Weakly $I_{\theta g}$-closed sets

THASLEEM RANI. H

Research Scholar, Department of Mathematics, Madurai Kamaraj University, Madurai, Tamil Nadu, India

ABSTRACT: In this paper we introduce and study another generalized class of τ^* called weakly $I_{\theta g}$-open sets in ideal topological spaces. The relationships of weakly $I_{\theta g}$-closed sets and various properties of ideal topological sets are discussed.

Keywords: $I_{\theta g}$-open set, $I_{\theta g}$-closed set, I_g-closed set.

1. Introduction

In 1999, Dontchev et al. studied the notion of generalized closed sets in ideal topological spaces called I_g-closed sets [2]. In 2008, Navaneethakrishnan and Paulraj Joseph studied some characterizations of normal spaces via I_g-open sets [6]. In 2009, Navaneethakrishnan et al. introduced I_{rg}-open sets to establish some new characterizations of mildly normal spaces [5]. In 2013, Ekici and Ozen [3] have introduced weakly I_{rg}-closed sets which is a generalized class of τ^*. The main aim of this paper is to introduce and study another generalized class of τ^* called weakly $I_{\theta g}$-open sets in ideal topological spaces.

The relationships of weakly $I_{\theta g}$-closed sets and various properties of ideal topological sets are discussed.

2. Preliminaries

Definition 2.1 A subset G of an ideal topological space (X, τ, I) is said to be

1. I_g-closed [2] if $G^* \subseteq H$ whenever $G \subseteq H$ and H is open in X.
2. I_g-open [2] if $X \backslash G$ is an I_g-closed set.
3. pre^*_I-open [4] if $G \subseteq int^*(cl(G))$.
4. pre^*_I-closed [4] if $X \backslash G$ is pre^*_I-open (or) $cl^*(int(G)) \subseteq G$.
5. I-R closed [1] if $G = cl^*(int(G))$.

3. Generalized classes of τ^*

Definition 3.1 Let (X, τ, I) be an ideal topological space. A subset G of (X, τ, I) is said to be

1. a weakly $I_{\theta g}$-closed set if $(int(G))^* \subseteq H$ whenever $G \subseteq H$ and H is a θ-open set in X.
2. an $I_{\theta g}$-closed set if $G^* \subseteq H$ whenever $G \subseteq H$ and H is a θ-open set in X.
3. a weakly I_g-closed set if $(int(G))^* \subseteq H$ whenever $G \subseteq H$ and H is an open set in X.

Example 3.2 Let (X, τ, I) be an ideal topological space such that $X = \{a, b, c\}$, $\tau = \{\theta, \{a\}, \{b\}, \{a, b\}, X\}$ and $I = \{\theta, \{a\}\}$. Then $\{a\}$ is weakly $I_{\theta g}$-closed set and $\{b\}$ is not a weakly $I_{\theta g}$-closed set.

Definition 3.3 Let (X, τ, I) be an ideal topological space and $G \subseteq X$. Then G is said to be

1. weakly $I_{\theta g}$-open if $X \setminus G$ is a weakly $I_{\theta g}$-closed set.
2. $I_{\theta g}$-open if $X \setminus G$ is an $I_{\theta g}$-closed set.

Remark 3.4 Let (X, τ, I) be an ideal topological space. The following diagram holds for a subset $G \subseteq X$.

$$
\begin{array}{ccc}
 & I\text{ g-closed} & \longrightarrow & I_{\theta g}\text{-closed} \\
 & \uparrow & & \downarrow \\
 & *\text{-closed} & \longrightarrow & \text{weakly } I_{\theta g}\text{-closed} \\
 & \uparrow & & \\
I\text{-R-closed} & & &
\end{array}
$$

These implications are not reversible as shown in the following examples and in [1, 2, 4, 6, 5, 7].

Example 3.5 Let (X, τ, I) be an ideal topological space such that $X = \{a, b, c, d\}$, $\tau = \{\theta, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$ and $I = \{\theta\}$. Then $\{c\}$ is a weakly $I_{\theta g}$-closed set but not an $I_{\theta g}$-closed set.

