A New Class of Multifunctions Defined on Cozero Sets

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Abstract

The notion of *z*-perfectly continuous function is extended to the framework of multifunction. Basic properties of upper and lower *z*- perfectly multifunctions are studied. Examples are included to reflect upon the distinctiveness of upper (lower) *z*-perfectly continuous multifunction from that of other variants of continuity of multifunctions in the mathematical literature.

Keywords: strongly continuous multifunctions, upper (lower) perfectly continuous multifunctions, upper (lower) *z*-supercontinuous multifunctions, upper (lower) *z*-continuous multifunctions, upper (lower) semicontinuous multifunctions

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1 Introduction

The concept of perfectly continuous function is introduced by Noiri [22]. Properties of perfectly continuous functions are elaborated in [14]. The class of perfectly continuous function properly contains the class of strongly continuous function of Levine [20] and is strictly contained in the class of cl-supercontinuous (≡clopen continuous) functions introduced by Reillyand Vamanamurthy [24]. Taking account of [15], we can mention some remarkable classes of function in descending order, each class properly contains the next class *z*-perfectly continuous function [17], pseudo perfectly continuous function [15], quasi-perfectly continuous functions [18], δ-perfectly continuous [13] strongly continuous functions of Levine [20].

In this paper we extend the notions and results of variants of continuity of functions to the realm of multifunctions (see for example [2], [3], [4], [6], [7], [8], [9], [10], [11], [19], [21], [22], [23], [24], [25], [27]).

In the present paper we extend the notion of *z*-perfectly continuous functions [17] to the realm of multifunctions and introduce the notion of upper and lower *z*perfectly continuous multifunctions and elaborate upon their place in the hierarchy of variants of continuity of multifunctions that already exist in the literature. It turns out the class of upper (lower) *z*-perfectly continuous multifunction properly contains the class of upper (lower) quasi-perfectly continuous multifunctions [18] and so includes all upper (lower) perfectly continuous multifunctions [9].

Section 2 is devoted to the preliminaries and basic definitions. In section 3 we define the notion of upper and lower *z*-perfectly continuous multifunctions. In Section 4 we discuss characterization and study basic properties of upper *z*-perfectly continuous multifunctions. It turns out that upper *z*-perfectly continuity of multifunctions is preserved under the composition and expansion of range. In section 5, we study lower *z*-perfectly continuous multifunctions and give their characteristics. It is shown that lower z-perfectly continuity of multifunction is preserved under the composition and expansion of range.

2 Preliminaries and Basic Definitions

Throughout the paper we essentially follow that notations and terminology of L. Gorniewicz [5]. Let X and Y be nonempty sets. Then $\phi : X \to Y$ is called a multifunction from X into Y if for each $x \in X$, $\phi(x)$ is a nonempty subset of Y. Let B be a subset of Y. Then the set $\phi_+^{-1}(B) = \{x \in X : \phi(x) \cap B \neq \varphi\}$ is called large inverse image [5] [1] of B and the set $\phi_-^{-1}(B) = \{x \in X : \phi(x) \subset B\}$ is called small inverse image of B. The set $\Gamma_\phi = \{(x, y) \in X \times Y | y \in \phi(x)\}$ is called the graph of the multifunction. Let A be subset of X. Then $\phi(A) = \bigcup\{\phi(x) : x \in A\}$ is called image of A. A multifunction $\phi : X \to Y$ is upper semicontinuous (respectively lower semicontinuous) if $\phi_-^{-1}(U)$ (respectively $\phi_+^{-1}(U)$) is an open set in X for every open set U in Y. A subset U of a topological space X is called a cl-open set if it can be expressed as the union of clopen sets. The complement of a cl-open set will be referred to as a cl-closed. A subset A of a space X is called regular open if it is the interior of its closure i.e. $A = \bar{A}^\circ$. A collection β of subsets of a space X is called an open complementary system [16] if β consists of open sets such that for each $B \in \beta$, there exist $B_1, B_2, \ldots \in \beta$ with $B = \bigcup\{X \setminus B_i : i \in Z^+\}$. A subset U of a space

