Performance Analysis of MIMO systems to achieve higher spectral efficiency in Nakagami fading environment

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Abstract : The proposed work deals with the performance analysis of 4 × 4 Multiple Input Multiple Output (MIMO) wireless in uplink (UL) and 8 × 8 MIMO for downlink (DL) to achieve desire spectral efficiency for Long Term Evolution Advance (LTE-A), one of the candidate for International Mobile Telecommunication - Advance (IMT-A) for 4G. Spatial multiplexing is the key technology used in channel modeling for MIMO system. Spatial multiplexing of 4 × 4 and 8 × 8 MIMO is carried out using Singular Value Decomposition (SVD). To achieve higher Spectral Efficiency (i.e. Bits/sec/Hz)) this scheme employs the Waterfilling algorithm for improving the power allocation in all subcarriers/ channels. Due to more degree of freedom gain in MIMO system higher bit rate is possible and is achieved. The MIMO Channel model is established in Rayleigh, Nakagami and Rician fading environment. Results are discussed with comparative analysis of different fading models.

IndexTerms **- MIMO, Waterfilling Algorithm, Nakagami fading, SVD, Spatial Multiplexing, Spectral efficiency, LTE-A, IMT-A, Degree of freedom gain**

I. Introduction

In recent years the International Telecommunications Union– Radio communications sector (ITU-R) has defined the system requirements for International Mobile Telecommunications- Advanced (IMT-A) for 4G [1]. One of the key features of IMT-A is to enhance peak data rates to support advanced services and applications in the order of 100 Mbit/s for high mobility and 1 Gbit/s for low mobility conditions. Keeping in mind the above requirements 3rd Generation Partnership Project (3GPP) group aims at further evolving Long Term Evolution-Advanced (LTE-A) to meet/exceed the IMT-A capabilities.

The use of multiple antennas at transmitter and receiver, popularly known as multiple-input multiple-output (MIMO) wireless is an emerging cost-effective technology that offers substantial leverage to make 1Gbps wireless links a reality [2]. Under suitable fading conditions, having both multiple transmit and multiple receive antennas provide an additional spatial dimension for communication and yields a degree-of-freedom gain. These additional degrees of freedom can be exploited by Spatially Multiplexing several data streams onto the MIMO channel, and lead to an increase in the capacity: the capacity of such a MIMO channel with n transmit and receive antennas is proportional to n [3].

The performance of MIMO systems can be evaluated either by measurements or by using MIMO channel models. Measurement campaigns are both time and cost consuming. Hence analytical and simulation techniques are generally used for this purpose [4]. This paper provides complete performance evaluation of 4×4 MIMO in UL and 8×8 MIMO in DL in different fading environments namely Rayleigh, Nakagami and Rician. MIMO channel modeling is carried out using spatially multiplexed channel and it's a vector Gaussian channel. The channel capacity is computed by decomposing the vector channel into a set of parallel, independent scalar Gaussian channels. This is carried out by Singular Value Decomposition (SVD). The optimal strategy called waterfilling or water pouring is used to allocate the power to the sub-carriers in the channel. Transmitter allocates more power to the stronger sub-carriers, taking advantage of the better channel conditions, and less power or even no power to the weaker ones.

II. Channel Model using SVD

A narrowband time –invariant wireless channel with n_t transmit and n_r receive antennas is described by an n_r by n_t deterministic matrix **H**. By looking at the capacity of the channel we determine how much spatial multiplexing that H can support.

The time variant channel is described by

$$\mathbf{y} = \mathbf{H}\,\mathbf{x} + \mathbf{w}\ , \qquad \ldots(1)$$

where $\mathbf{x} \in \mathcal{C}^{n_t}$, $\mathbf{y} \in \mathcal{C}^{n_r}$ and $\mathbf{w} \sim \mathcal{CN}(0, N_0 I_{n_r})$ denote the transmitted signal, received signal and white Gaussian noise respectively at a symbol time. The channel matrix $\mathbf{H} \in \mathcal{C}^{n_r \times n_t}$ is deterministic and assumed to be constant at all times and known to both receiver and transmitter [3].

