MATHEMATICAL MODELING IN ANCIENT INDIA

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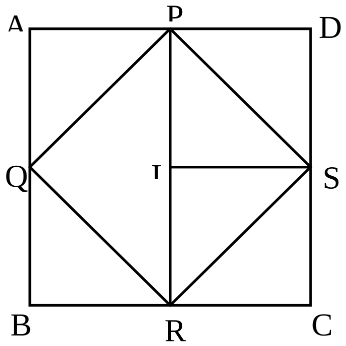
ABSTRACT

Mathematics was an independent branch of knowledge in ancient India. Ancient Indian mathematicians have contributed a lot of the development of mathematics and these are as old as the civilization of the people of India. Their works deal with different rules in mathematical contents. These rules may be put as mathematical modeling particularly in the construction of various types of sacrificial altars. These models are based on some rules given by them in the form of Sanskrit verses. When we go through these verses we find the ancient Indian Mathematicians were more advanced in mathematics than their contemporaries elsewhere in the world and they have given the birth to the modern mathematics.

Keywords : *mathematical modelling, Śulbasūtras, isoceless triangles, sthula*

In a series of different models for transforming a square into a rectangle in Śulbasūtras, Baudhāyana has given a method for transformation of a square into a rectangle. In this method a square is transformed into a rectangle such that the diagonal of the square equals the longer side of the rectangle. This method is also given by Kātyāyana ((Kśl 3.4).

According to the method the square PQRS is divided into two equal parts by drawing its diagonal. The portion PSR is given again divided into two equal parts by joining the vertex S with the mid point L of the diagonal and each is transformed to occupy the position PAQ and QBR to each side PQ and QR of the square respectively. Then PRBA is the required rectangle.



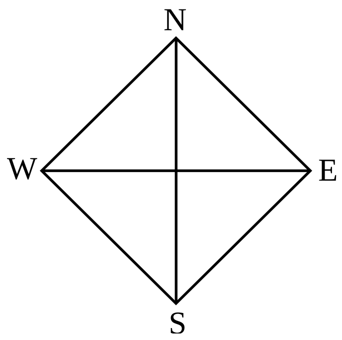
For

□ PQRS = Δ PQR + Δ PLS + Δ LRS
= Δ PQR + Δ PAQ + Δ QBR
= ▭ PABR

For drawing perpendicular bisector of a given line Kātyāyana has given the following models (rule).

तदन्तरं रज्वाभ्यस्य पाशौ कृत्वा शङ्क्वोः पाशौ प्रतिमुच्य।
दक्षिणायम्य मध्ये शंकुं निहन्ति। एवमुत्तरतः सोदीची।।

(Doubling the distance between them (the end points) on a cord and making ties one fixes the ties on the points, stretches (the cord) to the south and strikes a pin at the middle point (of the cord). Similarly to the north, that is the north-south line).



In this model we find that the process which has been adopted in Śulbasūtras is similar to the modern method of drawing the perpendicular bisector of a line. Only instead of drawing intersecting arcs to get two points equidistant from the ends of the line, isoceless triangles which are drawn on either side of the line as the base and their vertices are joint.

Aryabhatta has given a model for obtaining the area of a triangle which is as follows :

त्रिभुजस्य फलशरीरं समदलकोटीभुजार्ध संवर्गः।

(A.B. Ganitapāda 6)

"The area of a triangle is the product of the perpendicular and half the base."

According to Brahmagupta

स्थूलफलं त्रिचतुर्भुजजाहपतिबाहुयोगदलघातः।
भुजयोगार्धचतुष्टयभुजोनघातात् पदं सूक्ष्मम्।।

(Br.Sp.S1.XII.21)

"The product of half the sums of the sides and countersides of a triangle or a quadrilateral is the rough value at the area. Half the sum of the sides is severally lessend by the three or four sides, the square-root of the product of the remainders is the exact area." That is, if a, b, c, d are the sides of a quadrilateral taken in order, we have

$$\text{Area} = \frac{a+c}{2}, \frac{b+d}{2} \text{ roughly}$$

$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)} \text{ exatly}$$

$$\text{where } s = \frac{a+b+c+d}{2}$$

When one of the sides is zero, the quadrilateral becomes a triangle, whence the sthula and of a triangle

$$= \frac{\text{base}}{2} \cdot \frac{\text{sum of the sides}}{2} = \frac{a}{2} \cdot \frac{b+c}{2}$$

Area = $\frac{a}{2} \cdot \frac{b+c}{2}$; roughly

Area = $\sqrt{(s\text{-}a)\,(s\text{-}b)\,(s\text{-}c)\,(s\text{-}d)}$ exatly

Bhaskara II who flourished in the 12th century A.D. gives the rule for finding out the volume of a sphere prior to Newton (1642) in verse 41 of his Lilavati he gives the following model (rule) for finding out the volume of a

$$\text{Sphere} = \frac{\text{Surface of the sphere} \times \text{diameter}}{6} = \frac{4\pi r^2 \times 2r}{6} = \frac{4\pi r^3}{3}$$

More than five centuries after words Newton the celebrated Cambridge Professor of Mathematics was to arrive at the same result through a different method

$$\text{Volume of a sphere} = \frac{4 \times \text{diameter} \times \text{area of the circle}}{2} = \frac{4 \times 2r \times \pi r^2}{6} = \frac{4\pi r^3}{3}$$

It is not astonishing that ancient Indian Mathematician of the 12th century was able devise as accurate a model (rule) for determining the volume of a sphere as the British mathematical celebrity Newton was to lay down in the 17th century?

The Indian were the first to recognize the existence of absolutely negative quantities. By giving two models one model (idea) was "possession" for positive quantities. The conception also of opposite direction of a line, as a interpretation of +ve and –ve quantities, was not foreign to them. They observed that quadratic has always two roots. Thus Bhaskara (1150 AD) gives $x = 50$ and $x = -5$ for the roots of $x^2 - 45x = 250$.

He further says that the second value $x = -5$ in this case is not to be taken as it was not considered, confident enough, people do not approve of negative roots.

"It shows that negative roots were known but not admitted."

In the Śulbasūtras different mathematical models have been given for the construction of rectilinear figures. For the construction of such figures the squared relationship between the diagonal and the two sides of rectilinear figures has been given. The models were as follows :

(a) $n^2 + \left(\frac{3n}{4}\right)^2 = \left(\frac{5n}{4}\right)^2$

For $n = 4$, $4^2 + 3^2 = 5^2$

(b) $n^2 + \left(\frac{5}{12n}\right)^2 = \left(\frac{13}{12n}\right)^2$

For $n = 1$, $1^2 + \left(\frac{5}{12}\right)^2 = \left(\frac{13}{12}\right)^2$

(c) $1^2 + 3^2 = \left(\sqrt{10}\right)^2$

(b) $1^2 + \left(\sqrt{2}\right)^2 = \left(\sqrt{3}\right)^2$

This shows that Śulbasūtras exhibit a through familiarity with the properties of the right angled triangle or rather the properties of the sides and diagonals of figures with right angular corners. Our student know this property as Pythagoras theorem but the fact is that Pythagoras is believed to have lived from about 572 BC to 501 BC and the Śulbasūtras are, therefore, prior to him. Pythagoras was, therefore, not the first person who gave famous model named after him and there is no evidence to show that he gave any proof.

We may safely, conclude that ancient Indian Mathematicians have given models in different branches of mathematics in the form of rules which are in the form of Sanskrit verses and these models can be critically studied in terms of modern mathematical models.

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