Selection of car model through TOPSIS using intuitionistic fuzzy entropy weights

Sheikh Ahmed Hossain
Department of Mathematics,
Brahmananda Keshab Chandra College, Kolkata India.

Abstract : This paper proposes a new fuzzy TOPSIS decision making model using entropy weights for dealing with multiple attribute decision making (MADM) under intuitionistic fuzzy environment. The assignment of attribute weights is one of the most important issues in fuzzy multiple attribute decision making (MADM). In this study, attribute values are considered as intuitionistic fuzzy numbers while the information about attribute weights are completely unknown. Entropy is one of the weight measures based on objective evaluation. Non-probabilistic-type entropy measures for intuitionistic fuzzy sets have been developed and applied to measure attribute weights. Performance ratings are used to give a numerical measure of each attribute value. We have applied Intuitionistic Fuzzy TOPSIS to analyze performance matrix based on performance ratings and ranked them in order of preference.

***Index Terms* - Entropy weight; Intuitionistic Fuzzy Set; Multiple attribute decision making; TOPSIS.**

I. INTRODUCTION

It is evident that for a particular category of commodity several options are available. Some articles have criterion which are preferable to someone others may not and vice versa. It is sometimes very difficult to take decisions to such commodity. Finding the best alternative based on some criterion is an important part of decision making process. The underlying theory is called multi criterion decision making (MCDM) or multi attribute decision making (MADM). The theory consists of a set of alternatives and these are to be weighted on the basis of some criterion. A well-known classical method called TOPSIS (technique for order performance by similarity to deal solution) was developed by Hwang and Yawn [11]. The idea of this method is based on the classification of two types of criterion, called cost criterion and benefit criterion. After identifying both criterion we find shortest distant PIS (positive ideal solution) that maximizes benefit criterion and we find farthest distant from NIS (negative ideal solution) that minimizes cost criterion. The criterion we decide based on verbal queries and its answers, are qualitative in nature. These answers may carry some uncertainties. To meet these uncertaintinities we use intuitionistic fuzzy set (IFS) proposed by Atanassov [10]. Unlike fuzzy sets developed by Zadeh [2], the IFS consists of two characteristics, called degree of membership and non-membership. The idea of using IFS in this model is the higher degree of hesitancy in answering questions. To deal with vague sets Gau and Buchrer [8] developed a model where single membership function is not sufficient for an element in the set. The imprecise knowledge or information leads more hesitancy in the criterion which in turn leads to the effect on the ultimate decision making. Entropy is a measure of randomness or disorder of a system greater the randomness higher the entropy of the system. For a given decision making the entropy of a system is minimum for consistent judgement situation and maximum for inconsistent judgements situation.
In this study Shannon's entropy definition is used to find the weight of criterion. We have proposed IFS TOPSIS and Shannon weight criterion to find weight of each alternatives. Based on the weight of the weight of the alternative we rank the alternatives that is helpful to obtain decision. In section 2 IFS theory is discussed and entry of IFS is formulated. In section 3, IFS TOPSIS for making decision making in developed. In section 4, a numerical illustration is given in which five models of car are selected as alternatives and a set of three criteria.