This is a Vector Gaussian Channel. The capacity of this channel can be computed by decomposing this vector channel into a set of parallel, independent Gaussian sub-channels. Every linear transformation represented as a composition of three operations: a rotation, a scaling and another rotation. In the notations of matrices the matrix **H** has a singular value decomposition (SVD):

$$\mathbf{H} = \mathbf{U}\,\mathbf{\Lambda}\,\mathbf{V}^{*}\ , \qquad \ldots(2)$$

where $\mathbf{U} \in \mathcal{C}^{n_r \times n_r}$ and $\mathbf{V} \in \mathcal{C}^{n_t \times n_t}$ are unitary (rotation) matrices and $\mathbf{\Lambda} \in \mathfrak{R}^{n_r \times n_t}$ is a rectangular matrix whose diagonal elements are non-negative real numbers and whose off-diagonal elements are zero. The diagonal elements $\lambda_1 \geq \lambda_2 \geq \lambda_3 \cdots \lambda_{n_{\min}}$ are the ordered singular values of the matrix **H**, where $n_{\min} := \min(n_t, n_r)$. Since

$$\mathbf{HH^*} = \mathbf{U \Lambda \Lambda^t U^*}, \qquad \ldots (3)$$

the squared singular values λ_i^2 are the eigenvalues of the matrix **HH*** and also of **H*H ,** here we have $n_{\min}$ eigenvalues. We rewrite the SVD as

$$\mathbf{H} = \sum_{i=1}^{n_{\min}} \boldsymbol{\lambda}_i \, \mathbf{u}_i \, \mathbf{v}_i^* \,, \qquad \ldots (4)$$

i.e the sum of rank-one matrices $\boldsymbol{\lambda}_i \mathbf{u}_i \mathbf{v}_i^*$. It can be seen that the rank of **H** is precisely the number of non-zero singular values. Now we define,

$$\tilde{\mathbf{x}} := \mathbf{V}^* \mathbf{x}, \qquad \ldots (5)$$

$$\tilde{\mathbf{y}} := \mathbf{U}^* \mathbf{y}, \qquad \ldots (6)$$

$$\tilde{\mathbf{w}} := \mathbf{U}^* \mathbf{w} \qquad \ldots (7)$$

we can rewrite the channel (1) as

$$\tilde{\mathbf{y}} := \mathbf{\Lambda} + \tilde{\mathbf{w}} \,, \qquad \ldots (8)$$

where $\tilde{\mathbf{w}} \sim \mathcal{CN}(0, N_0 I_{n_r})$ has the same distribution as **w**, and $\|\tilde{\mathbf{x}}\|^2 = \|\mathbf{x}\|$.Thus, the energy is preserved and we have an equivalent representation as a parallel Gaussian channel:

$$\tilde{\mathbf{y}}_i := \boldsymbol{\lambda}_i \tilde{\mathbf{x}}_i + \tilde{\mathbf{w}}_i \,, \quad i = 1, 2, .. \, n_{\min} \qquad \ldots (9)$$

The equivalence is summarized in Figure: 1

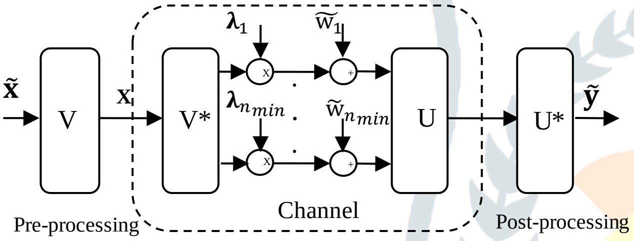


Figure:1 Converting MIMO channel into parallel channel through SVD

The SVD decomposition interpreted as two coordinate transformations: it says that if the input is expressed in terms of a coordinate system defined by the columns of **V** and the output is expressed in terms of a coordinate system defined by the column of **U**, then the input/output relationship is very simple. Equation (8) is a representation of the original channel (1) with the input and output expressed in terms of these new coordinates.