X is called strongly open F_σ-set [5] if there exists a countable open complementary system $\beta(U)$ with $U \in \beta(U)$. A subset H of a space X is called a regular G_δ-set [21] if H is the intersection of a sequence of closed sets whose interiors contain H, i.e if

$H = \bigcap_{i=1}^{\infty} F_i = \bigcap_{i=1}^{\infty} F_i^\circ$ where each F_i is a closed subset of X. The complement of a regular G_δ set is called a regular F_σ-set. Let X be a topological space and let $A \subset X$. A point $x \in X$ is called a ϑ-adherent point [26] of A if every closed neighbourhood of x intersects A. Let $cl_\vartheta A$ denote the set of all ϑ-adherent point of A. The set A is called ϑ-closed if $A = cl_\vartheta A$. The complement of a ϑ-closed set is referred to as a ϑ-open set. A point $x \in X$ is said to be a cl-adherent point of A if every clopen set containing x intersects A. Let $[A]_{cl}$ denote the set of all cl-adherent points of A. Then a set A is cl-closed if and only if $A = [A]_{cl}$. A subset of A of a space X is said to be cl-closed if it is the intersection of clopen sets. The complement of a cl-closed set is referred to as a cl-open set. Any union of cozeros sets is called z-open [12]. The complement of z-open set is referred to as z-closed set.

Definition 2.1 ([12]). A multifunction $\phi : X \to Y$ from a topological space X into a topological space Y is said to be

(1) strongly continuous if $\phi_-^{-1}(B)$ is clopen in X for every subset $B \subset Y$.

(2) upper perfectly continuous if $\phi_-^{-1}(V)$ is clopen in X for every open set $V \subset Y$.

[1] However, what we call "large inverse large $\phi_+^{-1}(B)$" some authors call it 'lower inverse' and denoteand employit bytheϕ^-(notation$B$) and similarly$\phi^+(B)$ fortheythecallsame. "small inverse image $\phi_-^{-1}(B)$" as 'upper inverse image'

(3) lower perfectly continuous if $\phi^{-1}(V)$ is clopen in X for every open set $V \subset Y$.

(4) upper completely continuous if $\phi_{-}^{+\,-1}(V)$ is regular open in X for every open set $V \subset Y$.

(5) lower completely continuous if $\phi_{+}^{-1}(V)$ is regular open in X for every open set $V \subset Y$.

Definition 2.2. A multifunction $\phi : X ¨ Y$ from a topological space X into a topological space Y is said to be

(1) upper z-supercontinuous [3] if for each $x \in X$ and each open set V containing $\phi(x)$, there exists a cozero set U containing x such that $\phi(U) \subset V$.

(2) lower z-supercontinuous [3] if for each $x \in X$ and each open set V with $\phi(x) \cap V = 6\ \varphi$, there exists a cozero set U containing x such that $\phi(z) \cap V\ 6 = \varphi$ for each $z \in U$.

(3) upper D_δ-supercontinuous [11] if for each $x \in X$ and each open set V containing $\phi(x)$, there exists a regular F_σ-set U containing x such that $\phi(U) \subset V$.

(4) lower D_δ-supercontinuous [11] if for each $x \in X$ and each open set V with $\phi(x) \cap V = 6\ \varphi$, there exists a regular F_σ-set U containing x such that $\phi(z) \cap V = 6\ \varphi$ for each $z \in U$.

(5) upper strongly ϑ-continuous [19] if for each $x \in X$ and each open set V containing $\phi(x)$, there exists a ϑ-open set U containing x such that $\phi(U) \subset V$.

(6) lower strongly ϑ-continuous [19] if for each $x \in X$ and each open set V with $\phi(x) \cap V = 6\ \varphi$, there exists a ϑ-open set U containing x such that $\phi(z) \cap U\ 6 = \varphi$ for each $z \in U$.