II. INTUITIONISTIC FUZZY SETS

Let $\widetilde{\mathfrak{U}}$ be a universe of discourse. A set $\tilde{\mathcal{A}}$ on $\widetilde{\mathfrak{U}}$ is said to be an intuitionistic fuzzy set if it is of the form $\tilde{\mathcal{A}} = \{(x, \mu_{\tilde{\mathcal{A}}}(x), \nu_{\tilde{\mathcal{A}}}(x))/x \in \widetilde{\mathfrak{U}}\}$ where $\mu_{\tilde{\mathcal{A}}}: \widetilde{\mathfrak{U}} \to [0,1], \nu_{\tilde{\mathcal{A}}}: \widetilde{\mathfrak{U}} \to [0,1]$ with the condition $0 \le \mu_{\tilde{\mathcal{A}}}(x) + \nu_{\tilde{\mathcal{A}}}(x) \le 1,\ \forall\, x \in \widetilde{\mathfrak{U}}$. The number $\mu_{\tilde{\mathcal{A}}}(x)$ is called the membership degree and $\nu_{\tilde{\mathcal{A}}}(x)$ is called non-membership degree of x in $\tilde{\mathcal{A}}$, respectively. Accordingly it is obvious that in case of ordinary fuzzy set we can write in the form of intuitionistic fuzzy set $\tilde{\mathcal{A}} = \{(x, \mu_{\tilde{\mathcal{A}}}(x), 1 - \mu_{\tilde{\mathcal{A}}}(x))/\, x \in \widetilde{\mathfrak{U}}\}$. Hence it may be interpreted that a ordinary fuzzy set is a particular case of intuitionistic fuzzy set. For a crisp set, an element $x \in A$ if $\mu_A(x) = 1, \nu_A(x) = 0$ or $\mu_A(x) = 0, \nu_A(x) = 1$. We define $\pi_{\tilde{\mathcal{A}}}(x) = 1 - \mu_{\tilde{\mathcal{A}}}(x) - \nu_{\tilde{\mathcal{A}}}(x),\ x \in \widetilde{\mathfrak{U}}$. Clearly $0 \le \pi_{\tilde{\mathcal{A}}}(x) \le 1$. We can interpret $\pi_{\tilde{\mathcal{A}}}(x)$ as the hesitation margin of the element x. Thus an IFS has three membership functions, membership, non-membership, hesitation.
In other words, the application of intuitionistic fuzzy sets instead of fuzzy sets means the introduction of another degree of freedom into a set description (i.e. in addition to μA we also have νA or πA). Since the intuitionistic fuzzy sets being a generalization of fuzzy sets give us an additional possibility to represent imperfect knowledge, they can make it possible to describe many real problems in a more adequate way. Basically, intuitionistic fuzzy sets based models may be adequate in situations when we face human testimonies, opinions, etc. involving two (or more) answers of the type: "yes", "no" or "I do not know", "I am not sure", etc. Voting can be a good example of such a situation as the human voters may be divided into three groups of those who: "vote for", "vote against", "abstain or giving invalid votes". This third "grey" area is of a great interest from a point of view of, say, customer behaviour analysis, voter behaviour analysis, etc. because people from this third undecided group after proper enhancement (eg. different marketing activities) can finally become sure, i.e. become persons voting for (or against), customers wishing to buy products advertised, etc. For convenience of notation, IFSs(U) is denoted as the set of all IFSs in U.
Definition: Multiplied IFS
The multiplied IFS, denoted by $\lambda\tilde{A}$ for any positive real number λ and is defined by

$$\lambda\tilde{A} = \left\{\left(x, 1-\left(1-\mu_{\tilde{A}}(x)\right)^{\lambda}, \left(\nu_{\tilde{A}}(x)\right)^{\lambda}, x \in \tilde{\mathfrak{U}}\right)\right\}. \quad (1)$$

III. Entropy of IFS

Let us consider a discrete probability distribution$(x_i, p_i), i = 1,2,\cdots,n, 0 < p_i < 1$. The randomness of the the distribution may be characterized by a measure called entropy $H(p_1, p_2, \cdots, p_n)$ developed by Shannon [12]. The entropy function is defined by $H(p_1, p_2, \cdots, p_n) = -\Sigma_{i=1}^{n} p_i \ln(p_i)$. Entropy measure is also an uncertainty measure in a discrete distribution based on the Boltzmann entropy of classical statistical mechanics, where pi(i=1,2,3 ... ,n) are the probabilities of random variable computed from a probability mass function P. Later, De Luca and Termini [13] defined a non-probabilistic entropy formula of a fuzzy set based on Shannon's function on a finite universal set X={x1,x2, ... , xn} as

$$E_{LT}(\tilde{A}) = -k\Sigma_{i=1}^{n}[\mu_{\tilde{A}}(x_i)\ln\mu_{\tilde{A}}(x_i) + \left(1-\mu_{\tilde{A}}(x_i)\right)\ln(1-\mu_{\tilde{A}}(x_i))], k > 0. \quad (2)$$