Example 3.6 Let X, τ and I be as in Example 3.5. Then $\{c\}$ is a weakly $I_{\theta g}$-closed set but not $*$-closed.

Proposition 3.7 Every weakly I_g-closed set is a weakly $I_{\theta g}$-closed but not conversely.

Example 3.8 Let X, τ and I be as in Example 3.2. Then $\{a, b\}$ is a weakly $I_{\theta g}$-closed set but not a weakly I_g-closed set.

Theorem 3.9 Let (X, τ, I) be an ideal topological space and $G \subseteq X$. The following properties are equivalent:

1. G is a weakly $I_{\theta g}$-closed set,

2. $cl^*(int(G)) \subseteq H$ whenever $G \subseteq H$ and H is a θ-open set in X.

Proof. (1)⇒ (2) : Let G be a weakly $I_{\theta g}$-closed set in (X, τ, I). Suppose that $G \subseteq H$ and H is a θ-open set in X. We have $(int(G))^* \subseteq H$. Since $int(G) \subseteq G \subseteq H$, then $(int(G))^* \cup int(G) \subseteq H$. This implies that $cl^*(int(G)) \subseteq H$.

(2) ⇒ (1) : Let $cl^*(int(G)) \subseteq H$ whenever $G \subseteq H$ and H is θ-open in X. Since $(int(G))^* \cup int(G) \subseteq H$, then $(int(G))^* \subseteq H$ whenever $G \subseteq H$ and H is a θ-open set in X. Thus G is a weakly $I_{\theta g}$-closed set.

Theorem 3.10 Let (X, τ, I) be an ideal topological space and $G \subseteq X$. If G is θ-open and weakly $I_{\theta g}$-closed, then G is $*$-closed.

Proof. Let G be a θ-open and weakly $I_{\theta g}$-closed set in (X, τ, I). Since G is θ-open and weakly $I_{\theta g}$-closed, cl*(G) = cl*(int_θ(G)) ⊆ cl*(int(G)) ⊆G. Thus, G is a ∗-closed set in (X, τ, I).

Theorem 3.11 Let (X, τ, I) be an ideal topological space and G ⊆ X. If G is a weakly $I_{\theta g}$-closed set, then (int(G))*\G contains no any nonempty θ-closed set.

Proof. Let G be a weakly $I_{\theta g}$-closed set in (X, τ, I). Suppose that H is a θ-closed set such that H⊆(int(G))*\G. Since G is a weakly $I_{\theta g}$-closed set, X\H is θ-open and G ⊆ X\H, then (int(G))* ⊆ X\H. We have H ⊆ X\(int(G))*. Hence, H ⊆(int(G))* ∩ (X\(int(G))*) = θ. Thus, (int(G))*\G contains no any nonempty θ-closed set.

Theorem 3.12 Let (X, τ, I) be an ideal topological space and G ⊆ X. If G is a weakly $I_{\theta g}$-closed set, then cl*(int(G))\G contains no any nonempty θ-closed set.

Proof. Suppose that H is a θ-closed set such that H ⊆(int(G))*\G. By Theorem 3.11, it follows from the fact that cl*(int(G))\G = ((int(G))* ∪ int(G))\G = ((int(G))* ∩ G^c) ∪ (int(G) ∩ G^c) ⊇ (int(G))*∩ G^c= (int(G))*\G. Hence the result.

Theorem 3.13 Let (X, τ, I) be an ideal topological space. The following properties are equivalent:

1. Every weakly $I_{\theta g}$-closed set is pre$*_I$-closed.

2. Each singleton {x} of X is a θ-closed set or {x} is pre$*_I$-open.

Proof. (1) ⇒(2) : Let G be weakly $I_{\theta g}$-closed set in X. Then we have cl*(int(G)) ⊆ G. Let x ∈ X. Assume that {x} is not a θ-closed set. It follows that X is the only θ-open set containing X\{x}. Then, X\{x} is a weakly $I_{\theta g}$-closed set in (X, τ, I). Thus, cl*(int(X\{x})) ⊆X\{x} and hence {x} ⊆ int*(cl({x})). Consequently, {x}is pre$*_I$-open.

