DESIGN OF MIMO PI CONTROLLER USING MULTIPLE INTEGRATION APPROACH FOR TWO TANK MIMO PROCESS

[1] C.Ravikumar, [2] D.Sivakumar
[1]Associate Professor, [2] Professor
[1]Department of Instrumentation and Control Engineering
[1]A.V.C College of Engineering, Mannampandal, Mayiladuthurai, 609305 Tamil Nadu. India.

Abstract : ***Process identification of Multi-Input Multi-Output (MIMO) process ranks as one of the important tasks in PI controller design. The closed loop behaviour of MIMO process decisively depends upon the identified process model. Nevertheless, the process identification of MIMO process is a demanding task, since there exists interaction, nonlinearities and process noise. However the controller design is given significance rather than obtaining a mathematical model of the process. In this paper, an attempt has been made to design a controller not based on process model, but simply from three areas of the process open loop step response. The proposed tuning method stands out to be pretty simple, with the simulation results depicting comparable or even better closed loop responses than the simple and comprehensive BLT (Biggest Log modulus Tuning) method.***

IndexTerms - Multi-Input Multi-Output, interaction, open loop step response, tuning, closed loop response.

I. INTRODUCTION

MIMO processes are quite frequent in chemical, power plants and process industries. The control of MIMO processes is much more complicated than SISO processes due to the interaction between the variables. The controller design method adopted for SISO process is not applied to MIMO process. Two control schemes are typically available for MIMO processes. The first is decentralized control scheme or multiloop control scheme, where single loop controllers are used. The second scheme is a full multivariable controller or centralized controller, where the controller is not a diagonal one [1]. If multivariable process exhibits stronger interaction between process inputs and outputs, multivariable controllers ought to be applied in order to achieve satisfactory performance. On the other hand if the interaction between the variables is negligible, multiloop controllers are preferred. Even though the multivariable control scheme gives closed loop response, it possesses two important drawbacks. First one is the cross coupling process variables that makes it difficult to design each loop independently, where the adjustment of controller parameter of one loop affecting the other loop. Second one for ‘n’ input and ‘m’ output MIMO systems 2 ‘n’ X ‘m’ parameters should be tuned for a full matrix PI controller. Due to these drawbacks and if the interaction between the variables are modest, multiloop controller is adequate to control a MIMO system.

Over the preceding years different methods have been proposed by authors for designing multiloop controllers for MIMO systems [2-4]. The Biggest Log modulus Tuning (BLT) method is one of the simplest methods to design a multiloop control system [5]. The generalised Ziegler-Nichol’s method [6], feed forward method [7], decentralised auto-tuner method [8], ISTE optimisation method [9] are other tuning methods generally adopted for a decentralised MIMO system. Wang et al [10] has proposed a controller design for MIMO system based on relay excitation auto-tuning method. A time domain approach of multivariable control scheme, based on multiple integrations of process step response has been proposed by Vrancic et al. [11]. Wang et al. [12] in their study have presented an approach for tuning MIMO systems by making use of decoupling controllers where the controller tuning was based on FOPDT (First Order Plus Dead Time) and SOPDT (Second Order Plus Dead Time)models derived from the process open loop step response. The paper aims to demonstrate the effective design of decentralised controller by adopting a multiple integration method for the coupled tank process.

II. MAGNITUDE OPTIMUM MULTIPLE INTEGRATION

In general the controller tuning methods can be divided primarily in to direct and indirect types [13, 14]. The direct tuning methods are not in need of a process model, whereas the indirect methods need the identified model of the process to calculate the controller parameters. In the direct method, the controller tuning is usually designed in a closed loop manner and the tuning method is relatively tedious and also needs initial controller parameters. The indirect tuning methods are usually based on process model obtained from step response and frequency response experiments. Consequently, the quality of the calculated controller parameters depends on the quality of the identified process model.