Definition 2.3. A multifunction $\phi : X ¨ Y$ from a topological space X into a topological space Y is said to be

(a) upper (lower) perfectly continuous (respectively almost perfectly continuous, respectively quasi perfectly continuous, respectively δ-perfectly continuous) [9] if $\phi_{-}^{-1}(U)(\phi_{+}^{-1}(U))$ is clopen in X for every open (respectively regular open, respectively ϑ-open, respectively δ-open) subset U of Y.

(b) upper quasi z-supercontinuous ([11]) if for each $x \in X$ and each ϑ-open set V containing $\phi(x)$, there exists a cozero set U containing x such that $\phi(U) \subset V$.

Definition 2.4. A multifunction $\phi : X ¨ Y$ from a topological space X into a topological space Y is said to be

(a) upper supercontinuoue (δ-continuous) [1] if for each $x \in X$ and each open (regular open) set V containing $\phi(x)$, there exists a regular open set U containing x such that $\phi(U) \subset V$.

(b) lower supercontinuous (δ-continuous) [1], if for each $x \in X$ and each open (regular open) set V with $\phi(x) \cap V \neq \varphi$, there exists a regular open set U containing x such that $\phi(z) \cap V \neq \varphi$, for each $z \in U$.

Definition 2.5. A multifunction $\phi : X \to Y$ from a topological space X into a topological space Y is said to be

(i) upper supercontinuous (δ-continuous) [2] if for each $x \in X$ and each open (regular open) set V containing $\phi(x)$, there exists a regular open set U containing x such that $\phi(U) \subset V$.

(ii) lower supercontinuous (δ-continuous) [2] if for each $x \in X$ and each open (regular open) set V with $\phi(x) \cap V \neq \varphi$, there exists a regular open set U containing x such that $\phi(z) \cap V \neq \varphi$ for each $z \in U$.

(iii) upper cl-supercontinuous [8] at $x \in X$ if for each open set V with $\phi(x) \subset V$, there exists a clopen set U containing x such that $\phi(U) \subset V$.

(iv) lower cl-supercontinuous [8] at $x \in X$ if for each open set V with $\phi(x) \cap V \neq \varphi$, there exists a clopen set U containing x such that $\phi(z) \cap V \neq \varphi$ for each $z \in U$.

3 Upper and lower *z*-perfectly continuous

Definition 3.1. A multifunction $\varphi : X \to Y$ from a topological space X into a topological space Y is

(a) upper z-perfectly continuous at $x \in X$ if for each cozero set V with $\phi(x) \subset V$, $\exists$ a clopen set U in X containing x such that $\phi(U) \subset V$.

The multifunction is said to be upper z-perfectly continuous if it is upper zperfectly continuous at each $x \in X$.

(b) lower z-perfectly continuous at $x \in X$ if for each cozero set V with $\phi(x) \cap V \neq \varphi$, there exists a clopen set U in X containing x such that $\phi(x) \cap V \neq \varphi$ for each $x \in U$.

The multifunction is said to be lower z-perfectly continuous if it is lower zperfectly continuous at each $x \in X$.

Examples

Example 3.1. Let $X = \{a, b, c\}$ with the topology as $\tau_X = \{\phi, X, \{a\}, \{b, c\}\}$. Let $Y = \{x, y\}$ with the topology $\tau_Y = \{\varphi, Y, \{y\}\}$. Define a multifunction $\varphi : (X,\tau_X) \to (Y,\tau_Y)$ by $\phi(a) = \{y\}$, $\phi(b) = \{x, y\}$, $\phi(c) = \{x\}$. Then the

multifunction is upper perfectly continuous and hence upper z-perfectly continuous but not lower perfectly continuous and lower z-perfectly continuous.

Example 3.2. Let *X* ={*a*, *b*, *c*} with the topology τ_x ={φ, *X*,{*a*},{*c*},{*a*, *c*},{*a*, *b*}} and let *Y* ={*x*, *y*} with the topology τ_Y ={φ, *Y*,{*y*}}.