(2) ⇒(1) : Let G be a weakly $I_{\theta g}$-closed set in (X, τ, I). Let x ∈ cl*(int(G)). Suppose that {x} is pre$*_I$-open. We have {x} ⊆int*(cl({x})). Since x ∈cl*(int(G)), then int*(cl({x})) ∩int(G) ≠ θ. It follows that cl({x}) ∩int(G) ≠ θ. We have cl({x} ∩int(G)) ≠ θ and then {x} ∩ int(G) ≠ θ. Hence, x ∈ int(G). Thus, we have x ∈ G. Suppose that {x}is a θ-closed set. By Theorem 3.12, cl*(int(G))\G does not contain {x}. Since x ∈cl*(int(G)), then we have x ∈G. Consequently, we have x ∈G. Thus, cl*(int(G)) ⊆ G and hence G is pre$*_I$-closed.

Theorem 3.14 Let (X, τ, I) be an ideal topological space and G ⊆ X. If cl*(int(G))\G contains no any nonempty ∗-closed set, then G is a weakly $I_{\theta g}$- closed set.

Proof. Let G ⊆ H and H be a θ-open set. Then H is a ∗-open set. Assume that cl*(int(G)) is not contained in H. It follows that cl*(int(G)) ∩ (X\H) is a nonempty ∗-closed subset of cl*(int(G))\G. This is a contradiction. Hence, G is a weakly $I_{\theta g}$-closed set.

Theorem 3.15 Let (X, τ, I) be an ideal topological space and G ⊆ X. If G is a weakly $I_{\theta g}$-closed set, then int(G) = H\K where H is I-R closed and K contains no any nonempty θ-closed set.

Proof. Let G be a weakly $I_{\theta g}$-closed set in (X, τ, I). Take K = (int(G))*\G. Then, by Theorem 3.11, K contains no any nonempty θ-closed set. Take H = cl*(int(G)). Then H = cl*(int(H)). Moreover, we have

H\K = [cl*(int(G))]\[(int(G))*\G] = [cl*(int(G))]\[(cl*(int(G))\G] = cl*(int(G))∩X\[(cl*(int(G))\G] = cl*(int(G)) ∩ {[X\cl*(int(G))] ∪G}= cl*(int(G)) ∩ G = [int(G) ∪(int(G))*] ∩ G = int(G).

Theorem 3.16 Let (X, τ, I) be an ideal topological space and G ⊆ X. Assume that G is a weakly $I_{\theta g}$-closed set. The following properties are equivalent:

1. G is pre*$_I$-closed,
2. cl*(int(G))\G is a θ-closed set,
3. (int(G))* \G is a θ-closed set.

Proof. (1) ⇒(2) : Let G be pre*$_I$-closed. We have cl*(int(G)) ⊆G. Then, cl*(int(G))\G = θ. Thus, cl*(int(G))\G is a θ-closed set.

(2) ⇒(1) : Let cl*(int(G))\G be a θ-closed set. Since G is a weakly $I_{\theta g}$-closed set in (X, τ, I), then by Theorem 3.12, cl*(int(G))\G = θ. Hence, we have cl*(int(G)) ⊆G. Thus, G is pre*$_I$-closed.

(2) ⇔(3) : It follows easily from that cl*(int(G))\G = (int(G))*\G.

Theorem 3.17 Let (X, τ, I) be an ideal topological space and G ⊆ X be a weakly $I_{\theta g}$-closed set. Then G ∪ (X\(int(G))*) is a weakly $I_{\theta g}$-closed set in (X, τ, I) .