One of the most efficient indirect tuning methods which optimise the closed loop tracking performance is magnitude optimum (MO) method [15] also called as Betrags optimum method [16]. The MO method results in a very good closed loop response for a large class of process models frequently encountered in the process and chemical industries. However, the method is very demanding since it requires reliable estimation of quite a large number of process parameters even when using relatively simple controller structures, the main reason why the method is not commonly used in practice. In recent times, the efficiency of the MO method has been improved by using acquired no-parametric process data in the time domain instead of using explicit parametric identification of the process. The proposed approach has resulted in a relatively simple experiment in the time-domain while retaining all the properties of MO method. Since, the method is based on a multiple integration of the process time response, it is called as “magnitude optimum multiple integration” (MOMI) method.

The ultimate intend of the control system design is to track the reference effectively. In other words, the closed loop system should have an infinite bandwidth and zero phase shift. However, this is impossible in practice as every system features some time delay and/or dynamics while the controller's gain is limited due to physical limitations. Since a system's dynamics cannot be ignored, a new design objective is needed. One possible design aim is to maintain the closed-loop magnitude response curve as flat and as close to unity for as large a bandwidth as possible for a given plant and controller structure [17]. Therefore, the idea is to find a controller that makes the frequency response from set-point to plant output as close to unity as possible for low frequencies, the technique known by magnitude optimum (MO) [17], modulus optimum [18], or Betrags optimum, which results in a fast and non-oscillatory closed-loop time response for a large class of process models [19].

If $G_{CL}(s)$ is the closed-loop transfer function from the set-point to the process output, the controller is determined in such a way that $G_{CL}(0)=1$,

$$\left.\frac{d^r|G_{CL}(j\omega)|}{d\omega^r}\right|_{\omega=0} = 0, r = 1,2, \dots \dots r_{max} \quad (1)$$

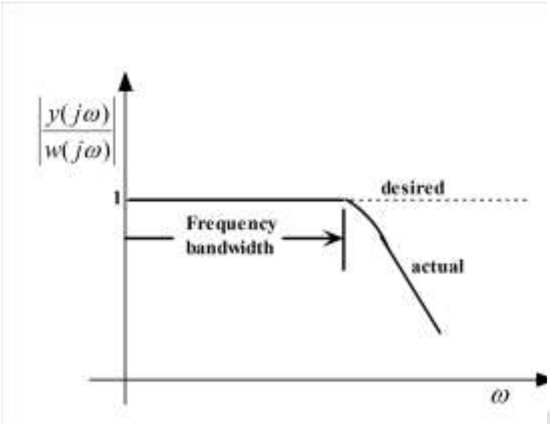

Fig. 1 Magnitude Optimum criterion

Let us assume the actual process is described by the following transfer function

$$G_P(s) = K_p \frac{1+b_1 s+b_2 s^2+\dots+b_m s^m}{1+a_1 s+a_2 s^2+\dots+a_n s^n} \quad (2)$$

where K_p is the process steady state gain, and a_1 to a_n and b_1 to b_n are the process parameters of the transfer function. In equation (2), 'm' must be less than or equal to 'n' and 'n' should be an arbitrary positive integer. The identification of the process parameters from the measurement is proved to be really difficult which can be overcome by using the concept of multiple integration. From the open loop response of the process y(t) , for the step change u in the process input at t=0, the following areas can be expressed by the integration;

$$A_1 = Y_1(\infty)$$
.
. (3)
.
$$A_K = Y_K(\infty)$$

where

$$y_0(t) = \frac{y(t)-y(0)}{\Delta u}$$

$$y_1(t) = \int_0^t [y_0(\infty) - y_0(\tau)]d\tau \quad (4)$$
.
.
.
$$y_k(t) = \int_0^t [A_{k-1} - y_{k-1}(\tau)]d\tau \quad (5)$$

The Symbol Δu in (4) represents the magnitude of the step change in the input. The graphic representations of the first two areas are shown in Fig. 2 and 3.

The controller structure is chosen to be of the PI type, described by the following transfer function:

$$G_c(s) = K\left(1 + \frac{1}{sT_i}\right) \quad (6)$$

The Fig. 4 depicts the closed loop structure with PI controller and the controller parameters can be obtained using MOMI method.

