{\mu}\tilde{E}_i(\underline{e})} \quad (4)$$

Here, (R, L) is the emerging set that decreases/ increases separately. T2IFLC completely covers the triangular T2IFSs in the feeling of upper and lower FM's. Subsequently, it can be constantly ensured that a crisp estimation of $\underline{e}$ has a place with two progressive T2IFSs, i.e. $\tilde{E}_i$ and $\tilde{E}_{i+1}$. Subsequently, for any crisp information, simply two rules (N = 2) are constantly operated as the points (R, L) are constantly equivalent to "1". A triangular shape FM of T2IFLC is obtained in the form of LMF and UMF. The reduced type set are determined as:

$$\underline{e}^{r'}{}_{fuz} = \frac{\underline{\mu}\tilde{E}_i(\underline{e})*B_i + \overline{\mu}\,\tilde{E}_{i+1}(\underline{e})^*B_{i+1}}{\underline{\mu}\tilde{E}_i(\underline{e}) + \overline{\mu}\,\tilde{E}_{i+1}(\underline{e})}\ , \quad \underline{e}^{l'}{}_{fuz} = \frac{\overline{\mu}\,\tilde{E}_i(\underline{e})^*B_i + \underline{\mu}\tilde{E}_{i+1}(\underline{e})*B_{i+1}}{\overline{\mu}\,\tilde{E}_i(\underline{e}) + \underline{\mu}\tilde{E}_{i+1}(\underline{e})} \tag{5}$$

After replacing Eq.5 in Eq.2, proposed T2IFLC closed form mapping is obtained. Following are the properties of the presented a T2IFLC output.

(i) In relation to '$\underline{e}$', '$\underline{e}'_{fuz}$' has a continual function.

(ii) In relation to the input '$\underline{e}$', '$\underline{e}'_{fuz}$' is proportional, i.e. $\underline{e}'_{fuz}(e) = -\underline{e}'_{fuz}(\underline{-e})$.

(iii) In an event that if an input error equivalent to zero then fuzzified error equivalents to be zero. It is mandatory to have zero error i.e. $(\underline{e}) = 0$ then $\underline{e}'_{fuz} = 0$

III. LINEAR QUADRATIC REGULATOR

The finite horizon, linear quadratic regulator (LQR) is given by

$$\dot{x} = Ax + Bu \qquad x \in \mathbb{R}^n, u \in \mathbb{R}^n, x_0 \ given \tag{6}$$

$$\tilde{J} = \frac{1}{2}\int_0^T (x^T Qx + u^T Ru)dt + \frac{1}{2}x^T(T)P_1 x(T) \tag{7}$$

where $Q \geq 0, R > 0, P_1 \geq 0$ are symmetric, positive (semi-) definite matrices. Note the factor of $\frac{1}{2}$ is left out, but we included it here to simplify the derivation. Gives same answer (with $\frac{1}{2}x$ cost).

Solve via maximum principle:

$$H = x^T Qx + u^T Ru + \lambda^T(Ax + Bu) \tag{8}$$

$$\dot{x} = \left(\frac{\partial H}{\partial \lambda}\right)^T = Ax + Bu \quad x(0) = x_0 \tag{9}$$

$$-\dot{\lambda} = \left(\frac{\partial H}{\partial \lambda}\right)^T = Qx + A^T\lambda \quad \lambda(T) = P_1 x(T) \tag{10}$$

$$0 = \frac{\partial H}{\partial u} = Ru + \lambda^T B \Longrightarrow \quad u = -R^{-1}B^T\lambda \tag{11}$$

This gives the optimal solution. Apply by solving two-point boundary value problem (hard).

Alternative: guess the form of the solution, $\lambda(t) = P(t)x(t)$. Then

$$\dot{\lambda} = \dot{P}x + \dot{P}x = \dot{P}x + P(Ax - BR^{-1}B^T P)x \tag{12}$$

$$-\dot{P}x - PAx + PBR^{-1}BPx = Qx + A^T Px \tag{13}$$

This equation is satisfied if we can find $P(t)$ such that

$$-\dot{P} = PA + A^T P - PBR^{-1}BP + Q$$

$$P(T) = P_1 \tag{14}$$

State feedback with reference trajectory-

Suppose we are given a system $x = f(x, u)$ and a feasible trajectory (x_d, u_d). We wish to design a compensator of the form $u = \alpha(x, x_d, u_d)$ such that $\lim_{t\to\infty} x - x_d = 0$. This is known as the trajectory tracking problem.To design the controller, we construct the error system. We will assume for simplicity that $f(x) + g(x)u$ (i.e., the system is nonlinear in the state, but linear in the input; this is often the case in applications). Let $e = x - x_d, v = u - u_d$ and compute the dynamics for the error:

$$\dot{e} = \dot{x} - \dot{x_d} = f(x) + g(x)u - f(x_d) + g(x_d)u_d \tag{15}$$

$$\dot{e} = f(e + x_d) - f(x_d) + g(e + x_d) - g(x_d)(v + u_d) - g(x_d)u_d \tag{16}$$

$$\dot{e} = F(e, v, x_d(t), u_d(t)) \tag{17}$$

In general, this system is time varying. For trajectory tracking, we can assume that e is small (if our controller is doing a good job) and so we can linearize around $e = 0$:

$$\dot{e} \approx A(t)e + B(t)v \tag{18}$$

where

$$A(t) = \left.\frac{\partial F}{\partial e}\right|_{(x_{d(t)}, u_{d(t)})} \qquad B(t) = \left.\frac{\partial F}{\partial v}\right|_{(x_{d(t)}, u_{d(t)})}$$

