Kantowski - Sachs Dark Energy Cosmological Model In Saez – Ballester Theory Of Gravitation

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Abstract : A Kantowski-Sachs universe with variable equation of state (EoS) parameter and constant deceleration parameter is obtained in a scalar-tensor theory of gravitation proposed by Saez-Ballester (Phys. Lett. A 113:467,1986). To determine solution of field equation we use special law of variation for Hubble parameter presented by Bermann which yields a cosmological model with negative constant deceleration parameter. The physical and kinematical properties of the model have been discussed.

Keywords: Cosmological model, Dark energy, Saez-Ballester Scalar-Tensor theory.

I. Introduction:

Recent observations like type Ia supernova (SN Ia), CMB anisotropy and large scale structure [1-8] indicates that our universe is flat and there exist dark energy, which constitutes about 70% of the total energy of Universe.
Saez-Ballester formulated a scalar tensor theory of gravitation in which the metric is coupled with a dimensionless character of the scalar field and antigravity regime appears. This theory also suggests a possible way to solve missing matter problem in non flat FRW cosmologies.
The field equations given by Saez-Ballester theory for the combined scalar and tensor fields are

$$G_{ij} - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -kT_{ij} \tag{1}$$

and the scalar field ϕ satisfies the equation

$$2\phi^n\phi^i_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{2}$$

Where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein tensor, R the scalar curvature, ω and n are constants, T_{ij} is the stress tensor of matter.
Also, we have energy conservation equation,

$$T^{ij}_{;i} = 0 \tag{3}$$

Here comma and semicolon denote partial and covariant differentiation respectively.
Cosmological models in the Saez-Ballester [9] scalar-tensor theory of gravitation have been studied by Singh and Agrawal [10], Ram and Tiwari [11], Singh and Ram [12], Mohanty and Sahu [13], Tripathi et al. [14], Reddy et al. [15], Rao et al. [16,17], Ram et al. [18] and many. Recently, Naidu et al. [19] obtained Bianchi type-III cosmological model in Saez-Ballester theory considering a variation law for Hubble's parameter which yields a constant value of deceleration parameter. In literature it is common to use a constant deceleration parameter as it duly gives a power law for metric function or corresponding quantity. But for a universe which was decelerating in the past and accelerating at present, the DP must show signature flipping [20-22]. So, in general, the DP is not a constant but time variable. Amirhashchi et al. [23], Pradhan et al. [24-26] and Yadav [27] investigated Bianchi type cosmological models with time-dependent deceleration parameter. Recently, Rahman et al [28] studied Bianchi type-III dark energy model with variable EoS parameter in the framework of Saez-Ballester scalar-tensor theory of gravitation.
In recent years various form of time dependent w have been used for variable models by Mukhopadhyay et al [29, 30]; Usmani et al [31]. Recently many reacherers [32- 36] have obtained dark energy models with variable EoS parameter in different contexts. Yadav et al. [37], Pradhan et al. [38, 39] have recently studied homogeneous and anisotropy Bianchi type – III space-time in context of massive strings. Recently Katore et al. [40] have obtained anisotropic DE models with constant deceleration parameter and dark energy cosmological model coupled with perfect fluid in Seaz-Ballester theory of gravitation have been discussed by Rao et al.[41].
Motivated by above works, in this paper we have investigated a Kantowski-Sachs universe with variable equation of state (EoS) parameter and constant deceleration parameter in a Saez-Ballester scalar-tensor theory of gravitation. Some physical and geometrical behaviors of the model are also discussed.

II. The Metric and Field equations:-

We consider Kantowski – Sachs space time given by

$$ds^2 = dt^2 - A^2dr^2 - B^2\left[d\theta^2 + Sin^2\theta d\psi^2\right] \tag{4}$$

where A, B, are the functions of t only.
The energy momentum tensor of fluid is taken as

$$T_i^j = diag[T_0^0, T_1^1, T_2^{\backslash 2}, T_3^3], \quad (5)$$

The simplest generalization of EoS parameter of fluid is to determine it separately on each spatial axis by preserving diagonal form of the energy momentum in a consistent way with the considered metric. Hence one can parameterize this as follows

$$T_i^j = diag[\rho, -p_x, -p_y, -p_z],$$

$$T_i^j = diag[1, -w_x, -w_y, -w_z]\rho,$$

$$T_i^j = diag[1, -(w+\delta)\rho, -w\rho, -(w+\eta)\rho] \quad (6)$$

Where ρ is the energy density of the fluid, p_x, p_y, p_z are pressures and w_x, w_y, w_z are the directional EoS parameters along the x, y, z axes respectively. $w(t) = \frac{p}{\rho}$ is the free EoS parameter of the fluid. We have parameterize the deviation from isotropy by setting $w_y = w$ and then introducing skewness parameter δ and η which is the deviation from along both x and y axes respectively.