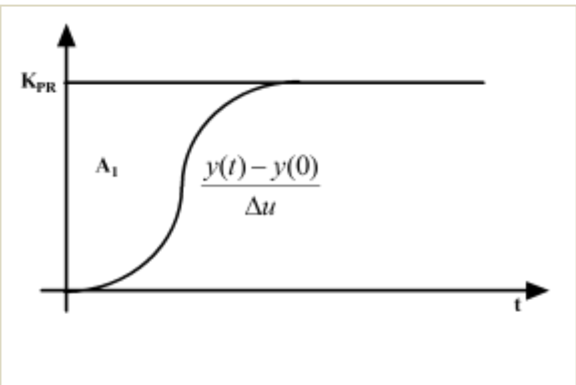

Fig.2 Graphic representation of area A_1

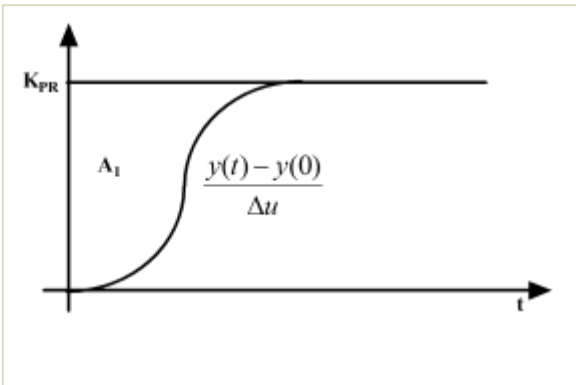

Fig.3 Graphic representation of area A_2

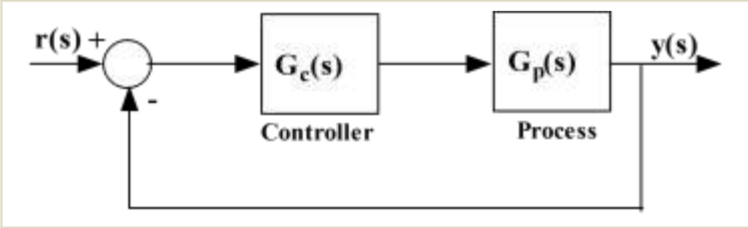

Fig. 4 Closed loop control structure

To achieve the demanding frequency criterion, the PI controllers can be calculated in the following way.

$$K = \frac{A_3}{2(A_1A_2 - A_0A_3)} \quad (7)$$

$$K_I = \frac{A_2}{2(A_1A_2 - A_0A_3)} \quad (8)$$

The proposed tuning method is implemented for a two tank interacting process.

III. TWO TANK INTERACTING SYSTEM

The implementation of MOMI method is very simple and straightforward where the process open loop step response is alone to be measured and some integration to be done in order to calculate the areas A_1 to A_3 to find the PI controller parameters. Two tank interacting system consists of two identical cylindrical tanks connected together by interconnection pipe as shown in Fig. 5. The flow between the tanks is adjusted by variable restriction and the flow through the pipe is proportional to the height of water. Water is pumped from reservoir into the tanks by variable speed pumps, which are driven by electric motor.

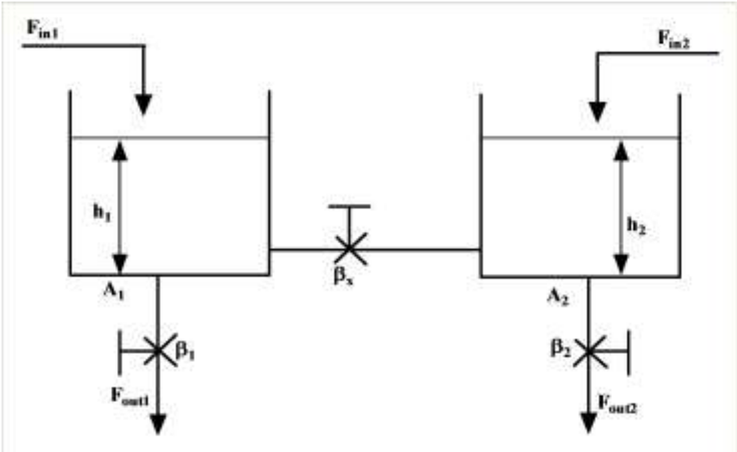

Fig. 5 Schematic of two tank interacting system

3.1 MODELLING

Consider the basic coupled tank apparatus as in Fig.5. Taking flow balances about each tank may derive the dynamical equation of the system. For the first tank,