It is often the case that $A(t) and\ B(t)$ depend only on x_d, in which case it is convenient to write $A(t) = A(x_d)\ and\ B(t) = B(x_d)$.Assume now that x_d and u_d are either constant or slowly varying (with respect to the performance criterion). This allows us to consider just the (constant) linear system given by $(A(x_d), B(x_d))$. If we design a state feedback controller $K(x_d)$for each x_d, then we can regulate the system using the feedback

$$v = K(x_d)e \tag{19}$$

Substituting back the definitions of e and v, our controller becomes

$$u = K(x_d)(x + x_d) + u_d \tag{20}$$

This form of controller is called a gain scheduled linear controller with feedforward u_d. In the special case of a linear system

$$\dot{x} = Ax + Bu \tag{21}$$

it is easy to see that the error dynamics are identical to the system dynamics (so $\dot{e} = Ae + Bv$) and in this case, we do not need to schedule the gain based on x_d; we can simply compute a constant gain K and write

$$u = K(x_d)(x - x_d) + u_d \tag{22}$$

Integral Action-

The controller based on state feedback achieves the correct steady state response to reference signals by careful computation of the reference input u_d. This requires a perfect model of the process in order to ensure that (x_d, u_d) satisfies the dynamics of the process. However, one of the primary uses of feedback is to allow good performance in the presence of uncertainty, and hence requiring that we have an exact model of the process is desirable. An alternative to calibration is to make use of integral feedback, in which the controller uses an integrator to provide zero steady state error.

The basic approach in integral feedback is to create a state within the controller that computes the integral of the error signal, which is then used as a feedback term. We do this by augmenting the description of the system with a new state z:

$$\frac{d}{dt}\begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} Ax + Bu \\ y - r \end{bmatrix} = \begin{bmatrix} Ax + Bu \\ Cx - r \end{bmatrix} \quad (23)$$

The state z is seen to be the integral of the error between the desired output, r, and the actual output, y. Note that if we find a compensator that stabilizes the system then necessarily we will have $\dot{z} = 0$ in steady state and hence $y = r$ in steady state. Given the augmented system, we design a state space controller in the usual fashion, with a control law of the form

$$u = -K(x - x_d) - K_i z + u_d \quad (24)$$

where K is the usual state feedback term, K_i is the integral term and x_d is used to set the reference input for the nominal model. For a step input, the resulting equilibrium point for the system is given as

$$x_e = -(A - BK)^{-1} B(u_d - K_i z_e) \quad (25)$$

Note that the value of z_d is not specified, but rather will automatically settle to the value that makes $\dot{z} = y - r = 0$, which implies that at equilibrium the output will equal the reference value. This holds independently of the specific values of A, B and K, as long as the system is stable (which can be done through appropriate choice of K and Ki).

The final compensator is given by

$$u = -K(x - x_e) - K_i z + u_d \quad (26)$$

$$\dot{z} = y - r \quad (27)$$

where we have now included the dynamics of the integrator as part of the specification of the controller.

IV. Design Strategy for Type 2- Interval Fuzzy PID and Type 2- Interval Fuzzy with LQR Controller

In this section, the outlined procedure for singular input T2IFLC is presented. The output is expressed in the error ($\underline{e}$) domain since the T2IFLC has singular input. This clarifies the design method of T2IFLC to the generation of non-linear control curve. In this strategy, the height (l_i) of lower-side FM's of $\tilde{E}_i$are considered as design parameters. Firstly, design parameters effect on T2IFLC are scrutinized and then an autonomous design method is proposed to generated control curves/curves. The parameters for T2IFLC structure are set as $\underline{B}_{-1} = -1$, $\underline{B}_{+1} = +1$, $\underline{B}_0 = 0$, $\underline{d}_{-1} = -1$, $\underline{d}_{+1} = +1$ and $\underline{d}_0 = 0$. For the information Left-wing ($\underline{e} \in [-1, 0]$), the end purposes of the type decreased set would then be able to be inferred as takes after-

$$\underline{e}^{l\prime}{}_{fuz} = \frac{-\overline{\mu}\tilde{E}_{-1}(\mathrm{e})}{\overline{\mu}\tilde{E}_{-1}(\mathrm{e}) + \underline{\mu}\tilde{E}_0(\mathrm{e})}, \quad \underline{e}^{r\prime}{}_{fuz} = \frac{-\overline{\mu}\tilde{E}_{-1}(\mathrm{e})}{\underline{\mu}\tilde{E}_{-1}(\mathrm{e}) + \overline{\mu}\tilde{E}_0(\mathrm{e})} \quad (28)$$