Define a multifunction $\phi : (X,\tau_X) \to (Y,\tau_Y)$ by $\phi(a) = \{y\}$, $\phi(b) = \{x, y\}$, $\phi(c) = \{y\}$. Then clearly ϕ is lower perfectly continuous and lower *z*- perfectly perfectly continuous. But ϕ is not upper perfectly continuous and upper *z*-perfectly continuous as $\{y\}\subset Y$ is cozero set in *Y* but $\phi_-^{-1}(\{y\})=\{a, c\}$ is not clopen so ϕ

is not upper *z*-perfectly continuous.

4 Properties of upper *z*-perfectly continuous multifunctions

Theorem 4.1. *Let* $\phi : X \to Y$ *be a multifunction from a topological space X into a topological space Y. Then the following statements are equivalent*

(a) *ϕ is upper z-perfectly continuous*

(b) *$\phi_-^{-1}(B)$ is a cl-open set in X for every z-open set B in Y.*

(c) *$\phi_+^{-1}(B)$ is a cl-closed set in X for every z-closed set B in Y.*

Proof. (a) ⇒(b): Let *B* be a *z*-open subset of *Y* .

To show that $\phi_-^{-1}(B)$ is cl-open set in *X*.

Let $x \in \phi_-^{-1}(B)$. Then $\phi(x)\subset B$. Since *B* is a *z*-open subset of *Y* , so $B = \bigcup_{\alpha\in\Lambda} U_\alpha$, each U_α is a cozero set.

Therefore $\phi(x)\in U_\alpha$ for some $\alpha\in\Lambda$. Since ϕ is upper *z*-perfectly continuous, therefore ∃ a clopen set H_α containing *x* such that $\phi(H_\alpha)\subset U_\alpha$. Now

$$x\in H_\alpha \subset \varphi_-^{-1}(U_\alpha) \subset \bigcup_{\alpha\in\Lambda} \varphi_-^{-1}(U_\alpha) \subset \varphi_-^{-1}\left(\bigcup_{\alpha\in\Lambda} U_\alpha\right) = \varphi_-^{-1}(B).$$

Since H_α is clopen, so H_α is cl-open.

Hence, $x \in H_\alpha \subset \phi_-^{-1}(B)$ and so $\phi_-^{-1}(B)$ is a cl-open set in *X*.

(b) ⇒(c): Let *B* be a *z*-closed subset of *Y* . Then $Y \setminus B$ is a *z*-open subset of *Y*

By (b) $\phi_-^{-1}(Y \setminus B)$ is cl-open set in *X*.

Since $\phi_-^{-1}(Y \setminus B)= X \setminus \phi_+^{-1}(B)$, so $\phi_+^{-1}(B)$ is a cl-closed set in *X*. □

Theorem 4.2. *If a multifunction $\phi : X \to Y$ is upper z-perfectly continuous and $\phi(X)$ is endowed with the subspace topology then the multifunction $\phi : X \to \phi(X)$ is upper z-perfectly continuous.*

Proof. Since ϕ is upper z-perfectly continuous, so for every cozero set V of Y, $\phi^{-1}(V \cap \phi(X)) = \phi_-^{-1}(V) \cap \phi_-^{-1}(\phi(X)) = \phi_-^{-1}(V) \cap X = \phi_-^{-1}(V)$ is cl-open and hence $\phi : X \to \phi(X)$ is upper z-perfectly continuous.

□

Theorem 4.3. *If $\phi : X \to Y$ is upper z-perfectly continuous and $\psi : Y \to Z$ is upper z-continuous then $\psi \circ \phi$ is upper z-perfectly continuous.*

Proof. Let W be a cozero set in Z.

Since ψ is upper z-continuous, $\psi_-^{-1}(W)$ is a z-open set in Y.