F_{in1}-Q_1-F_{out1} = rate of change of fluid volume in tank1 = dv_1/dt=Adh_1/dt
F_{in1} = flow rate of fluid to tank1=$K_{pp1}u_1$
Q_1= flow rate of fluid from tank1 to tank2
h_1=height of fluid in tank1
A_1=cross sectional area of tank1.
For the second tank,
F_{in2}+Q_1-F_{out2} = rate of change of fluid volume in tank2 = dv_2/dt=A_2dh_2/dt
where v_2= volume of the fluid in tank2
h_2=height of fluid in tank2
F_{out2}=flow rate of fluid out of tank2

If the inter tank restriction and the drain from tank2 is assumed to behave like orifices, then the following equations follow from the characteristic relations for the orifices:

$Q_1=a_{12}\beta_x \text{ sqrt}(2g(h_1-h_2))$
$F_{in2}=K_{pp2}u_2$
$F_{out2}= a_2\ \beta_2\text{sqrt}(2gh_2)$

where g=gravitational constant,
a_{12}=cross sectional area of inter connection pipe

a_2=cross sectional area of pipe from tank2

From the system dynamic equations it is clear that the system is nonlinear. The controlled variables are the level in tanks and the manipulated variable is flow to the tanks. Using equations, the mass balance equation of two tank interacting process is,

$$\frac{dh_1}{dt} = \frac{k_{pp1}u_1}{A_1} - a_1\frac{\beta_1}{A_1}\sqrt{2gh_1} - a_{12}\frac{\beta_x}{A_1}\sqrt{2g(h_1 - h_2)}$$

$$\frac{dh_2}{dt} = \frac{k_{pp2}u_2}{A_2} - a_2\frac{\beta_2}{A_2}\sqrt{2gh_2} + a_{12}\frac{\beta_x}{A_2}\sqrt{2g(h_1 - h_2)}$$

The parameters of the process and its operating point are given in Table 1 and 2.

Table 1. Parameters of a Laboratory Coupled Tank MIMO Process

A_1, A_2	a_2, a_{12}	β_1	β_2	β_x
154	0.5	0.7498	0.8040	0.2245

Table 2. Operating conditions of a Laboratory Coupled Tank MIMO Process

u_1	u_2	h_1	h_2	k_{pp1}	k_{pp2}
2.5	2.0	18.32	12.23	33.336	25.002

3.2 CONTROLLER DESIGN

The control objective of a two tank system is to maintain the level in the tanks which can be achieved by manipulating the inflow to the tanks. According to the tuning procedure initially the step change ΔU at the first process input is applied and the corresponding process outputs are measured. The responses of sub-processes g_{11} and g_{21} can be thus obtained. The procedure is yet again repeated for the second input and the responses of sub-processes g_{12} and g_{22} are obtained. The process gain K_{PRij} (A_{0ij}) and the three areas (A_{1ij}, A_{2ij}, and A_{3ij}) are obtained by numerical integration, according to expression (3). The parameters of the PI controller g_{c1} and g_{c2} are calculated from areas of sub-processes g_{11} and g_{22}, respectively, by the expressions (7) and (8). The open loop responses of a two tank system are shown in Fig.6 and Fig.7 for a change in input of 0.5 from its nominal steady state value.