For the Right-wing ($\underline{e} \in [0, +1]$), the end point of the type deduced sets reduces to:

$$\underline{e}^{l\prime}{}_{fuz} = \frac{\underline{\mu}\tilde{E}_1(\underline{e})}{\overline{\mu}\tilde{E}_0(\underline{e}) + \underline{\mu}\tilde{E}_1(\underline{e})}, \quad \underline{e}^{r\prime}{}_{fuz} = \frac{\underline{\mu}\tilde{E}_1(\underline{e})}{\underline{\mu}\tilde{E}_0(\underline{e}) + \overline{\mu}\tilde{E}_1(\underline{e})} \quad (29)$$

Table 1: Expressions of $\underline{e}^{l}{}_{fuz}$ and $\underline{e}^{r}{}_{fuz}$

	Left wing '$\underline{e}$' ∈ [−1, 0]	Right wing '$\underline{e}$' ∈ [0,+1]
$\underline{e}^{l\prime}{}_{fuz}$	$\frac{-\overline{\mu}\tilde{E}_{-1}(\underline{e})}{\overline{\mu}\tilde{E}_{-1}(\underline{e}) + \underline{\mu}\tilde{E}_0(\underline{e})}$	$\frac{\underline{\mu}\tilde{E}_1(\underline{e})}{\overline{\mu}\tilde{E}_0(\underline{e}) + \underline{\mu}\tilde{E}_1(\underline{e})}$
$\underline{e}^{r\prime}{}_{fuz}$	$\frac{-\overline{\mu}\tilde{E}_{-1}(\underline{e})}{\underline{\mu}\tilde{E}_{-1}(\underline{e}) + \overline{\mu}\tilde{E}_0(\underline{e})}$	$\frac{\underline{\mu}\tilde{E}_1(\underline{e})}{\underline{\mu}\tilde{E}_0(\underline{e}) + \overline{\mu}\tilde{E}_1(\underline{e})}$

To inspect the impact of the design parameters (l_{-1}, l_0, l_1) on the output effectively the inferred explanatory articulations of $\underline{e}^{l\prime}{}_{fuz}$ and $\underline{e}^{r\prime}{}_{fuz}$ for a "three rule" interval T2IFLC are organized in Table 1. In this examination, the just Right-wing will be assessed in detail. Because of symmetrical and uniformly distributed nature of input and output FM, the evaluation of Right-wing can be extended for Left-wing in detail. The accompanying meta-rules inferred to shape a control activity from the determined articulations of $\underline{e}^{l\prime}{}_{fuz}$ and $\underline{e}^{r\prime}{}_{fuz}$ for Right-wing, to get a palatable framework execution.

a. The values of '$\underline{e}^{l\prime}{}_{fuz}$' decrements/increments, if the values of $\overline{\mu}\tilde{E}_1(\underline{e})$ (i.e.l_1) decrements/increments respectively.
b. The values of '$\underline{e}^{r\prime}{}_{fuz}$' increments/decrements, if the values of $\overline{\mu}\tilde{E}_0(\underline{e})$ (i.e. l_0) decrements/increments respectively. The defuzzified output of a T2IFLC ($\underline{e}'_{fuz}$) is the mean value of '$\underline{e}^{r\prime}{}_{fuz}$' and '$\underline{e}^{l\prime}{}_{fuz}$' values.
c. If the values of 'l_1' is incremented while 'l_0' is decremented then the values of '$\underline{e}'_{fuz}$' is incremented since the values of both '$\underline{e}^{r\prime}{}_{fuz}$' and '$\underline{e}^{l\prime}{}_{fuz}$' are incremented. Hence, we obtain an aggressive control action.
d. If the values of 'l_1' is decremented while 'l_0' is incremented then the values of '$\underline{e}'_{fuz}$' is decremented since the values of both '$\underline{e}^{r\prime}{}_{fuz}$' and '$\underline{e}^{l\prime}{}_{fuz}$' are decremented. Hence, we obtain a smooth control action.