Therefore $\psi_-^{-1}(W) = \bigcup_{\alpha \in \Lambda} W_\alpha$, where each W_α is a cozero set in Y. Again since ϕ is upper z-perfectly continuous, so each $\phi_-^{-1}(W_\alpha)$ is clopen

$$(\psi \circ \varphi)_-^{-1}(W) = \varphi_-^{-1}(\psi_-^{-1}(W)) = \varphi_-^{-1}\left(\bigcup_{\alpha \in \Lambda} W_\alpha\right) = \bigcup_{\alpha \in \Lambda} \varphi_-^{-1}(W_\alpha)$$

which is clopen in X.

Therefore $\psi \circ \phi : X \to Z$ is upper z-perfectly continuous. □

Theorem 4.4. *Composition of two upper z-perfectly continuous function is upper zperfectly continuous.*

Proof. The result follows as every upper z-perfectly continuous multifunction is upper z- continuous. □

Theorem 4.5. *If $\phi : X \to Y$ is upper z-perfectly continuous and $\psi : Y \to Z$ is upper semi-continuous then $\psi \circ \phi : X \to Z$ is upper z-perfectly continuous multifunction.*

Proof. Since every upper semi-continuous function is upper z-perfectly continuous so $\psi \circ \varphi$ is upper z-perfectly continuous. □

Theorem 4.6. *Let $\phi : X \to Y$ be upper slightly continuous and $\psi : Y \to Z$ be upper z-perfectly continuous then the multifunction $\psi \circ \phi : Y \to Z$ is upper z-perfectly continuous.*

Proof. The result follows by Theorem 4.3 as every upper slightly continuous function is upper z- continuous. □

Corollary 4.7. *Let $\phi : X ¨ Y$ be upper z-perfectly continuous and if Z is a space containing Y as a subspace then $\psi : X ¨ Z$ defined by $\psi(x)=\phi(x)$ for each $x \in X$*

is upper z-perfectly continuous.

Proof. Let W be a co-zero set in Z then $W \cap Y$ is a cozero set in Y . Since $\phi : X \to Y$ is upper z-perfectly continuous, So $\phi^{-1}(W \cap Y)$ is clopen set in X.

$_{-}{}^{1} W)=\{x \in X : \psi(x) \subset W\}=\{x \in X : \psi(x_{-}) \subset W \cap Y\}$. Now $\psi^{-}($

Thus $\psi : X \to Z$ is upper z-perfectly continuous. □

5 Properties of lower *z*-perfectly continuous multifunction

Theorem 5.1. *Let $\phi : X \to Y$ be a multifunction from a topological space X into a topological space Y. Then the following statements are equivalent:*

(a) *ϕ is lower z-perfectly continuous*

(b) *$\phi_{+}{}^{-1}(B)$ is a cl-open set in X for every z-open set in Y.*

(c) *$\phi_{-}{}^{-1}(B)$ is a cl-closed set in X for every z-closed set B in Y*

Proof. (a)⇒(b): Let B be a z-open subset of Y . To show that $\phi_{+}{}^{-1}(B)$ is cl-open in X. Let $x \in \phi_{+}{}^{-1}(B)$. Then $\phi(x) \cap B =6 \varphi$. Since B is a z-open subset of Y , so $B = \bigcup_{\alpha \in \Lambda} U_\alpha$ whereeach U_α is a cozero set.

Therefore $\phi(x) \cap U_\alpha =6 \varphi$ for some $\alpha \in \Lambda$.

Since ϕ is lower z-perfectly continuous, therefore there exists a clopen set H containing x such that $\phi(h) \cap B =6 \varphi$ for each $h \in H$. Hence $x \in H \subset \phi_{+}{}^{-1}(B)$ and so $\phi^{-}_{+}(B)$ is a cl-open set in X being a union of clopen sets.