III. Waterfilling algorithm

In slow fading environment we are interested in achieving a target data rate within a coherence time period of the channel. In the fast fading case, one is concerned with the rate averaged over *many* coherence time periods. With transmitter channel knowledge, what will be the capacity of the fast fading channel can be understood by simple block fading model :

$$y[m] = h[m]x[m] + w[m], \qquad \ldots (10)$$

where $h[m] = h_l$ remains constant over the l^{th} coherence period of T_c ($T_c \gg 1$) symbols and is i.i.d. across different coherence periods. The channel over L such coherence periods can be modeled as a *parallel channel* with L sub-channels that fade independently. For a given realization of the channel gains $h_1, \ldots \ldots h_L$, the capacity (in bits/symbol) of this parallel channel is

$$\max_{P_1, \ldots\ldots, P_L} \frac{1}{L} \sum_{l=1}^{L} \log\left(1 + \frac{P_l |h_l|}{N_0}\right) \qquad \ldots (11)$$

subject to

$$\frac{1}{L} \sum_{l=1}^{L} P_l = P \,, \qquad \ldots (12)$$

where P is the average power constraint. Optimal power allocation is Waterfilling:

$$P_l^* = \left(\frac{1}{\lambda} - \frac{N_0}{|h_l|^2}\right)^+, \qquad \ldots (13)$$

where λ satisfies

$$\frac{1}{L}\sum_{l=1}^{L}\left(\frac{1}{\lambda}-\frac{N_0}{|h_l|^2}\right)^{+}=P, \qquad \dots (14)$$

Figure:2 gives a pictorial view of the optimal power allocation strategy. Think of the values $N_0/|h_l|^2$ plotted as a function of the sub-channel index, $h_1, \dots\dots h_L$, as tracing out the bottom of a vessel. If P units of water per sub-channel are filled into the vessel, the depth of the water at sub-channel L is the power allocated to that sub -channel, and 1/λ is the height of the water surface. Thus, this optimal strategy is called *waterfilling* or *waterpouring* [3].

Weak channels power allocation = 0

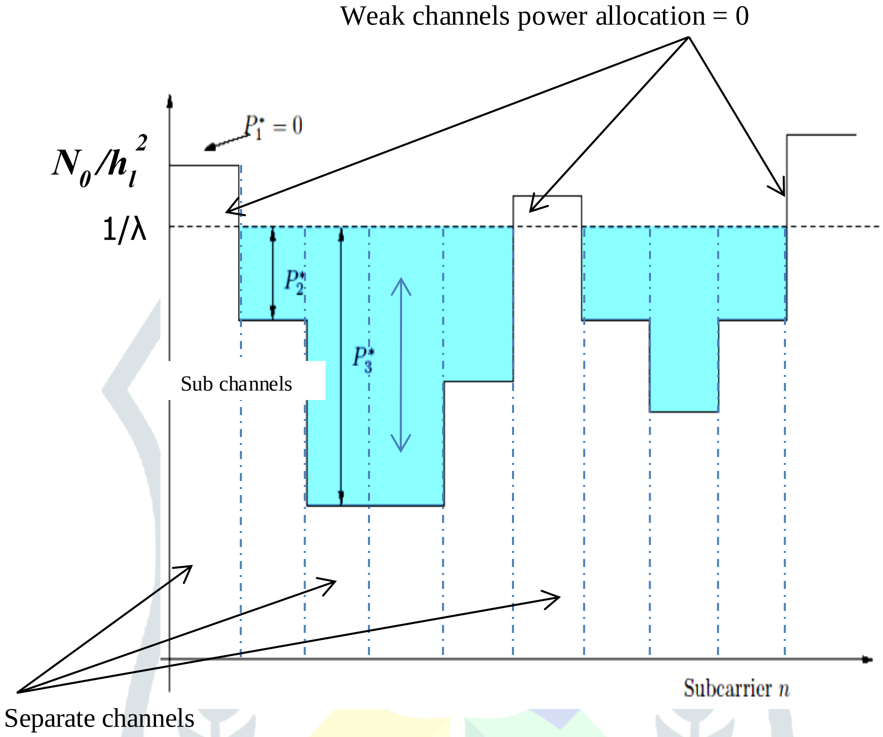


Figure:2 Waterfilling power allocation over the sub channels.