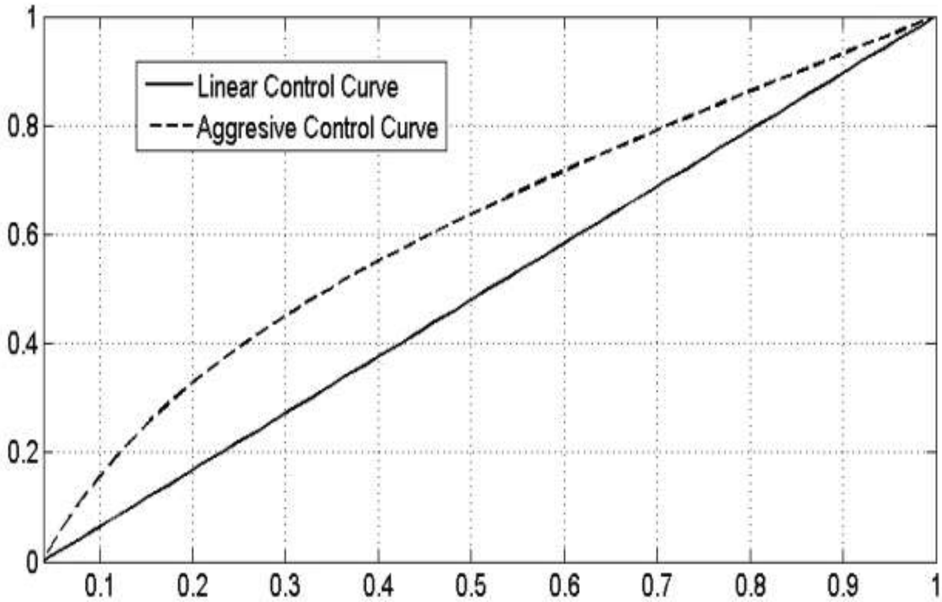

Figure 2: Linear control curve versus Aggressive control curve

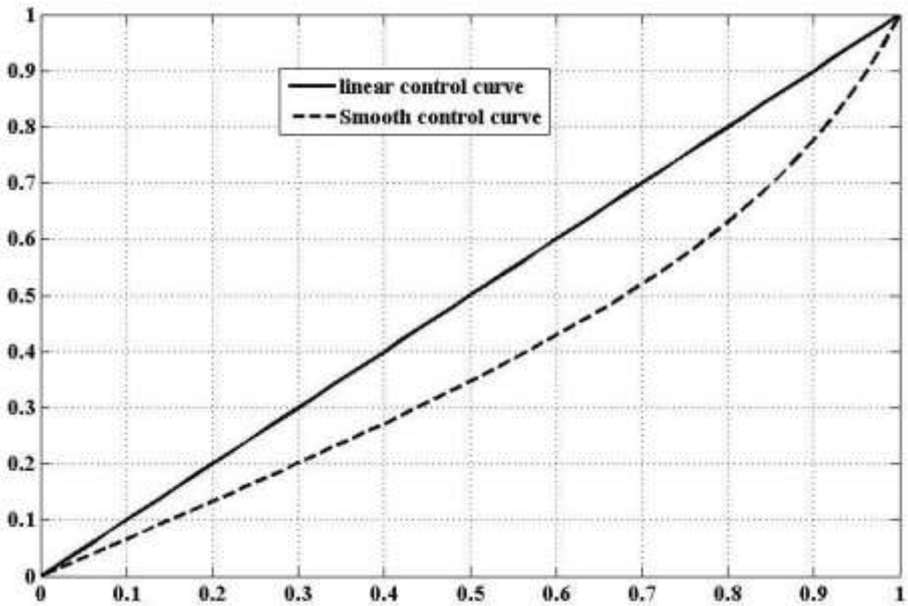

Figure 3: Linear control curve versus Smooth control curve

By choosing l_1 moderately greater then l_0, a aggressive control action can be produced i.e. $1 \geq l_1 = l_{-1} \geq l_0 \geq 0$. To get aggressive control curve, selecting $\underline{m}_1 = \underline{m}_{-1} = 0.9$ and $\underline{m}_0 = 0.2$, the control curve showed in Fig.2. At the point when $\underline{e}$ is near zero, the control curve has a high affect ability. For quick transient framework response, aggressive control curve is favored. At set point value, i.e. $\underline{e}$ equals to zero aggressive control action is sensitive to noises.

Smooth control action is acquired by choosing l_0 moderately greater than l_1, i.e. $1 \geq l_0 \geq l_1 = l_{-1} \geq 0$. To acquire smooth control curve given in Fig.3, selecting $l_1 = l_{-1} = 0.2$ and $l_0 = 0.9$. At the point when $\underline{e}$ is near to zero, it has low affectability. For robust control performance against parameter vulnerabilities and additionally background noises, smooth control curve is favored. The control law for T2IFPID controller is given by

$$u(t) = K_P(t)\underline{e}_{fuz}(t) + K_I(t)\int \underline{e}_{fuz}(t) + K_D\frac{d\underline{e}_{fuz}(t)}{dt} \qquad (30)$$

The control law for T2IFLQR controller is given by

$$u(t) = -K(t)\underline{e}_{fuz}(t) \qquad (31)$$

where,

K_P = Proportional gain
K_D = Derivative gain
K_I = Integral gain

K = LQR feedback gain

V. Stability Analysis

Robust Stability Analysis for fuzzy based systems is carried out through Popov-Lyapunov method. In this approach initially, expand a nonlinear system into a Taylor series. As presented in [13] the system is described in terms of Perturbed Lur'e system. Following are the steps to determine the robust stability.