Proof. Let G be a weakly $I_{\theta g}$-closed set in (X, τ, I). Suppose that H is a θ-open set such that G ∪(X\(int(G))*) ⊆ H. We have X\H⊆X\(G∪(X\(int(G))*)) = (X\G)∩(int(G))* = (int(G))*\G. Since X\H is a θ-closed set and G is a weakly $I_{\theta g}$-closed set, it follows from Theorem 3.11 that X\H = θ. Hence, X = H. Thus, X is the only θ-open set containing G∪(X\(int(G))*). Consequently, G∪(X\(int(G))*) is a weakly $I_{\theta g}$-closed set in (X, τ, I). **Corollary 3.18** Let (X, τ, I) be an ideal topological space and G ⊆ X be a weakly $I_{\theta g}$-closed set. Then (int(G))*\G is a weakly $I_{\theta g}$-open set in (X, τ, I) .

Proof. Since X\((int(G))*\G) = G∪(X\(int(G))*), it follows from Theorem 3.17 that (int(G))*\G is a weakly $I_{\theta g}$-open set in (X, τ, I).

Theorem 3.19 Let (X, τ, I) be an ideal topological space and G ⊆ X. The following properties are equivalent:

1. G is a ∗-closed and θ-open set,
2. G is I -R closed and a θ-open set,
3. G is a weakly $I_{\theta g}$-closed and θ-open set.

Proof. (1) ⇒(2) ⇒(3) : Obvious.

(3) ⇒(1) : It follows from Theorem 3.10.

Proposition 3.20 Every pre*$_I$-closed set is weakly $I_{\theta g}$-closed but not conversely.

Proof. Let H ⊆G and G is a θ-open set in X. Since H is pre*$_I$-closed, cl*(int(H)) ⊆ H ⊆ G. Hence H is weakly $I_{\theta g}$-closed.

Example 3.21 Let (X, τ, I) be an ideal topological space such that X = {a, b, c, d}, τ = {θ, {c}, {d}, {a, c}, {c, d}, {a, c, d}, X} and I = {θ, {b}}. Then {b, c} is a weakly $I_{\theta g}$-closed set but not pre*$_I$-closed.

4. Further properties

Theorem 4.1 Let (X, τ, I) be an ideal topological space. The following properties are equivalent:

1. Each subset of (X, τ, I) is a weakly $I_{\theta g}$-closed set,

2. G is pre$*_I$-closed for each θ-open set G in X.

Proof. (1) ⇒(2) : Suppose that each subset of (X, τ, I) is a weakly $I_{\theta g}$-closed set. Let G be a θ-open set in X. Since G is weakly $I_{\theta g}$-closed, then we have $cl^*(int(G)) \subseteq G$. Thus, G is pre$*_I$-closed.

(2) ⇒(1) : Let G be a subset of (X, τ, I) and H be a θ-open set in X such that $G \subseteq H$. By (2), we have $cl^*(int(G)) \subseteq cl^*(int(H)) \subseteq H$. Thus, G is a weakly $I_{\theta g}$-closed set in (X, τ, I).

Theorem 4.2 Let (X, τ, I) be an ideal topological space. If G is a weakly $I_{\theta g}$-closed set and $G \subseteq H \subseteq cl^*(int(G))$, then H is a weakly $I_{\theta g}$-closed set.

Proof. Let $H \subseteq K$ and K be a θ-open set in X. Since $G \subseteq K$ and G is a weakly $I_{\theta g}$-closed set, then $cl^*(int(G)) \subseteq K$. Since $H \subseteq cl^*(int(G))$, then $cl^*(int(H)) \subseteq cl^*(int(G)) \subseteq K$. Thus, $cl^*(int(H)) \subseteq K$ and hence, H is a weakly $I_{\theta g}$-closed set.

Corollary 4.3 Let (X, τ, I) be an ideal topological space. If G is a weakly $I_{\theta g}$-closed and open set, then $cl^*(G)$ is a weakly $I_{\theta g}$-closed set.

Proof. Let G be a weakly $I_{\theta g}$-closed and open set in (X, τ, I). We have $G \subseteq cl^*(G) \subseteq cl^*(G) = cl^*(int(G))$. Hence, by Theorem 4.2, $cl^*(G)$ is a weakly $I_{\theta g}$-closed set in (X, τ, I).