Note that there are some sub-channels where the bottom of the vessel is above the water and no power is allocated to them. In these sub-channels, the channel condition is too poor for it to be worthwhile to transmit information. The transmitter allocates more power to the stronger sub-channels, taking advantage of the better channel conditions, and less or even no power to the weaker ones.
By the laws of large numbers, (14) converges to

$$\mathbb{E}\left[\left(\frac{1}{\lambda}-\frac{N_0}{|h|^2}\right)^{+}\right]=P \qquad \dots (15)$$

n_{min}
Information
streams

for almost all realization of the fading process$\{h[m]\}$. Here, the expectation is taken with respect to the stationary distribution of the channel state. The parameter $\boldsymbol{\lambda}$ now converges to a constant, depending only on the channel statistics but not on the specific realization of the fading process. Hence, the optimal power at any time depends on the channel gain h at that time :

$$P^*(h)=\left(\frac{1}{\lambda}-\frac{N_0}{|h|^2}\right)^{+} \qquad \dots (16)$$

The capacity of the fast fading channel with transmitter knowledge is

$$C=\mathbb{E}\left[\log\left(1+\frac{P^*(h)|h|^2}{N_0}\right)\right] \quad \text{bits/sec/Hz.} \qquad \dots (17)$$

Equations (17),(16) and (15) together allow us to compute the capacity of channel using Waterfilling algorithm.

IV. Complete channel model using spatial multiplexing and Waterfilling algorithm

We have seen Gaussian Parallel channels and talked about capacities of time-invariant frequency-selective channels and about time-varying fading channels with full channel state information (CSI). The time-invariant MIMO channel is same example. Here, the spatial dimension plays the same role as the time and frequency dimensions in those examples. The capacity is now familiar

$$C = \sum_{i=1}^{n_{\min}} \log\left(1 + \frac{P_i^* \lambda_i^2}{N_0}\right) \quad \text{bits/sec/Hz} \quad \ldots(18)$$

where $P_1^* \ldots\ldots P_{n_{\min}}^*$ are the waterfilling power allocations :

$$P_i^* = \left(\mu - \frac{N_0}{\lambda_i^2}\right)^+, \quad \ldots(19)$$

with μ chosen to satisfy the total power constraint $\sum_i P_i^* = P$. Each λ_i corresponds to an *eigenmode* of the channel (also called *eigenchannel*). Each non-zero eigenchannel can support a data stream: thus, the MIMO channel can support the spatial multiplexing of multiple streams.
Figure:3, pictorially depicts the SVD-based architecture for reliable communication.

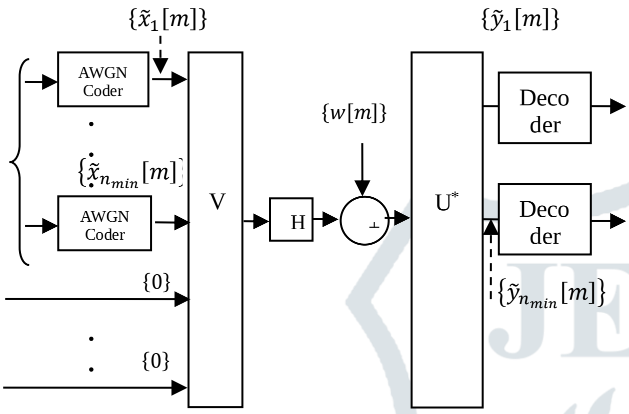


Figure :3 The SVD architecture for MIMO communication

The key parameters that determine the performance are low and high SNR regimes. At high SNR , the water level is deep and the policy of allocating equal amount of power on the non-zero eigenmode is asymptotically optimal:

$$C \approx \sum_{i=1}^{k} \log\left(1 + \frac{P\lambda_i^2}{kN_0}\right)$$

$$\approx k \log \mathbf{SNR} + \sum_{i=1}^{k} \log\left(\frac{\lambda_i^2}{k}\right) \quad \text{bits/sec/Hz} \ldots(20)$$

