Particular cases of Partition Identities

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Abstract:- Partition theory is one of the branches of mathematics subject which gives a new direction to the subject of mathematics. Not every person can live without its appreciation, which has some information and interest in it. "The discrete disciplines or classified, atomic studies of molecular and substances, proving the theory of numbers, or problems associated with all sources etc" are the main applications of the principle of division. In this article we consider some partition identities with numerical examples so that we can understand and develop and interest in this important topic partition theory.

Keywords:- Partition, distinct, congruent, modulo.

Introduction:-

Th^m 1**.(Sylvester's theorem)** "Let $A_k(n)$ be the number of partitions of n containing odd parts (repetitions allowed) with exactly k distinct parts appearing. Let $B_k(n)$ denote the number of partitions of n into distinct parts such that exactly k sequences of consecutive integers appear in each partition. Then

$$A_k(n) = B_k(n)".$$

Solution :- we prove this by particular example, Here first we consider partition of n=15 i.e.

P(15) =176 such that { 15 ,14 1 ,13 2 ,13 1 1 ,12 3 ,12 2 1 ,12 1 1 1 ,11 4 ,11 3 1 ,11 2 2 ,11 2 1 1 ,11 1 1 1 1 ,10 5 ,10 4 1 ,10 3 2 ,10 3 1 1 ,10 2 2 1 ,10 2 1 1 1 ,10 1 1 1 1 1 ,9 6 ,9 5 1 ,9 4 2 ,9 4 1 1 ,9 3 3 ,9 3 2 1 ,9 3 1 1 1 ,9 2 2 2 ,9 2 2 1 1 ,9 2 1 1 1 1 ,9 1 1 1 1 1 1 ,8 7 ,8 6 1 ,8 5 2 ,8 5 1 1 ,8 4 3 ,8 4 2 1 ,8 4 1 1 1 ,8 3 3 1 ,8 3 2 2 ,8 3 2 1 1 ,8 3 1 1 1 1 ,8 2 2 2 1 ,8 2 2 1 1 1 ,8 2 1 1 1 1 1 ,8 1 1 1 1 1 1 1 ,7 7 1 ,7 6 2 ,7 6 1 1 ,7 5 3 ,7 5 2 1 ,7 5 1 1 1 ,7 4 4 ,7 4 3 1 ,7 4 2 2 ,7 4 2 1 1 ,7 4 1 1 1 1 ,7 3 3 2 ,7 3 3 1 1 ,7 3 2 2 1 ,7 3 2 1 1 1 ,7 3 1 1 1 1 1 ,7 2 2 2 2 ,7 2 2 2 1 1 ,7 2 2 1 1 1 1 ,7 2 1 1 1 1 1 1 ,7 1 1 1 1 1 1 1 1 ,6 