In co-moving coordinate system Saez-Ballester independent field equations for the metric (4) with the help of (5) and (6) can be written as

$$\frac{2B_{44}}{B} + \frac{B_4^{\ 2}}{B^2} + \frac{1}{B^2} - \frac{\omega}{2}\phi^n \phi_4^{\ 2} = -(w+\delta)\rho \quad (7)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} - \frac{\omega}{2}\phi^n \phi_4^{\ 2} = -w\rho \quad (8)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} - \frac{\omega}{2}\phi^n \phi_4^{\ 2} = -(w+\eta)\rho \quad (9)$$

$$\frac{2A_4}{A}\frac{B_4}{B} + \frac{B_4^{\ 2}}{B^2} + \frac{1}{B^2} + \frac{\omega}{2}\phi^n \phi_4^{\ 2} = \rho \quad (10)$$

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right) + \frac{n}{2}\frac{\phi_4^{\ 2}}{\phi} = 0 \quad (11)$$

$$\rho_4 + (w+\delta)\,\rho\frac{A_4}{A} + (w+\eta)\,\rho\frac{B_4}{B} + w\rho\left[\frac{\cot\theta}{B^2} + \frac{B_4}{B}\right] + \rho\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right) = 0 \quad (12)$$

Here suffix 4 indicate ordinary differentiation with respect to t.

From equation (8) and (9), we have

$$\eta = 0 \quad (13)$$

Using equation (13), the set of field equations reduces to

$$\frac{2B_{44}}{B} + \frac{B_4^{\ 2}}{B^2} + \frac{1}{B^2} - \frac{\omega}{2}\phi^n \phi_4^{\ 2} = -(w+\delta)\rho \quad (14)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} - \frac{\omega}{2}\phi^n \phi_4^{\ 2} = -w\rho \quad (15)$$

$$\frac{2A_4}{A}\frac{B_4}{B} + \frac{B_4^{\ 2}}{B^2} + \frac{1}{B^2} + \frac{\omega}{2}\phi^n \phi_4^{\ 2} = \rho \quad (16)$$

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right) + \frac{n}{2}\frac{\phi_4^{\ 2}}{\phi} = 0 \quad (17)$$

$$\rho_4 + (w+\delta)\rho\frac{A_4}{A} + w\rho\left(\frac{\cot\theta}{B^2} + \frac{2B_4}{B}\right) + \rho\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right) = 0 \quad (18)$$

III. Solutions of Field equations and the Model:-

The field equations (14) – (18) of a system of five independent equations being the consequence of the field equation with six unknowns $A, B, \phi, \omega, \rho, and, \delta$. Therefore one additional constraint relating these unknowns is required to obtain explicit solutions of the system.

With the help of special law of variation for Hubble's parameter presented by Berman (1983) that yields constant deceleration parameter models of the universe. We consider only constant deceleration parameter models defined by

$$q = -\frac{RR_{44}}{R_4^{\,2}} = \text{constant} \tag{19}$$

Where the overall scale factor R , is given by

$$R = (AB^2)^{\frac{1}{3}} \tag{20}$$

Here the constant is taken as negative (i.e. it is an accelerating model of the universe).
On solving equation (20), we get

$$R = (at+b)^{\frac{1}{1+q}} \tag{21}$$

where $a \neq 0$ and b are constants of integration .

This equation implies that the condition of expansion is $(1+q) > 0$ (because the scale factor R cannot be negative as well as we know that if $q > 0$ then $\frac{dR}{dt}$ is slowing down and if $q < 0$ then $\frac{dR}{dt}$ is speeding up).

Also the equations being non-linear, we assume a relation between metric coefficients given by

$$A = \alpha B \tag{22}$$

Now with the help of equations (20) – (22), the field equations of Saez-Ballester theory admit an exact solution given by

$$A = k_1 (at+b)^{\frac{1}{1+q}} \quad \text{Where } k_1 = \alpha^{\frac{2}{3}} \tag{23}$$

$$B = k_2 (at+b)^{\frac{1}{1+q}} \quad \text{Where } k_2 = \alpha^{\frac{-1}{3}} \tag{24}$$

$$\phi = k_4 (at+b)^{\frac{2q-4}{(1+q)(n+2)}} \quad \text{where } k_4 = \left[\frac{k_3}{a}\left(\frac{q+1}{q-2}\right)\left(\frac{n+2}{2}\right)\right]^{\frac{2}{n+2}} \tag{25}$$