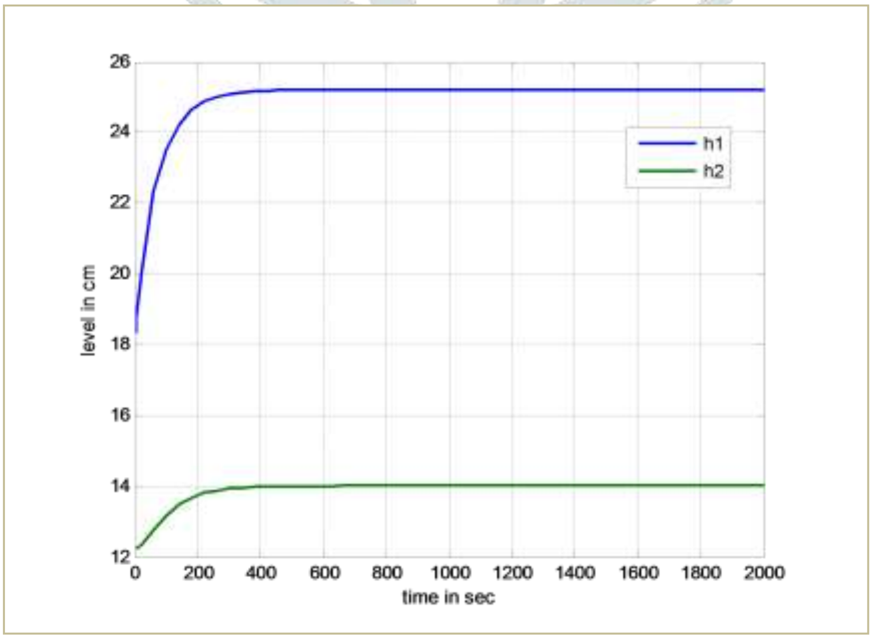

Fig. 6 Open loop response for change in inflow1

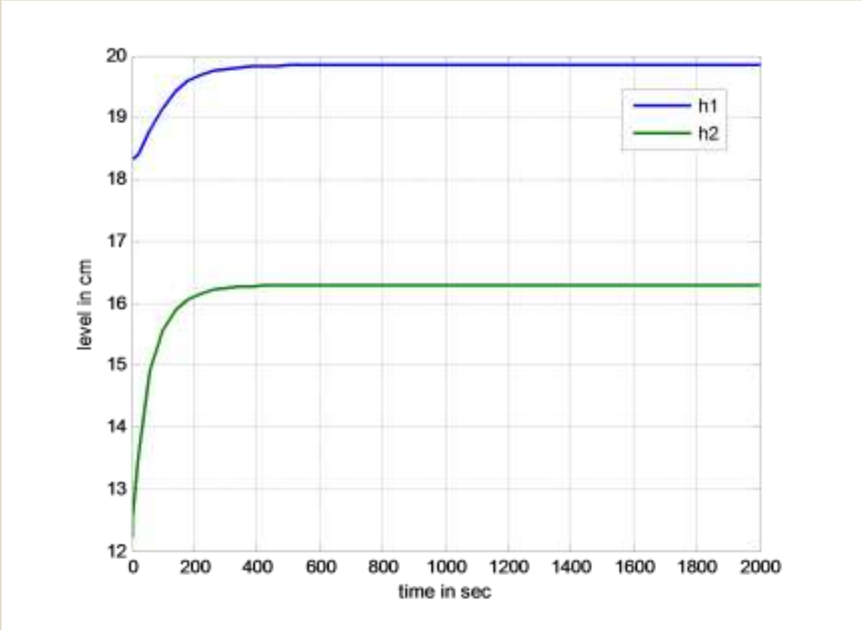

Fig .7 Open loop response for change in inflow2

From the open loop responses, the process steady state gains are measured as: K_{pr11}=$A0_{11}$=12.8, K_{pr12}=$A0_{12}$=3.0, K_{pr21}=$A0_{21}$=4.0, K_{pr22}=$A0_{22}$=6.0. The areas A_1 to A_3 are then calculated from the step responses obtained using numerical integration method and are tabulated in Table 3.

Table 3 Areas for two tank sub-processes

Sub processes	A_0	A_1	A_2	A_3
g_{11}	16.99	3864	8.284e5	1.77e8
g_{12}	6.691	1860	3.986e5	7.89e7
g_{21}	9.231	2698	7.048e5	2.107e8
g_{22}	11.38	2216	3.809e5	7.992e7

The PI controller parameters are calculated by using the areas from process g_{11} and g_{12} respectively as,

K_{c1}= 0.4569 K_{i1}=0.0021

K_{c2} =0.2259 K_{i2}=0.0015

The closed loop responses for both the cases are shown in Fig.8 and Fig.9.