1. Plot the Popov's plot for the nominal system. If the Popov's criteria are pleased, then find the significant slope (r). The r should be scalar such that value of $r > 0$.
2. Calculate 'v' and 'γ' from [13].
3. Choose a symmetric positive-definite matrix E and a positive real number 'ϵ', then the '$\underline{P}$' matrix is acquired by solving the following Ricatti equation:

$$\underline{A}_r^T \underline{P} + \underline{P}\underline{A}_r - \underline{P}\underline{R}_r + \underline{Q}_r = 0 \qquad (32)$$

where

$$\underline{A}_r = A - \frac{1}{\gamma} bv^T, \underline{Q}_r = \epsilon E + \frac{vv^T}{\gamma}, \underline{R}_r = -\frac{bb^T}{\gamma}$$

4. Choose a positive-definite matrix 'E_0' such that $\epsilon E = \epsilon E_0 + \boldsymbol{\delta} I$, then the proper value of '$\boldsymbol{\delta}$' can be obtained.
5. '$\underline{\beta}$' i.e. measurement of robustness can be obtained can be obtained from inequality [13].

VI. Simulation results and Stability Analysis of CSTR

For calculating transfer function of CSTR cooling process, the step response is taken into consideration. For the step response, the input is step input, the initial temperature is taken as 57^oC and the set point is taken as 45^oC. The process has very large dead time and is highly damped. Therefore, the step response can be fitted into a simple first-order model with dead-time.

Therefore, the transfer function of the CSTR process is given by

$$\underline{G}(s) = \frac{0.12e^{-2s}}{3s+1} \qquad (33)$$

Tuning of prop-integ-deriv for CSTR is done, it is resulting as follows: K_P=6.667, $K_D = 2$ and $K_I = 1.9$. Simulation results of CSTR with unit step are shown in Fig.4, Fig.5 and Fig.6. Figure 4 shows the comparison between PID and LQR controller. Figure 5 shows the comparison between T2IFPID and T2IFLQR with smooth control action and Fig.6 shows the comparison between T2IFPID and T2IFLQR with aggressive control action. It can be seen that different types of controller are able to track the reference with different rise time and settling time. Response due to Aggressive control action Fuzzy controller has less rise time compared to other controller; while response due to Smoother control action Fuzzy controller has higher settling time. Table 2 shows step response characteristics of PID controller, LQR controller, T2IFPID controller and T2IFLQR controller with smooth and aggressive control action. To find out the stability of fuzzy system using Popov-Lyapunov method first system is converted into perturbed Lur'e system, which is represented by transfer function as follows-

$$\underline{G}_{T2IFPID}(s) = \frac{-0.1067s+0.32}{s^2+1.269s+0.525} \qquad (34)$$

$$\underline{G}_{T2IFLQR}(s) = \frac{-0.04s+0.04}{s^2+1.333s+0.333} \qquad (35)$$