where $\boldsymbol{k}$ is the number of non-zero λ_i^2 ,i.e., the rank of **H**, and SNR $:= P/N_0$. *The parameter* $\boldsymbol{k}$ *is the number of spatial degrees of freedom per second per hertz.* It represents the dimension of the transmitted signal as modified by the MIMO channel, i.e., the dimension of the image of **H**. This is equal to the rank of the matrix **H** and with full rank; it can be seen that a MIMO channel provides $n_{\min}$ spatial degrees of freedom. At low SNR , the optimal policy is to allocate power only to the strongest eigenmode (the bottom of the vessel to waterfill. The resulting capacity is

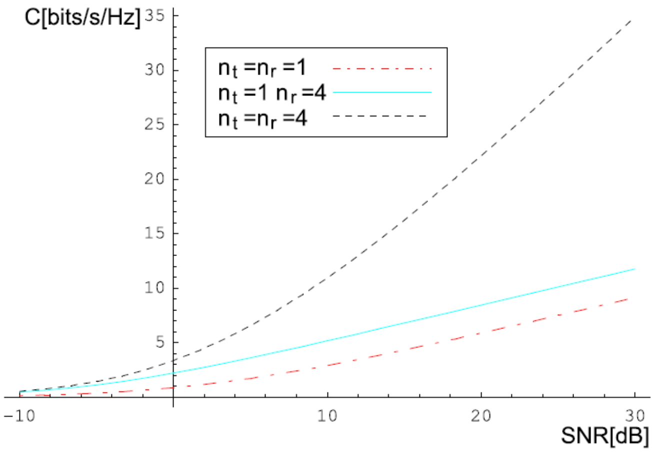
$$C \approx \frac{P}{N_0}\left(\max_i \lambda_i^2\right) \log_2 \mathrm{e} \quad \text{bits/sec/Hz} \quad \ldots(21)$$

V. Performance gains

The capacity of the MIMO fading channel is a function of the distribution of the singular value, $\boldsymbol{\lambda_i}$, of the random channel matrix **H**. By Jensen's inequality, we have

$$\sum_{i=1}^{n_{min}} \log\left(1 + \frac{SNR}{n_t}\lambda_i^2\right) \le n_{min} log\left(1 + \frac{SNR}{n_t}\left[\frac{1}{n_{min}}\sum_{i=1}^{n_{min}} \lambda_i^2\right]\right), \quad \ldots(22)$$

with equality if and only if the singular values are all equal. One can expect a high capacity if the channel matrix **H** is sufficiently random and statistically well conditioned, with the overall channel gain well distributed across the singular values. One can expect such a channel to attain the full degrees of freedom at high SNR.



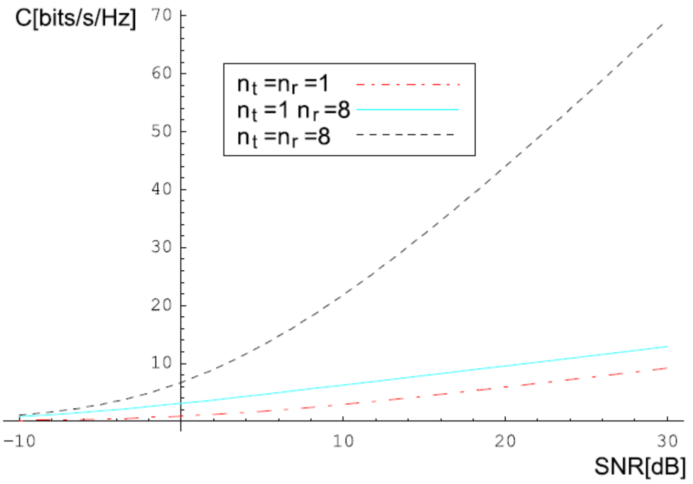
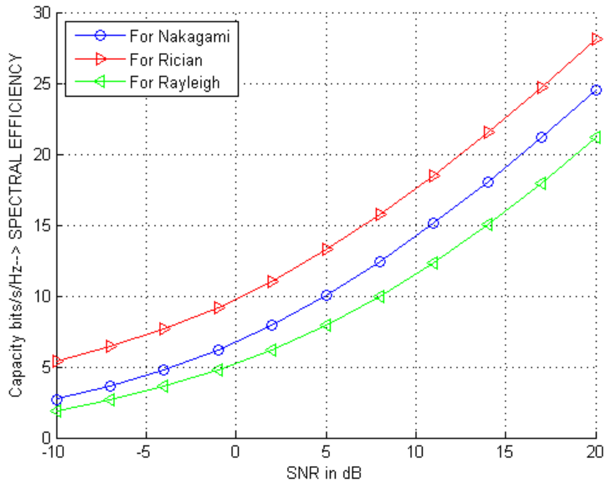