Theorem 4.4 Let (X, τ, I) be an ideal topological space and $G \subseteq X$. If G is a nowhere dense set, then G is a weakly $I_{\theta g}$-closed set.

Proof. Let G be a nowhere dense set in X. Since $int(G) \subseteq int(cl(G)) = \theta$, then $int(G) = \theta$. Hence, $cl^*(int(G)) = \theta$. Thus, G is a weakly $I_{\theta g}$-closed set in (X, τ, I).

Remark 4.5 The reverse of Theorem 4.4 is not true in general as shown in the following example.

Example 4.6 Let X, τ and I be as in Example 3.2. Then $G = \{a\}$ is a weakly $I_{\theta g}$-closed set but not a nowhere dense set.

Remark 4.7 The union of two weakly $I_{\theta g}$-closed sets in an ideal topological space need not be a weakly $I_{\theta g}$-closed set.

Example 4.8 Let (X, τ, I) be an ideal topological space such that $X = \{a, b, c, d\}$, $\tau = \{\theta, \{b\}, \{c, d\}, \{b, c, d\}, X\}$ and $I = \{\theta, \{a\}, \{b\}, \{a, b\}\}$. Clearly $A = \{c\}$ and $B = \{d\}$ are weakly $I_{\theta g}$-closed sets. But their union $A \cup B = \{c, d\}$ is not a weakly $I_{\theta g}$-closed set.

Lemma 4.9 Let G be an open subset of a topological space (X, τ).

If $K (\subseteq G)$ is θ-open in (G, τ_G), then there exists a θ-open set L in (X, τ) such that $K = L \cap G$.

Theorem 4.10 Let (X, τ, I) be an ideal topological space and $H \subseteq G \subseteq X$. If G is an open set in X and H is a weakly $I_{\theta g}$-closed in G, then H is a weakly. $I_{\theta g}$-closed in X.

Proof. Let K be a θ-open set in X and $H \subseteq K$. We have $H \subseteq K \cap G$. By Lemma 4.9, $K \cap G$ is a θ-open set in G. Since H is a weakly $I_{\theta g}$-closed set in G, then $cl^*_G(int_G(H)) \subseteq K \cap G$. Also, we have $cl^*(int(H)) \subseteq$

$cl^*_G(int(H)) = cl^*_G(int_G(H)) \subseteq K \cap G \subseteq K$. Hence, $cl^*(int(H)) \subseteq K$. Thus, H is a weakly $I_{\theta g}$-closed in (X, τ, I).

Theorem 4.11 Let (X, τ, I) be an ideal topological space and $H \subseteq G \subseteq X$. If G is an open set in X and H is a weakly $I_{\theta g}$-closed set in X, then H is a weakly $I_{\theta g}$-closed set in G.

Proof. Let $H \subseteq K$ and K be a θ-open set in G. By Lemma 4.9, there exists a θ-open set L in X such that $K = L \cap G$. Since H is a weakly $I_{\theta g}$-closed set in X, then $cl^*(int(H)) \subseteq K$. Also, we have $cl^*_G(int_G(H)) = cl^*_G(int(H)) = cl^*(int(H)) \cap G \subseteq K \cap G = K$. Thus, $cl^*_G(int_G(H)) \subseteq K$. Hence, H is a weakly $I_{\theta g}$-closed set in G.

Theorem 4.12 Let (X, τ, I) be an ideal topological space and $G \subseteq X$. Then G is a weakly $I_{\theta g}$-open set if and only if $H \subseteq int^*(cl(G))$ whenever $H \subseteq G$ and H is a θ-closed set.