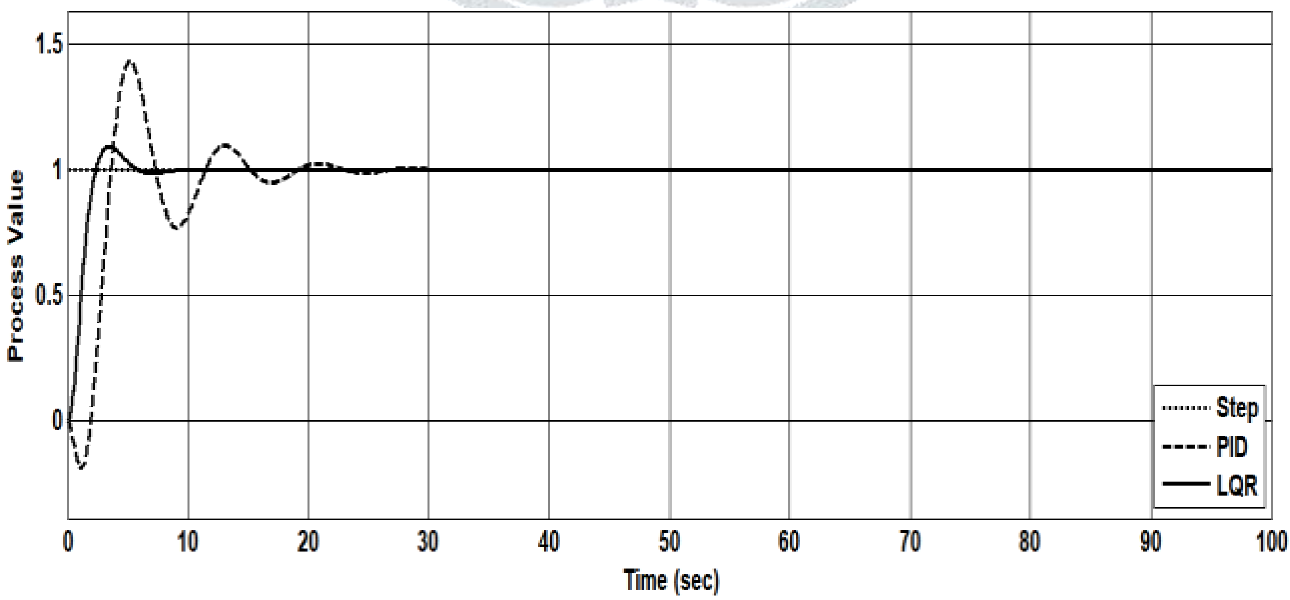

Figure 4: Setpoint tracking of PID controller and LQR controller

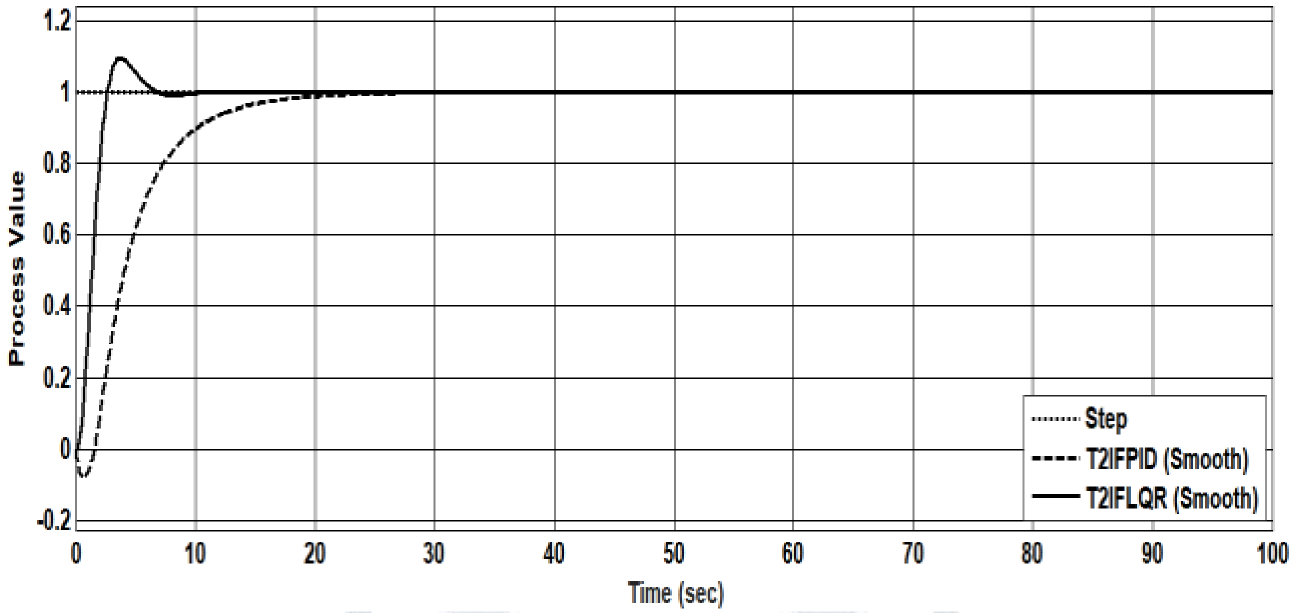

Figure 5: Setpoint tracking of T2IFPID (Smooth) controller and T2IFLQR (Smooth) controller

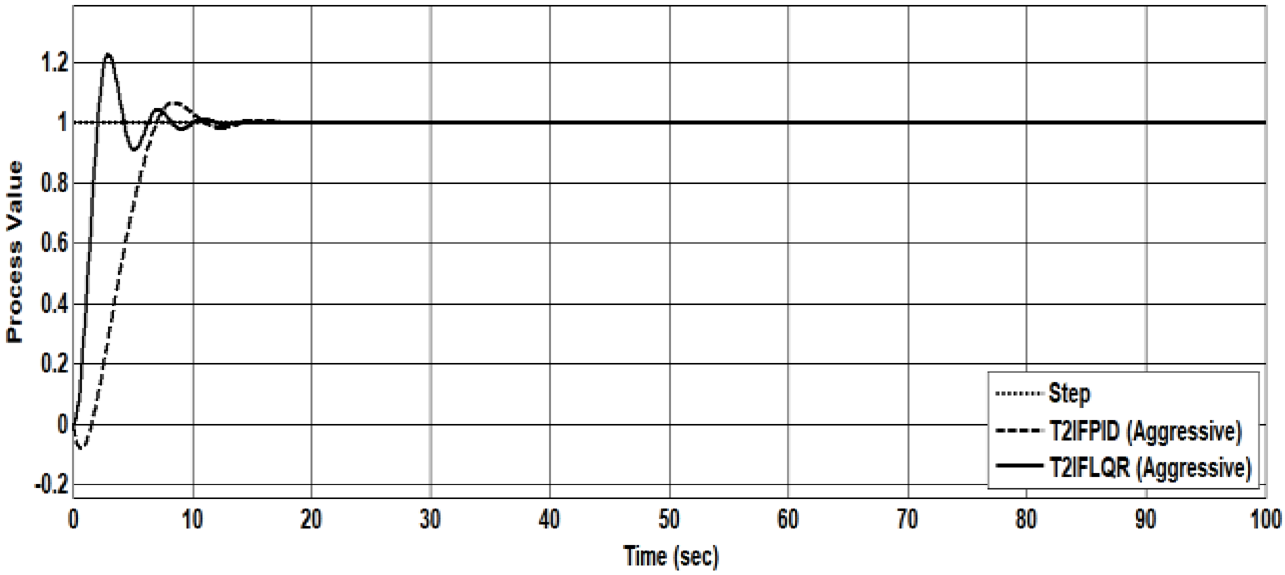

Figure 6: Setpoint tracking of T2IFPID (Aggressive) controller and T2IFLQR (Aggressive) controller