Figure:4 Capacity of i.i.d. Rayleigh fading channel.
Upper: 4 by 4 channel, lower: 8 by 8 channel

We plot the capacity for the i.i.d. Rayleigh fading model in Figure:4 for different numbers of antennas[3]. Indeed, we see that for such a random channel the capacity of MIMO system can be very large. At moderate to high SNR , the capacity of an ***n*** by ***n*** channel is about ***n*** times the capacity of a 1 by 1 system. The asymptotic slope of capacity versus SNR in dB scale is proportional to ***n***, which means that the capacity scales with SNR like n log SNR**.**

VI. Simulation Results



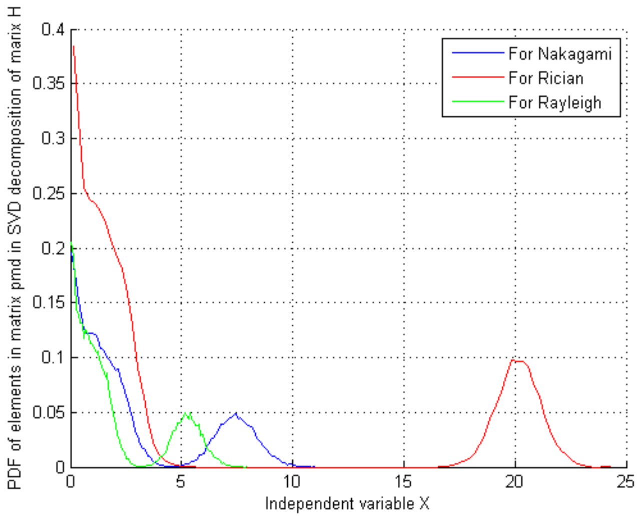
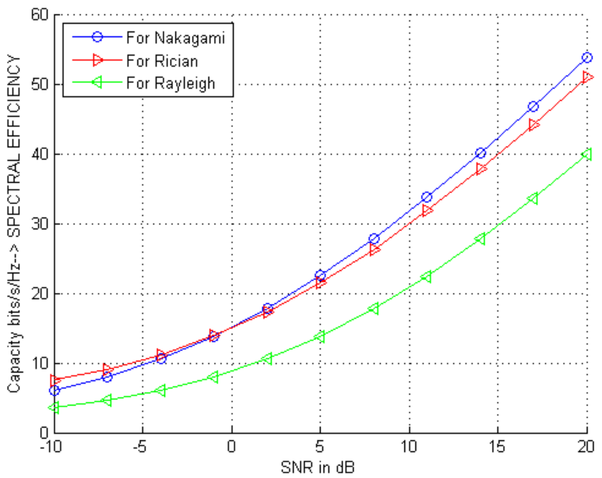


Figure : 5 Simulation results for 4 x 4 MIMO (Uplink) in Rayleigh , Nakagami and Rician Fading Environment. Upper: Channel Capacity results Lower: Different Fading curves.