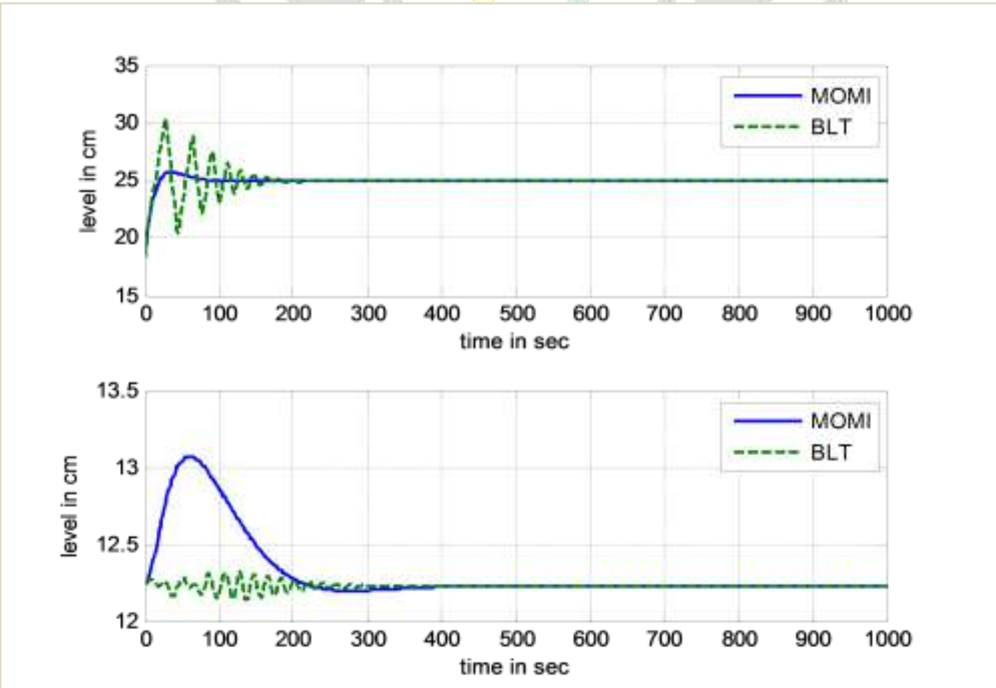

Fig.8 Closed loop response for set point change in level1

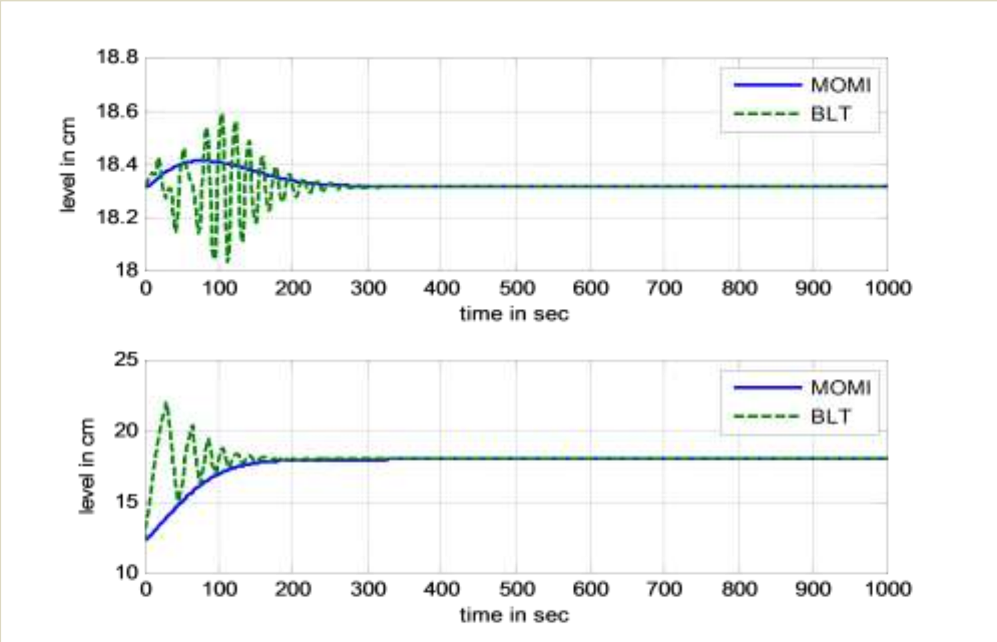

Fig.9 Closed loop response for set point change in level2

IV. RESULTS AND DISCUSSION

The magnitude optimum multiple integration technique has been applied to the two tank process. From the open loop step response of the process, three areas are calculated using numerical integration method and the decentralised controller is designed. The simulation is carried out using MATLAB toolbox and the results have shown that a setpoint change in the loop1 gives better performance compared with BLT whereas the interaction performance is better in BLT. In case of setpoint change in loop2 the ISE and IAE values are better in BLT compared with MOMI method and the interaction effect is less in MOMI. In MOMI method the decentralised controller is designed with ease without even knowing the process model.