After a suitable choice of co-ordinates and constants, the dark energy model corresponding to equations (23) and (24) can be written as

$$ds^2 = \frac{dT^2}{a^2} - k_1^{\,2} T^{\frac{2}{1+q}} dr^2 - k_2^{\,2} T^{\frac{2}{1+q}} \left[d\theta^2 + Sin^2\theta d\psi^2\right] \tag{26}$$

IV.Physical Properties:-

Kintowski-Sachs cosmological model with dark energy in Saez-Ballester theory is represented by equation (26).It is observed that the model has no initial singularity i.e. at $T = 0$ and it represent an accelerating model of the universe

The physical and kinematical parameters in this model that are important in cosmology are spatial volume V^3, expansion scalar θ, shear scalar σ^2, Hubble parameter H and Average anisotropy parameter A_m which have the following expansion for the the models (26)

$$V^3 = T^{\frac{3}{1+q}} \tag{27}$$

$$H = \frac{a}{(1+q)T} \tag{28}$$

$$\theta = \frac{3a}{(1+q)T} \tag{29}$$

$$\sigma^2 = 0 \tag{30}$$

$$A_m = 0 \tag{31}$$

The energy density ρ , the skewness parameter δ, η and the EoS parameter ω , and the overall density parameter Ω are given by

$$\rho = \frac{3a^2}{(1+q)^2 T^2} + \frac{1}{k_2^{\,2} T^{\frac{2}{1+q}}} + \frac{\omega \phi^n k_4^{\,2} a^2 (2q-4)^2}{2(1+q)^2 (n+2)^2} T^{\frac{-2n(1+q)-12}{(1+q)(n+2)}} \tag{32}$$

$$\eta = 0 \tag{33}$$

$$\omega = \frac{\left[\frac{a^2(2q-1)}{(1+q)^2T^2} + \frac{\omega\phi^n k_4{}^2 a^2(2q-4)^2}{2(1+q)^2(n+2)^2} T^{\frac{-2n(1+q)-12}{(1+q)(n+2)}}\right]}{\left[\frac{3a^2}{(1+q)^2T^2} + \frac{1}{k_2{}^2 T^{\frac{2}{1+q}}} + \frac{\omega\phi^n k_4{}^2 a^2(2q-4)^2}{2(1+q)^2(n+2)^2} T^{\frac{-2n(1+q)-12}{(1+q)(n+2)}}\right]} \tag{34}$$

$$\delta = \frac{\frac{-1}{k_2{}^2 T^{\frac{2}{1+q}}}}{\left[\frac{3a^2}{(1+q)^2T^2} + \frac{1}{k_2{}^2 T^{\frac{2}{1+q}}} + \frac{\omega\phi^n k_4{}^2 a^2(2q-4)^2}{2(1+q)^2(n+2)^2} T^{\frac{-2n(1+q)-12}{(1+q)(n+2)}}\right]} \tag{35}$$

$$\Omega = 1 + \frac{(1+q)^2}{3a^2k_2{}^2} T^{\frac{2q}{1+q}} + \frac{\omega\phi^n k_4{}^2(2q-4)^2}{6(n+2)^2} T^{\frac{4(q-2)}{(1+q)(n+2)}} \tag{36}$$

It may be observed that at initial moment , when $T = 0$, the spatial volume will be zero and the overall density parameter Ω will be 1, while the energy density ρ and the pressure p diverge, when $T \to 0$ then expansion scalar θ, shear scalar σ^2 and Hubble parameter H tends to ∞. For large values of $T(T \to \infty)$, we observe that spatial Volume becomes infinite , expansion scalar θ, shear scalar σ^2, Hubble parameter H, pressure p and density ρ becomes zero. Also, since $1+q>0$ the model is accelerating. It, therefore, follows that our dark energy model in Saez-Ballester theory is consistent with the recent observations of Type-I a supernova (Permutter et al. [1]; Reiss et al. [4]). Also $A_m = 0$ which shows that the model is isotropy and this model is shear free.

V. Conclusions:

Hence in this paper we have studied Kantowski-Sachs dark energy models in the frame work of Saez-Ballester scalar-tensor theory of gravitation. The model obtained represents accelerating model of the universe which is consistent with the recent observations of type –Ia supernova. The model presents a new dark energy model in Saez-Ballester theory with variable EoS parameter. The model obtained is expanding and non-singular. This model is shear free. For this model, we can see that, $A_m = 0$ which indicates that the model is isotropic and represents the present stage of the universe.

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