Szmidt and Kacprzyk [14] extended De Luca and Termini axioms presenting the four definitions with regard to entropy measure on IFSs(X). Recently, Vlachos et al. [15] presented the measure of intuitionistic fuzzy entropy which was proved to satisfy the four axiomatic requirements as follows:

$$E_{LT}^{IFS}(\tilde{A}) = -\frac{1}{n\ln 2}\sum_{i=1}^{n}[\mu_{\tilde{A}}(x_i)\ln\mu_{\tilde{A}}(x_i) + \nu_{\tilde{A}}(x_i)\ln\nu_{\tilde{A}}(x_i)$$
$$-(1-\pi_{\tilde{A}}(x_i))\ln(1-\pi_{\tilde{A}}(x_i)) - \pi_{\tilde{A}}(x_i)\ln 2]. \quad (3)$$

It is noted that $E_{LT}^{IFS}(\tilde{A})$ is composed of the hesitancy degree and the fuzziness degree of the IFS A.

IV. Proposed fuzzy TOPSIS Decision Making Model

The procedures of calculation for this proposed model can be described as follows:

Step1. We construct an intuitionistic fuzzy performance matrix

$$\tilde{P} = \begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ A_2 & \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A_m & \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{m2} \end{array} \quad (4)$$

W=(w_1,w_2, ..., w_n) be the weighting vector of criteria, where $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$.

A={A_1, A_2,..., A_m} is the set of all possible alternatives among which decision makers have to choose. C={C_1,C_2,...,C_m} is set of attribute or criteria with which alternative performance are measured. x_{ij} is the rating of alternative A_i with respect to criterion C_j. w_j is the weight of criterion C_j. In our study the performances or ratings $\tilde{x}_{ij}$ which are key ingredients of decision making do contains some hesitancy. So we replace these quantities with Intuitionistic Fuzzy Numbers $(\mu_{\tilde{A}}, \nu_{\tilde{A}}, \pi_{\tilde{A}})$.

$$\tilde{P} = \begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & (\mu_{11}(C_1),\nu_{11}(C_1)) & (\mu_{12}(C_2),\nu_{12}(C_2)) & \cdots & (\mu_{1n}(C_n),\nu_{1n}(C_n)) \\ A_2 & (\mu_{21}(C_1),\nu_{21}(C_1)) & (\mu_{22}(C_2),\nu_{22}(C_2)) & \cdots & (\mu_{2n}(C_n),\nu_{2n}(C_n)) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A_m & (\mu_{m1}(C_1),\nu_{m1}(C_1)) & (\mu_{m2}(C_2),\nu_{m2}(C_2)) & \cdots & (\mu_{mn}(C_n),\nu_{mn}(C_n)) \end{array} \quad (5)$$

where $\mu_{ij}(C_j) and \nu_{ij}(C_j)$ indicate the degrees that the alternative A_i satisfies and does not satisfies the criteria C_j respectively. The intuitionistic index $\pi_{ij}(C_j) = 1 - \mu_{ij}(C_j) - \nu_{ij}(C_j)$ is such that the larger $\pi_{ij}(C_j)$ the higher a hesitation margin of the DM about the alternative A_i with respect to the criteria C_j.

***Step2*. Determine the criteria weights using the entropy-based method.**

The well-known entropy method [1, 16] can obtain the objective weights, i.e. called entropy weights. The smaller entropy values to which all alternatives A_i(i= 1, 2, ...,m) with littler similar criteria values with respect to a set of criteria can be obtained. According to the idea mentioned as above, for the decision matrix, $\tilde{P} = [\tilde{x}_{ij}]_{m\times n}$, i=1,2, ..., m, j=1,2, ..., n under intuitionistic fuzzy environment, the expected information content emitted from each criterion C_j can be measured by the entropy value, denoted as $E_{LT}^{IFS}(C_j)$, as

$$E_{LT}^{IFS}(C_j) = -\frac{1}{m\ln 2}\sum_{i=1}^{n}[\mu_{ij}(C_j)\ln\mu_{ij}(C_j)+\nu_{ij}(C_j)\ln\nu_{ij}(C_j) - (1-\pi_{ij}(C_j))\ln(1-\pi_{ij}(C_j)) - \pi_{ij}(C_j)\ln 2]. \quad (6)$$