(b)⇒(c): Let B be a z-clsoed subset of Y . Then $Y \setminus B$ is a z-open subset of Y . By $(b)\phi_{-}{}^{-1}(Y \setminus B)$ is cl-open set in X. Since $\phi_{-}{}^{-1}(Y \setminus B) = X \setminus \phi_{+}{}^{-1}(B)$, so $\phi_{+}{}^{-1}(B)$ is a cl-closed set in X. □

Theorem 5.2. *If a multifunction $\phi : X ¨ Y$ is lower z-perfectly continuous and $\phi(X)$ is endowed with the subspace topology then the multifunction $\phi : X ¨ \phi(X)$ is lower z-perfectly continuous.*

Proof. Since ϕ is lower z-perfectly continuous, so for every cozero set V of Y , $\phi^{-1}(V \cap \phi(x)=\phi_{+}{}^{-1}(V) \cap \phi_{+}{}^{-1}(\phi(X))=\phi_{+}{}^{-1}(V) \cap X =\phi_{+}{}^{-1}(V)$ is cl-open. $_{+}$

Hence $\phi : X$ ¨$\phi(X)$ is lower z-perfectly continuous. □

Theorem 5.3. *If ϕ : X ¨ Y is lower z-perfectly continuous and ψ : Y ¨ Z is lower z-continuous, then $\psi\circ\phi$ is lower z-perfectly continuous. In particular, composition of two lower z-perfectly continuous multifunction is lower z-perfectly continuous.*

Proof. Let W be a cozero set in Z. Since ψ is lower z-continuous $\psi_+^{-1}(W)$ is a $z$open set in Y . Therefore $\psi^{-}{}_+^{1}(W)$ = S W_α, where each W_α is a cozero set in Y .

Again, since ϕ is lower z-perfectly continuous, $^{\alpha}\epsilon^{\Lambda}$ so each $\phi_+{}^{-1}(W_\alpha)$ is clopen.

Now $(\psi\circ\phi)^{-}{}_+^{1}(W)$=$\phi_+^{-}(\psi^{-}{}_+(W))$=$\phi_+^{-}$ S W_α $= \bigcup_{\alpha\in\Lambda} \varphi_+^{-}(W_\alpha)$ which is clopen in X. So $\psi\circ\phi$: X ¨ Z is lower z-perfectly continuous. $\alpha\epsilon\Lambda$ □

Corollary 5.4. *Composition of two lower z-perfectly continuous function is lower zperfectly continuous.*

Proof. The result follows as every lower z-perfectly continuous multifunction is lower z- perfectly. □

Corollary 5.5. *If ϕ : X ¨ Y is lower z-perfectly continuous and ψ : Y ¨ Z is lower semi-continuous then $\psi\circ\phi$: X ¨ Z is lower z-perfectly continuous multifunction.*

Proof. Since every lower semi-continuous multifunction is lower z- perfectly continuous, so $\psi\circ\phi$ is lower z-perfectly continuous. □

Theorem 5.6. *Let ϕ : X ¨ Y be lower slightly continuous and ψ : Y ¨ Z be lower z-perfectly continuous then the multifunction $\psi\circ\phi$: X ¨ Z is lower z-perfectly*

continuous.

Proof. The result follows by Theorem 5.3 as every lower slightly continuous function is lower z- continuous. □

The following corollary shows that lower z-perfectly continuity of a multifunction is preserved under the expansion of its range.

Corollary 5.7. *Let ϕ : X ¨ Y be lower z-perfectly continuous and if Z is a space containing Y as subspace then ψ : X ¨ Z defined by $\psi(x)$=$\phi(x)$ for each x ∈X is*

lower z-perfectly continuou.

Proof. Let W be a cozero set in Z then $W \cap Y$ is a cozero set in Y . Since ϕ : X ¨ Y is upper z-perfectly continuous, So $\phi^{-1}(W \cap Y)$ is clopen set in X. $_+$

Now $\psi^{-+1}(W)=\{x \in X : \psi(x) \in W\}=\{x \in X : \psi(x) \subset W \cap Y\}$.

Thus $\psi : X$ ¨ Z is upper z-perfectly continuous. □

References

[1] N. Ajmal and J.K. Kohli, Properties of hyperconnected spaces, their mappings into Hausdorff spaces and embeddings into hyperconnected spaces, Acta Math. Hungar. 60 (1-2) (1992), 41-49.

[2] M. Akdag,ˇ On supercontinuous multifunctions, Acta Math. Hungar 99 (1-2) (2003), 143-153.