6 3 ,6 6 2 1 ,6 6 1 1 1 ,6 5 4 ,6 5 3 1 ,6 5 2 2 ,6 5 2 1 1 ,6 5 1 1 1 1 ,6 4 4 1 ,6 4 3 2 ,6 4 3 1 1 ,6 4 2 2 1 ,6 4 2 1 1 1 ,6 4 1 1 1 1 1 ,6 3 3 3 ,6 3 3 2 1 ,6 3 3 1 1 1 ,6 3 2 2 2 ,6 3 2 2 1 1 ,6 3 2 1 1 1 1 ,6 3 1 1 1 1 1 1 ,6 2 2 2 2 1 ,6 2 2 2 1 1 1 ,6 2 2 1 1 1 1 1 ,6 2 1 1 1 1 1 1 1 ,6 1 1 1 1 1 1 1 1 1 ,5 5 5 ,5 5 4 1 ,5 5 3 2 ,5 5 3 1 1 ,5 5 2 2 1 ,5 5 2 1 1 1 ,5 5 1 1 1 1 1 ,5 4 4 2 ,5 4 4 1 1 ,5 4 3 3 ,5 4 3 2 1 ,5 4 3 1 1 1 ,5 4 2 2 2 ,5 4 2 2 1 1 ,5 4 2 1 1 1 1 ,5 4 1 1 1 1 1 1 ,5 3 3 3 1 ,5 3 3 2 2 ,5 3 3 2 1 1 ,5 3 3 1 1 1 1 ,5 3 2 2 2 1 ,5 3 2 2 1 1 1 ,5 3 2 1 1 1 1 1 ,5 3 1 1 1 1 1 1 1 ,5 2 2 2 2 2 ,5 2 2 2 2 1 1 ,5 2 2 2 1 1 1 1 ,5 2 2 1 1 1 1 1 1 ,5 2 1 1 1 1 1 1 1 1 ,5 1 1 1 1 1 1 1 1 1 1 ,4 4 4 3 ,4 4 4 2 1 ,4 4 4 1 1 1 ,4 4 3 3 1 ,4 4 3 2 2 ,4 4 3 2 1 1 ,4 4 3 1 1 1 1 ,4 4 2 2 2 1 ,4 4 2 2 1 1 1 ,4 4 2 1 1 1 1 1 ,4 4 1 1 1 1 1 1 1 ,4 3 3 3 2 ,4 3 3 3 1 1 ,4 3 3 2 2 1 ,4 3 3 2 1 1 1 ,4 3 3 1 1 1 1 1 ,4 3 2 2 2 2 ,4 3 2 2 2 1 1 ,4 3 2 2 1 1 1 1 ,4 3 2 1 1 1 1 1 1 ,4 3 1 1 1 1 1 1 1 1 ,4 2 2 2 2 2 1 ,4 2 2 2 2 1 1 1 ,4 2 2 2 1 1 1 1 1 ,4 2 2 1 1 1 1 1 1 1 ,4 2 1 1 1 1 1 1 1 1 1 ,4 1 1 1 1 1 1 1 1 1 1 1 ,3 3 3 3 3 ,3 3 3 3 2 1 ,3 3 3 3 1 1 1 ,3 3 3 2 2 2 ,3 3 3 2 2 1 1 ,3 3 3 2 1 1 1 1 ,3 3 3 1 1 1 1 1 1 ,3 3 2 2 2 2 1 ,3 3 2 2 2 1 1 1 ,3 3 2 2 1 1 1 1 1 ,3 3 2 1 1 1 1 1 1 1 ,3 3 1 1 1 1 1 1 1 1 1 ,3 2 2 2 2 2 2 ,3 2 2 2 2 2 1 1 ,3 2 2 2 2 1 1 1 1 ,3 2 2 2 1 1 1 1 1 1 ,3 2 2 1 1 1 1 1 1 1 1 ,3 2 1 1 1 1 1 1 1 1 1 1 ,3 1 1 1 1 1 1 1 1 1 1 1 1 ,2 2 2 2 2 2 2 1 ,2 2 2 2 2 2 1 1 1 ,2 2 2 2 2 1 1 1 1 1 ,2 2 2 2 1 1 1 1 1 1 1 ,2 2 2 1 1 1 1 1 1 1 1 1 ,2 2 1 1 1 1 1 1 1 1 1 1 1 ,2 1 1 1 1 1 1 1 1 1 1 1 1 1 ,1 1 1 1 1 1 1 1 1 1 1 1 1 1 1}

let $n = 15$, $k = 3$. Then the partitions enumerated by $A_3(15)$ are

11+3+1, 9+5+1, 9+3+1+1+1, 7+5+3, 7+5+1+1+1,

7+3+3+1+1, 7+3+1+1+1+1+1, 5+5+3+1+1, 5+3+3+3+1,

5+3+3+1+1+1+1, 5 + 3 + 1 + 1 + 1 + 1 + 1 + 1 + 1}.