Where j=1,2, ... , n and $\frac{1}{m\ln 2}$ is a constant which assures $0 \le E_{LT}^{IFS}(C_j) \le 1.$

Therefore, the degree of divergence (d_j) of the average intrinsic information provided by the corresponding performance ratings on criterion C_j can be defined as

$$d_j = 1 - E_{LT}^{IFS}(C_j),\ j = 1,2,....,n. \quad (7)$$

The value of d_j represents the inherent contrast intensity of criterion C_j, then the entropy weight of the *j*th criterion is

$$w_j = \frac{d_j}{\sum_{j=1}^{n} d_j}. \quad (8)$$

Step3. Construction of weighted intuitionistic fuzzy decision matrix.

A weighted intuitionistic fuzzy decision matrix $\tilde{Z}$ can be obtained by aggregating the weight vector *W* and the intuitionistic fuzzy decision matrix $\tilde{P}$ as:

$$\tilde{Z} = W^T \otimes \tilde{P} = W^T \otimes [\tilde{x}_{ij}] = [\hat{x}_{ij}],$$

where $W = (w_1, w_2, \cdots, w_j, \cdots, w_n)$, $\hat{x}_{ij} = (\hat{\mu}_{ij}, \hat{\nu}_{ij}) = (1-(1-\mu_{ij})^{w_j}, \nu_{ij}^{w_j}),\ w_j > 0.$

Step4. Determine intuitionistic fuzzy positive-ideal solution (IFPIS, A^+) and intuitionistic fuzzy negative-ideal solution (IFNIS, A^-).

In general, the evaluation criteria can be categorized into two kinds, benefit and cost. Let *G* be a collection of benefit criteria and *B* be a collection of cost criteria. According to IFS theory and the principle of classical TOPSIS method, IFPIS and IFNIS can be defined as:

$$A^+ = \left\{\left(C_j, \begin{pmatrix} (\max_i \hat{\mu}_{ij}(C_j) \mid j \in G), (\min_i \hat{\mu}_{ij}(C_j) \mid j \in B), \\ (\min_i \hat{\nu}_{ij}(C_j) \mid j \in G), (\max_i \hat{\nu}_{ij}(C_j) \mid j \in B) \end{pmatrix} \right) \middle| i = 1(1)m \right\} \quad (9)$$

$$A^- = \left\{\left(C_j, \begin{pmatrix} (\min_i \hat{\mu}_{ij}(C_j) \mid j \in G), (\max_i \hat{\mu}_{ij}(C_j) \mid j \in B), \\ (\max_i \hat{\nu}_{ij}(C_j) \mid j \in G), (\min_i \hat{\nu}_{ij}(C_j) \mid j \in B) \end{pmatrix} \right) \middle| i = 1(1)m \right\} \quad (10)$$

Step 5. Calculation of the distance measures of each alternative A_i from IFPIS and IFNIS.

We use the measure of intuitionistic Euclidean distance (refer to Szmidt and Kacprzyk [14]) to help determining the ranking of all alternatives.

$$d_{IFS}(A_i, A^+) = \sqrt{\sum_{j=1}^{n}[(\mu_{A_i}(C_j) - \mu_{A^+}(C_j))^2 + (\nu_{A_i}(C_j) - \nu_{A^+}(C_j))^2 + (\pi_{A_i}(C_j) - \pi_{A^+}(C_j))^2]} \quad (11)$$

$$d_{IFS}(A_i, A^-) = \sqrt{\sum_{j=1}^{n}[(\mu_{A_i}(C_j) - \mu_{A^-}(C_j))^2 + (\nu_{A_i}(C_j) - \nu_{A^-}(C_j))^2 + (\pi_{A_i}(C_j) - \pi_{A^-}(C_j))^2]} \quad (12)$$

Step 6. Calculate the relative closeness coefficient (*CC*) of each alternative and rank the preference order of all alternatives.