[3] M. Akdag,ˇ On upper and lower *z*-supercontinuous multifunctions, Kyungpook Math. J. 45 (2005), 221-230.

[4] M. Akdag,ˇ On upper and lower D_δ-supercontinuous multifunctions, Miskolc Mathematical Notes 7 (2006), 3-11.

[5] L. Górniewicz, Topological Fixed Point Theory of Multivalued Mappings, Kluwer Academic Publishers, Dordrect, The Netherlands 1999.

[6] L'. Holda V. Balaz and Neubrunn, Remarks on c-upeer semicontinuous multifunctions, Acta Mat. Univ. Comenianae 50/51 (1987), 159-165.

[7] L'. Hola, Remarks on almost continuous multifunctions, Math. Slovaca 38 (1988), 325-331.

[8] J.K. Kohli and C.P. Arya, Upper and lower cl-supercontinuous multifunctions, Applied General Topology (to appear).

[9] J.K. Kohli and C.P. Arya, Strongly and perfectly continuous multifunctions, "Vasile Alescandri" University of Bacau,ˇ Faculty of Sciences, Scientific Studies and Research, Series Mathematics and Informatics, Vol. 20, No. 1 (2010) , 103-118.

[10] J.K. Kohli and C.P. Arya, Upper and lower almost cl-supercontinuous multi functions, Demonstratio Mathematica 44 (2) (2011), 407-421.

[11] J.K. Kohli and C.P. Arya, Generalizations of *z*-supercontinuous and $D_\delta$supercontinuous multifunctions, (preprint).

[12] J.K. Kohli and R. Kumar, *z*-supercontinuous functions, Indian J. Pure Appl. Math. 33 (7) (2002), 1097-1108.

[13] J.K. Kohli and D. Singh, δ-perfectly continuous functions, Demonstratio Mathematica 42 (1) (2009), 221-231.

[14] J.K. Kohli, D.Singh and C.P. Arya, Perfectly continuous functions, Studii Si Cerctari Ser. Mat. 18 (2008), 99-110.

[15] J.K. Kohli, D. Singh, J. Aggarwal and M.Rana, Pseudo perfectly continuous functions and closedness/compactness of their functions spaces, Applied General Topology. 14 (1) (2013), 115-134.

[16] N.C. Heldermann, Developability and some new regularity axiom, Canad. J. Math. 33(1981), 1981-643

[17] D. Singh and Manoj Kumar Rana, Z-perfectly continuous function and their function spaces, Scientific Studies and Research series Mathematics and Informatics 24 (2014), 135-152.

[18] D. Singh, J.K. Kohli and B.K. Tyagi, Quasi perfectly continuous function and their function Spaces, Scientific and Research Series Mathematics and Informatics 21 (4) (2011), 23-46.

[19] Y. Kucuk, On strongly ϑ-continuous multifunctions, Pure and Appl. Math. Sci. 40 (194), 43–54.

[20] N. Levine, Strong continuity in topological spaces, Amer. Math. Monthly, 67 (1960), 269.

[21] J. Mack, Countable paracompactness and weak normality properties, Trans Amer. Math. Soc. 148 (1970), 265-272.

[22] T. Noiri, On δ-continuous functions, J. Korean Math. Soc. 16 (1980), 161-166.

[23] T. Noiri, Supercontinuity and some strong forms of continuity, Indian J. Pure Appl. Math. 15 (3) (1984), 241-250.

[24] I.L. Reilly and M.K. Vamanamurthy, On supercontinuous mappings, Indian J. Pure Appl. Math 14 (6) (1983), 767-772.

[25] V. Popa and T. Noiri, On upper and Lower continuous multifunctions, Math Slovaka 43 (1993), 477-491.

[26] N.V. Velicko, H-closed topological spaces, Amer. Math. Soc. Transl. 78 (2) (1986), 103-118.

[27] R.E. Smitnson, Multifunctions, Nieuw Archief Voor Wiskunde 3XX (1972) , 31-53.