References

[1] Lieslehto, J. (1996). MIMO controller design using SISO controller design methods. *IFAC Proceedings Volumes*, *29*(1), 1152-1156.
[2] Åström, K. J., & Hägglund, T. (1995). PID controllers: theory, design, and tuning. *Instrument Society of America, Research Triangle Park, NC*, *10*.
[3] Koivo, H. N., & Tanttu, J. T. (1991). Tuning of PID Conrollers: Survey of Siso and Mimo Techniques. IFAC Proceedings Volumes, 24(1), 75-80.
[4] Zhuang, M., & Atherton, D. P. (1994). PID controller design for a TITO system. *IEE Proceedings-Control theory and applications*,*141*(2), 111-120.
[5] Luyben, W. L. (1986). Simple method for tuning SISO controllers in multivariable systems. *Industrial & Engineering Chemistry Process Design and Development*, *25*(3), 654-660.
[**6**] Niederlinski, A. (1971). A heuristic approach to the design of linear multivariable interacting control systems, *Automatica 7*(6), 691-701.
[**7**] Gawthrop, P. J. (1987). Continuous-time self-tuning control-a unified approach. *IFAC Proceedings Volumes*, *20*(2), 19-24.
[**8**] Wang, Q. G., Zou, B., Lee, T. H., & Bi, Q. (1997). Auto-tuning of multivariable PID controllers from decentralized relay feedback. *Automatica*, *33*(3), 319-330.
[**9**] Majhi, S. (2001). SISO controller for TITO systems. In *Int., Conf., on Energy, Automation and Information Tech*.
[**10**] Wang, Q. G., Zou, B., Lee, T. H., & Bi, Q. (1997). Auto-tuning of multivariable PID controllers from decentralized relay feedback. *Automatica*, *33*(3), 319-330.
[**11**] Vrančić, D., Lieslehto, J., & Strmčnik, S. (2001). Designing a MIMO PI controller using the multiple integration approach. *Process Control and Quality*, *11*(6), 455-468.
[**12**] Wang, Q. G., Huang, B., & Guo, X. (2000). Auto-tuning of TITO decoupling controllers from step tests. *ISA transactions*, *39*(4), 407-418.
[**13**] Åström, K. J., Hägglund, T., Hang, C. C., & Ho, W. K. (1993). Automatic tuning and adaptation for PID controllers-a survey. *Control Engineering Practice*, *1*(4), 699-714.
[**14**] Gorez, R. (1997). A survey of PID auto-tuning methods: Feature issue controller tuning. *Journal A*, *38*(1), 3-10.
[**15**] Hanus, R. (1975). Determination of controllers parameters in the frequency domain. *Journal A*, *16*(3), 128.
[**16**] Kessler, C. (1955). Über die Vorausberechnung optimal abgestimmter Regelkreise Teil III. Die optimale Einstellung des Reglers nach dem Betragsoptimum. *Regelungstechnik*, *3*, 40-49.
[**17**] Umland, J. W., & Safiuddin, M. (1990). Magnitude and symmetric optimum criterion for the design of linear control systems: What is it and how does it compare with the others?. *IEEE Transactions on Industry Applications*, *26*(3), 489-497.
[**18**] Åström, K. J., & Hägglund, T. (1995). *PID controllers: theory, design, and tuning* (Vol. 2). Research Triangle Park, NC: Isa.
[**19**] Vrančić, D., Peng, Y., & Strmčnik, S. (1999). A new PID controller tuning method based on multiple integrations. *Control Engineering Practice*, *7*(5), 623-633.
[**20**] Senthilkumar, M., & Lincon, S. A. (2015). Multiloop PI Controller for achieving simultaneous Time and Frequency Domain Specifications. *Journal of Engineering Science and Technology*, *10*(8), 1103-1115.