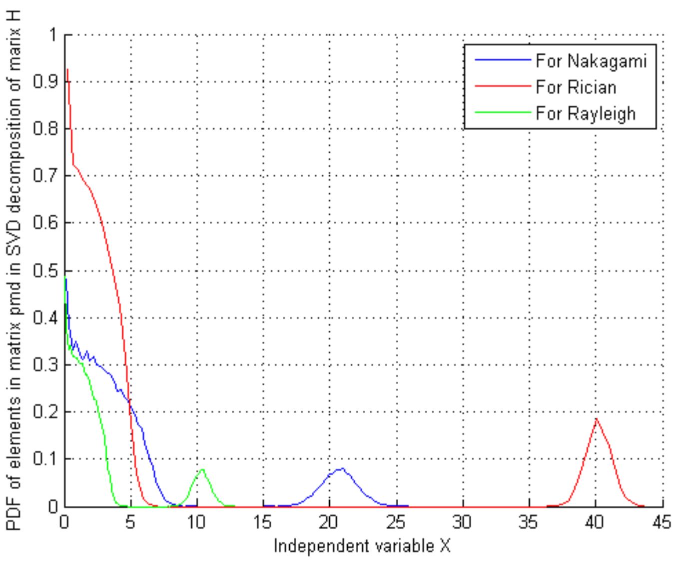


Figure : 6 Simulation results for 8 x 8 MIMO (Downlink) in Rayleigh, Nakagami and Rician Fading Environment. Upper: Channel Capacity results Lower: Different Fading curves.

VII. Discussion on Simulation results:

Simulations are carried out in order to achieve the proposed bit rate for IMT-A which, LTE-A exceeded in their proposals submitted in release 10 [6]. Table 1 show the requirements and target for IMT-A and LTE-A respectively.

Item	Sub-category	LTE-A (4G) target	IMT-Advanced (4G) requirement
Peak Spectral Efficiency (b/s/Hz)	Downlink	30 (upto 8x8 MIMO)	15 (4x4 MIMO)
	Uplink	15 (upto 4x4 MIMO)	6.75(2x4 MIMO)

Table:1 Requirement proposed by IMT-A and target proposal of LTE-A (Rel: 10)

In our simulations, observations taken at 15 dB SNR, LTE-A's proposed targeted peak spectral efficiency with Rayleigh fading channel is achieved for both downlink (using 8 x 8 MIMO) and uplink (using 4 x 4 MIMO). Flexible spectrum usage is proposed in LTE-A. 30 b/s/Hz Spectral efficiency of having 100 MHz of flexible spectrum bandwidth will give about maximum 2.7 Gbps bit rate in this simulation. In general, it is found that actual results are moreover less than the simulated ones so 1Gbps data rate is possible in indoor (local) environment. Which satisfies the 4G requirements suggested by IMT-A. This Model is also tested for Nakagami fading environment. According to the literature studies, Nakagami distributions reduce the Rayleigh fading problems [7]. Here in our simulations it is verified and can be visualized in figures 5 and 6. Maximum spectral efficiency will be achieved in Line of Site environment which is simulated as Rician fading environment.

VII. Conclusion

In this paper the MIMO system in the indoor environment is analyzed. According to the theory given in different references and after studying various algorithms it is observed that Waterfilling algorithm is most efficient for power management in sub carriers / sub channels for improvement of spectral efficiency of the MIMO system. However, the limitations lie with requirement of the knowledge of CSI at either transmission end or at the receiver end. By adapting the power one can increase the SNR to a certain level which overall improves the bit rate and reduces the BER .

Table:2 Comparative Spectral Efficiency for Rayleigh, Nakagami and Rician fading models for 8x8 and 4x4 MIMO systems.

SNR at 15 dB	Sub-category	Rayleigh Fading	Nakagami Fading	Rician Fading
Peak Spectral Efficiency (b/s/Hz)	Downlink (8x8 MIMO)	30	42	40
	Uplink (4x4 MIMO)	16	18	22

Spectral efficiency is defined as bit per second per hertz. Performance is observed in Matlab software using Monte-carlo simulations and summarized in Table:2.This model's simulation results satisfy the requirement of 4G.Here it exceeds the requirement of IMT-A which is the targeted requirement of LTE-A.

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