Proof. Let H be a θ-closed set in X and $H \subseteq G$. It follows that X\H is a θ-open set and $X\backslash G \subseteq X\backslash H$. Since X\G is a weakly $I_{\theta g}$-closed set, then $cl^*(int(X\backslash G)) \subseteq X\backslash H$. We have $X\backslash int^*(cl(G)) \subseteq X\backslash H$. Thus, $H \subseteq int^*(cl(G))$. Conversely, let K be a θ-open set in X and $X\backslash G \subseteq K$. Since X\K is a θ-closed set such that $X\backslash K \subseteq G$, then $X\backslash K \subseteq int^*(cl(G))$. We have $X\backslash int^*(cl(G)) = cl^*(int(X\backslash G)) \subseteq K$. Thus, X\G is a weakly $I_{\theta g}$-closed set. Hence, G is a weakly $I_{\theta g}$-open set in (X, τ, I).

Theorem 4.13 Let (X, τ, I) be an ideal topological space and $G \subseteq X$. If G is a weakly $I_{\theta g}$-closed set, then $cl^*(int(G))\backslash G$ is a weakly $I_{\theta g}$-open set in (X, τ, I) .

Proof. Let G be a weakly $I_{\theta g}$-closed set in (X, τ, I). Suppose that H is a θ-closed set such that $H \subseteq cl^*(int(G))\backslash G$. Since G is a weakly $I_{\theta g}$-closed set, it follows from Theorem 3.12 that H = θ. Thus, we have $H \subseteq int^*(cl(cl^*(int(G))\backslash G))$. It follows from Theorem 4.12 that $cl^*(int(G))\backslash G$ is a weakly $I_{\theta g}$-open set in (X, τ, I).

Theorem 4.14 Let (X, τ, I) be an ideal topological space and $G \subseteq X$. If G is a weakly $I_{\theta g}$-open set, then H = X whenever H is a θ-open set and $int^*(cl(G)) \cup (X\backslash G) \subseteq H$.

Proof. Let H be a θ-open set in X and $int^*(cl(G)) \cup (X\backslash G) \subseteq H$. We have $X\backslash H \subseteq (X\backslash int^*(cl(G))) \cap G = cl^*(int(X\backslash G))\backslash(X\backslash G)$. Since X\H is a θ-closed set and X\G is a weakly $I_{\theta g}$-closed set, it follows from Theorem 3.12 that X\H = θ. Thus, we have H = X.

Theorem 4.15 Let (X, τ, I) be an ideal topological space. If G is a weakly $I_{\theta g}$-open set and $int^*(cl(G)) \subseteq H \subseteq G$, then H is a weakly $I_{\theta g}$-open set.

Proof. Let G be a weakly $I_{\theta g}$-open set and $int^*(cl(G)) \subseteq H \subseteq G$. Since $int^*(cl(G)) \subseteq H \subseteq G$, then $int^*(cl(G)) = int^*(cl(H))$. Let K be a θ-closed set and $K \subseteq H$. We have $K \subseteq G$. Since G is a weakly $I_{\theta g}$-open set, it follows from Theorem 4.12 that $K \subseteq int^*(cl(G)) = int^*(cl(H))$. Hence, by Theorem 4.12, H is a weakly $I_{\theta g}$-open set in (X, τ, I).

Corollary 4.16 Let (X, τ, I) be an ideal topological space and $G \subseteq X$. If G is a weakly $I_{\theta g}$-open and closed set, then $int^*(G)$ is a weakly $I_{\theta g}$-open set.

Proof. Let G be a weakly $I_{\theta g}$-open and closed set in (X, τ, I). Then $int^*(cl(G)) = int^*(G) \subseteq int^*(G) \subseteq G$. Thus, by Theorem 4.15, $int^*(G)$ is a weakly $I_{\theta g}$-open set in (X, τ, I).

Definition 4.17 A subset A of an ideal topological space (X, τ, I) is called ${}_{\theta}E_I$-set if A = M∪N where M is θ-closed and N is pre*$_I$-open.

Remark 4.18 Every pre*$_I$-open (resp. θ-closed) set is ${}_{\theta}E_I$-set but not conversely.