The relative closeness coefficient (CC) of each alternative with respect to the intuitionistic fuzzy ideal solutions is calculated as:

$$CC_i = \frac{d_{IFS}(A_i, A^-)}{d_{IFS}(A_i, A^+) + d_{IFS}(A_i, A^-)}. \quad (13)$$

where $0 \le CC_i \le 1$, *i=1,2, ... , m.*

The larger value of *CC* indicates that an alternative is closer to IFPIS and farther from IFNIS simultaneously. Therefore, the ranking order of all the alternatives can be determined according to the descending order of *CC* values. The most preferred alternative is the one with the highest value *CC.*

V. Illustrative Example

In this section, in order to demonstrate the calculation process of the proposed approach we illustrate with an example. Let us consider a customer who intends to buy a car. Five types of cars (alternatives) A_j (j = 1, 2, 3, 4, 5) are available. The customer takes into account three attributes to decide which car to buy: (1) C_1 is fuel economy; (2) C_2 is comfort; (3) C_3 is price, where C_1 and C_2 are benefit criteria, and C_3 is cost criterion.

Assume that the characteristics of the alternatives A_j (j = 1, 2, 3, 4, 5) are represented by the intuitionistic fuzzy perfomance rating matrix $\tilde{P} = (\tilde{r}_{ij})_{5\times 3}$, where $\tilde{r}_{ij} = (\mu_{ij}(C_j), \nu_{ij}(C_j))$ is an IFN, $\mu_{ij}(C_j)$ indicates the degree that the alternative A_i satisfies the attribute C_j, and $v_{ij}(C_j)$ indicates the degree that the alternative A_i does not satisfy the attribute C_j.

Table 1. **Intuitionistic fuzzy Performance matrix** $\tilde{P}$

	C_1	C_2	C_3
A_1	⟨0.70, 0.20⟩	⟨0.85, 0.10⟩	⟨0.30, 0.50⟩
A_2	⟨0.90, 0.05⟩	⟨0.70, 0.25⟩	⟨0.40, 0.50⟩
A_3	⟨0.80, 0.10⟩	⟨0.85, 0.10⟩	⟨0.30, 0.60⟩
A_4	⟨0.90, 0.00⟩	⟨0.80, 0.10⟩	⟨0.20, 0.70⟩
A_5	⟨0.80, 0.15⟩	⟨0.75, 0.20⟩	⟨0.50, 0.40⟩

Applying Eq.(8) the criteria weighting vector can be expressed as: W = (0.543, 0.385, 0.071).
In this case, criteria and belong to benefit criteria, and criterion belong to cost criterion. Using Eqs. (9) and (10), each alternative's IFPIS (A^+) and IFNIS (A^-) with respect to criteria can be determined as

A^+=((0.7136,0.0000)(0.5183,0.4121)(0.0157,0.9705))

A^-=((0.4799,0.4173)(0.3709,0.5864)(0.0480,0.9370))

Table 2. **The distance measure, relative closeness coefficient and ranking Alternatives.**

Alternatives	$d_{IFS}(A_i,A^+)$	$d_{IFS}(A_i,A^-)$	CC_i	Rank
A_1	0.5350	0.2585	0.3257	4
A_2	0.5351	0.3414	0.3895	3
A_3	0.3628	0.4346	0.5450	2
A_4	0.0862	0.7760	0.9000	1
A_5	0.6541	0.1912	0.2262	5

Therefore, we can see that the order of rating among five alternatives is $A_4 \succ A_3 \succ A_2 \succ A_1 \succ A_5$ where "$\succ$"indicates the relation "preferred to".

VI. CONCLUSION

In this present work, we propose an entropy-based MADM model, in which the characteristics of the alternatives are represented by IFSs. In information theory, the entropy is related with the average information quantity of a source. The main difference of this method from classical TOPSIS consists in the introduction of objective entropy weight under intuitionistic fuzzy environment.

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