Hence $A_3(15) = 11$. The partitions enumerated by $B_3(15)$ are

11+3+1, 10+4+1, 9+5+1, 9+4+2, 8+6+1, 8+5+2, 8+4+2+1,

7+5+3, 7+5+2+1, 7+4+3+1, 6+5+3+1.

Hence $B_3(15) = 11$.

Th^m 1.2 (EULER-THEOREM), "The number of partitions of n into distinct parts (i.e. no part is repeated more than once) is equal to the number of partitions of n into odd parts".

Solution we prove this by particular example, Here first we consider partition of n=5 i.e.

$P(5) = 7$ i.e.{ 5 = 5 , 4 1 , 3 2 ,3 1 1 , 2 2 1 ,2 1 1 1 , 1 1 1 1 1}

Consider $P_d(n)$ = n{(1, 4), (2, 3) & (5) } = no. of partitions of non negative n (distinct parts).

⇨ $P_d(5) = 3$

& consider $P_O(n)$ = n{ (1, 1, 1, 1, 1), (1, 1, 3) & (5) }= no of partitions of non negative n (odd parts)

⇨ $P_O(5) = 3,$

Which implies our reqd. result i.e. $P_d(n) = P_O(n)$

Th^m (1ST ROGERS-RAMANUJAN). "The number of partitions of n such that the difference between any two parts is at least 2 is equal to the number of partitions of n such that each part is congruent to 1 or 4 modulo 5".

Solution . we prove this by particular example, Here first we consider partition of n=5 i.e.

P(9) =22 i.e. {9 = 9 ,8 1,7 2,7 1 1,6 3,6 2 1,6 1 1 1,5 4,5 3 1,5 2 2,5 2 1 1,5 1 1 1 1
4 4 1,4 3 2,4 3 1 1,4 2 2 1,4 2 1 1 1,4 1 1 1 1 1,3 3 3,3 3 2 1,3 3 1 1 1,3 2 2 2,
3 2 2 1 1,3 2 1 1 1 1,3 1 1 1 1 1 1,2 2 2 2 1,2 2 2 1 1 1,2 2 1 1 1 1 1,2 1 1 1 1 1 1 1
,1 1 1 1 1 1 1 1 1}

No.of partitions = 4 i.e.four partitions s.t. difference between every part .atleast two i.e.set having four member {(1, 3, 5), (1, 8),(2, 7), (3, 6)}.

Also No.of partitions = 4 i.e. four partitions s.t. every part ≡ 1 or 4(mod 5)

i.e..set having four member {(1,1,1,1,1,1,1,1,1), (1,1,1,1,1,4), (1,1,1,6), (9)}.

Th^m (2nd ROGERS-RAMANUJAN), "The number of partitions of n such that 1 is not a part and the difference between any two parts is at least 2 is equal to the number of partitions of n such that each part is congruent to 2 or 3 modulo 5".

Solutition. Here first we consider partition of n=5 i.e.

P(9) =22 i.e. {9 = 9 ,8 1,7 2,7 1 1,6 3,6 2 1,6 1 1 1,5 4,5 3 1,5 2 2,5 2 1 1,5 1 1 1 1
4 4 1,4 3 2,4 3 1 1,4 2 2 1,4 2 1 1 1,4 1 1 1 1 1,3 3 3,3 3 2 1,3 3 1 1 1,3 2 2 2,
3 2 2 1 1,3 2 1 1 1 1,3 1 1 1 1 1 1,2 2 2 2 1,2 2 2 1 1 1,2 2 1 1 1 1 1,2 1 1 1 1 1 1 1
,1 1 1 1 1 1 1 1 1}

No.of partitions of 1st type = 2 i.e.Set having two member {(2, 7), (3, 6)}.

Also for 2nd requirement , we have

No.of partitions of 2nd type = 2 i.e.Set having two member s.t.every part ≡ 2 or 3 (mod5) =n{(2, 2, 2, 3), (2, 7)}.

References

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