Example 4.19 Let (X, τ, I) be an ideal topological space such that X = {a, b, c, d}, τ = {θ, {a}, {b}, {a, b}, {b, c}, {a, b, c}, X} and I = {θ, {d}}. Then {d} is a ${}_{\theta}E_I$-set but not a pre*$_I$-open set. Also {a} is a ${}_{\theta}E_I$-set but not a θ-closed set.

Theorem 4.20 For a subset H of (X, τ, I), the following are equivalent.

1. H is pre*$_I$-open.
2. H is a ${}_{\theta}E_I$-set and weakly $I_{\theta g}$-open.

Proof. (1) ⇒(2): By Remark 2.4.18, H is a ${}_{\theta}E_I$-set and H is a weakly $I_{\theta g}$-open by Proposition 3.20.

(2) ⇒(1): Let H be a ${}_{\theta}E_I$-set and weakly $I_{\theta g}$-open. Then there exist a θ-closed set M and a pre*$_I$-open set N such that H =M∪N. Since M ⊆ H and H is weakly $I_{\theta g}$-open, by Theorem 4.12, M ⊆ int*(cl(H)). Also, we have N ⊆ int*(cl(N)). Since N ⊆ H, N ⊆ int*(cl(N)) ⊆int*(cl(H)). Then H = M ∪ N ⊆ int*(cl(H)). So H is pre*$_I$-open.

The following Example shows that the concepts of weakly $I_{\theta g}$-open set and ${}_{\theta}E_I$-set are independent.

Example 4.21 Let X, τ and I be as in Example 4.19 Then {d} is a ${}_{\theta}E_I$-set but not a weakly $I_{\theta g}$-open set. Also {a, c} is a weakly $I_{\theta g}$-open set but not a ${}_{\theta}E_I$-set.

Definition 4.22 A subset A of an ideal topological space (X, τ, I) is called F_I-set if A = M ∪ N where M is regular closed and N is pre*$_I$-open.

Remark 4.23 Every pre*$_I$-open (resp. regular closed) set is F_I-set but not conversely.

Example 4.24 Let X, τ and I be as in Example 4.19. Then {b, c, d} is a F_I-set but not a pre*$_I$-open set. Also {a, b} is a F_I-set but not a regular closed set.

Corollary 4.25 For a subset H of (X, τ, I), the following are equivalent.

1. H is pre*$_I$-open.
2. H is a F_I-set and weakly $I_{\theta g}$-open.

The following Example shows that the concepts of weakly $I_{\theta g}$-open set and F_I- set are independent.

Example 4.26 Let X, τ and I be as in Example 4.19. Then {a, d} is a F_I-set but not a weakly I_{rg}-open set. Also {c} is a weakly $I_{\theta g}$-open set but not a F_I-set.

References:

1. A. Acikgoz and S. Yuksel, Some new sets and decompositions of A_{I-R}- continuity, α-I-continuity, continuity via idealization, Acta Math. Hungar., 114(1-2)(2007), 79-89.
2. J. Dontchev, M. Ganster and T. Noiri, Unified operation approach of generalized closed sets via topological ideals, Math. Japonica, 49(1999), 395-401.

3. E. Ekici and S. Ozen, A generalized class of τ* in ideal spaces, Filomat, 27(4)(2013), 529-535.
4. E. Ekici, On AC_I-sets, BC_I-sets, β^*_I-open sets and decompositions of continuity in ideal topological spaces, Creat. Math. Inform, 20(2011), 47-54.
5. M. Navaneethakrishnan, J. Paulraj Joseph and D. Sivaraj, I_g-normal and I_g-regular spaces, Acta Math. Hungar., 125(4)(2009), 327-340.
6. M. Navaneethakrishnan and J. Paulraj Joseph, g-closed sets in ideal topological spaces, Acta Math. Hungar., 119(4)(2008), 365-371.
7. M. Rajamani, V. Inthumathi and S. Krishnaprakash, $I_{\pi g}$-closed sets and $I_{\pi g}$-continuity, Journal of Advanced Research in Pure Mathematics, 2(4)(2010), 63-72.