Table 2: Step response characteristic of continuous stirred tank reactor with different controller

	PID	**LQR**	**Smooth Type 2 Fuzzy PID**	**Smooth Type 2 Fuzzy LQR**	**Aggressive Type 2 Fuzzy PID**	**Aggressive Type 2 Fuzzy LQR**
Rise Time (sec)	4.470	1.350	7.749	1.700	4.470	1.350
Settling Time (sec)	38.273	20.573	25.668	11.778	30.972	19.573
Overshoot (%)	40.164	10.290	0.491	9.341	6.989	22.840
IAE	5.106	2.691	5.482	1.691	4.412	1.824

Transfer function $G_{T2IFPID}(s)$ represent perturbed Lur'e system of fuzzy system which implement T2IFPID controller while Transfer function $G_{T2IFLQR}(s)$ represent perturbed Lur'e system of fuzzy system which implement T2IFLQR. Fig.7 represent popov plot of the system represented by transfer function Eq.34, from that plot it can be concluded as given system satisfies popov criterion for slope of line $r \geq 1.35$. Similarly, popov plot, of system represented by Eq.35 is shown in Fig.8, from that it can be seen that given system satisfies popov criterion for slope of line $r \geq 1.44$. Appling Lyapunov stability method $\underline{P}$ matrices are given as follows-

$$\underline{P}_{T2IFPID} = \begin{bmatrix} 1.4766 & -0.1342 \\ -0.1342 & 0.4102 \end{bmatrix} \quad (36)$$

$$\underline{P}_{T2IFLQR} = \begin{bmatrix} 1.3040 & 1.2982 \\ 1.2982 & 3.7925 \end{bmatrix} \quad (37)$$

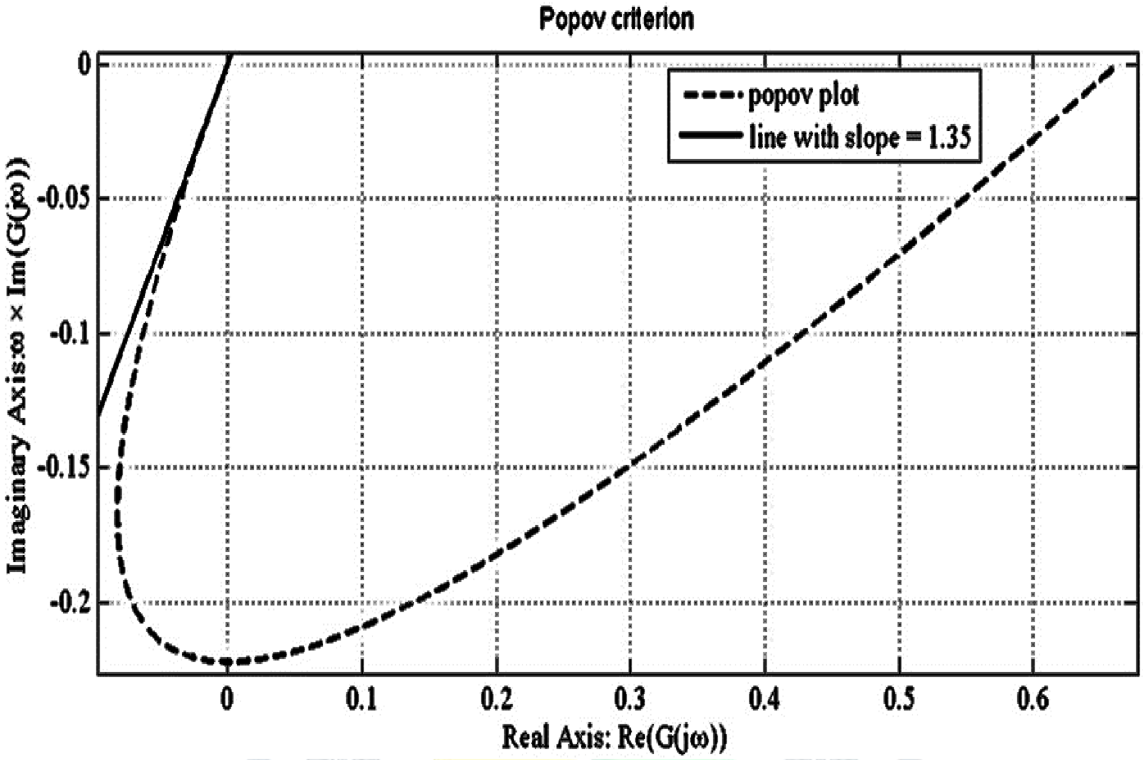

Figure 7: Popov plot of $G_{S-IT2}(s)$

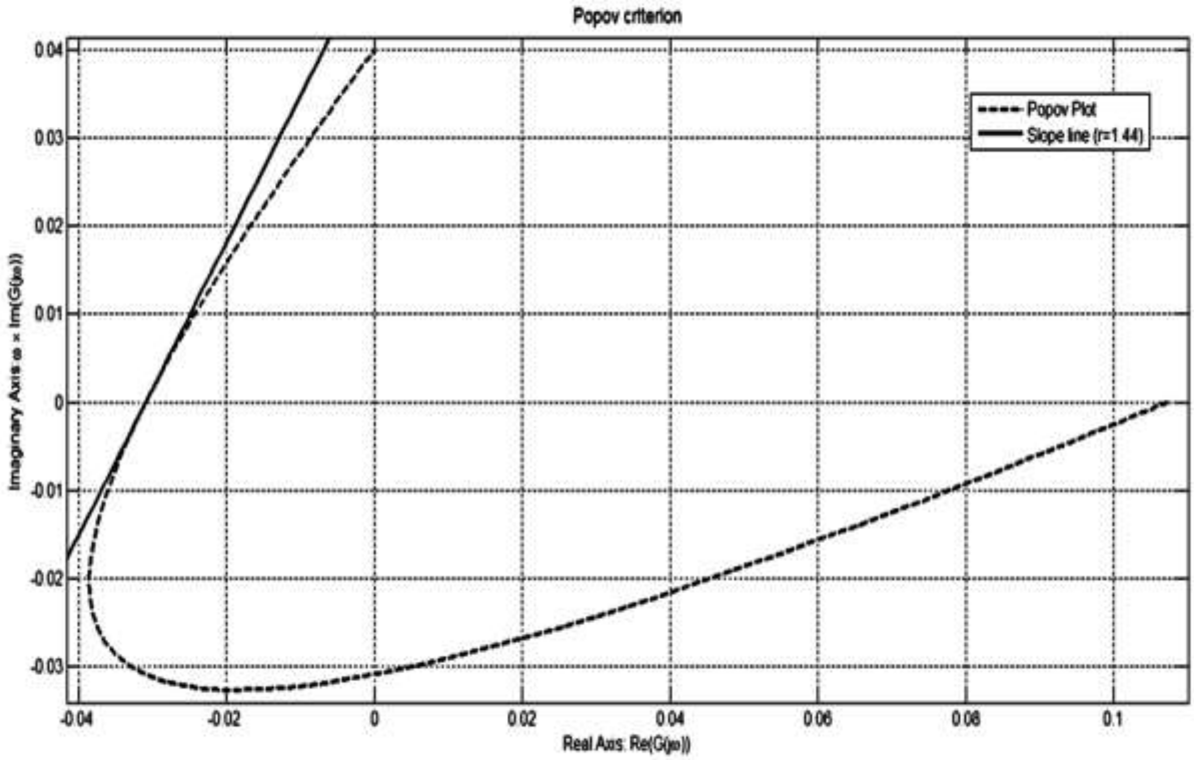

Figure 8: Popov plot of $G_{A-IT2}(s)$

Given $\underline{P}$ matrices are symmetric positive-definite, from this popov criterion and Lyapunov stability method its can be concluded that equilibrium point zero is uniformly asymptotically stable. Robustness measure $\underline{\beta}$ can be calculated, which are $\underline{\beta}_{S-IT2} = 0.7015$ for smoother control action fuzzy system and $\underline{\beta}_{A-IT2} = 0.4627$ for aggressive control action fuzzy system. From this it is stated that smoother control action fuzzy system is more robust than aggressive control action fuzzy system.

VII. CONCLUSION

The proposed design method is the simplest method to design T2IFLC. The proposed controller possesses the properties of LQR controller. The T2IFLC controller output is completely expressed in error. Hence, an autonomous design method is proposed to generate nonlinear control curves. The performance of the T2IFLC is compared with different controllers to investigate transient state performance and disturbance rejection. Stability analysis is carried through Popov-Lyapunov method to define the robustness of the proposed controller.

Aggressive control action is favored for quick transient reaction since it is delicate to the commotion, particularly around setpoint esteem. At relentless state, controller, which has a smooth control surface, is conceivably more powerful against non-linearity and vulnerabilities. T2IFLC gives a diverse reaction to the various footprint of uncertainty parameters. T2IFLQR controller beats the PID, LQR, T2IFPID controller.

VIII. ACKNOWLEDGMENT

I wish to acknowledge, Walchand college of Engineering. Sangli, Maharashtra for their support and provision of required Lab accessories. For my pleasure I am presenting deepest gratitude especially towards my father Mr. Dilip Gangurde, my mother Manisha Gangurde, my sister Miss. Trupti Gangurde and all my friends who have contributed their valuable effort in completing my project work as curricular activity for their guidance and support.

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