$f\tilde{e}$ -continuous and $f\tilde{e}$ -irresolute Mappings

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Abstract

In this paper the concept of $f\tilde{e}$-continuous, $f\tilde{e}$-irresolute, $f\tilde{e}$-open and $f\tilde{e}$-closed mappings are introduced. Some interesting properties and characterizations of them are investigated. Interrelations among the concepts are introduced are studied.

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1. Introduction

The concept of fuzzy set was introduced by Zadeh [11] in his classical paper. Fuzzy sets have applications in many fields such as information [7] and control[9]. In 1985, Sostak [8] established a new form of fuzzy topological structure. The concept of fuzzy e-open set was introduced and studied by Seenivasan [6]. The concept of fuzzy e-$T_{\frac{1}{2}}$-space was introduced and studied by[6]. The concept of g-border, g-frontier were studied in[2]. Balasubramaniyan [1] introduced the concepts of r-fuzzy $\tilde{e}$-border, r-fuzzy $\tilde{e}$-exterior, r-fuzzy $\tilde{e}$-frontier in the sense of Sostak [8] and Ramadan [4] are introduced. In this paper the concept of $f\tilde{e}$-continuous, $f\tilde{e}$-irresolute, $f\tilde{e}$-open and $f\tilde{e}$-closed mappings are introduced. Some interesting properties and characterizations of them are investigated. Interrelations among the concepts are introduced are studied.

Throughout this paper, let X be a non-empty set, $I=[0,1]$ and $I_0=(0,1]$.

2. Preliminaries

Definition 2.1 *[5]A function $T:I^X \to I$ is called a smooth topology on X if it satisfies the following conditions:*

(i) $T(\bar{0})=T(\bar{1})=1$.

(ii) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$ for any $\mu_1, \mu_2 \in I^X$.

(iii) $T(\bigvee_{i\in\Gamma}\mu_i) \geq \bigwedge_{i\in\Gamma}T(\mu_i)$ for any $\{\mu_i\}_{i\in\Gamma} \in I^X$.

The pair (X,T) is called a smooth topological space

Remark 2.1 *Let (X,T) be a smooth topological space. Then, for each $r\in I_0, T_r=\{\mu\in I^X; T(\mu)\geq r\}$ is Chang's fuzzy topology on X.*

Proposition 2.1 *[5] Let (X,T) be a smooth topological space. For each $\lambda\in I^X, r\in I_0$ an operator $C_\tau: I^X\times I_0 \to I^X$ is defined as follows:*

$C_\tau(\lambda,r)=\bigwedge\{\mu:\mu\geq\lambda, T(\bar{1}-\mu)\geq r\}$. For $\lambda,\mu\in I^X$ and $r,s\in I_0$ it satisfies the following conditions:

(1) $C_\tau(\bar{0},r)=\bar{0}$.
(2) $\lambda \le C_\tau(\lambda,r)$.
(3) $C_\tau(\lambda,r)\vee C_\tau(\mu,r)=C_\tau(\lambda\vee\mu,r)$.
(4) $C_\tau(\lambda,r)\le C_\tau(\lambda,s)$ if $r\le s$.
(5) $C_\tau(C_\tau(\lambda,r),r)=C_\tau(\lambda,r)$.

Proposition 2.2 *[4] Let (X,T) be a smooth topological space. For each $\lambda\in I^X, r\in I_0$ an operator $I_\tau : I^X\times I_0\to I^X$ is defined as follows:*

$I_\tau(\lambda,r)=\bigvee\{\mu:\mu\le\lambda,T(\mu)\ge r\}$. For each $\lambda,\mu\in I^X$ and $r,s\in I_0$ it satisfies the following conditions:

(1) $I_\tau(\bar{1}-\lambda,r)=\bar{1}-C_\tau(\lambda,r)$
(2) $I_\tau(\bar{1},r)=\bar{1}$.
(3) $I_\tau(\lambda,r)\le\lambda$
(4) $I_\tau(\lambda,r)\wedge I_\tau(\mu,r)=I_\tau(\lambda\wedge\mu,r)$.
(5) $I_\tau(\lambda,r)\ge I_\tau(\lambda,s)$ if $r\le s$.
(6) $I_\tau(I_\tau(\lambda,r),r)=I_\tau(\lambda,r)$.

Definition 2.2 *[3] Let (X,τ) be a fuzzy topological space, $\lambda\in I^X$ and $r\in I_0$. Then*

(1) A fuzzy set λ is called r-fuzzy regular open (for short, r-fro) if $\lambda=I_\tau(C_\tau(\lambda,r),r)$.
(2) A fuzzy set λ is called r-fuzzy regular closed (for short, r-frc) if $\lambda=C_\tau(I_\tau(\lambda,r),r)$.

Definition 2.3 *[3] Let (X,τ) be a fts. For $\lambda,\mu\in I^X$ and $r\in I_0$.*

(1) The r-fuzzy δ closure of λ, denoted by $\delta-C_\tau(\lambda,r)$, and is defined by $\delta-C_\tau(\lambda,r)=\bigwedge\{\mu\in I^X \mid \mu\ge\lambda,\mu$ is r-frc $\}$.

(2) The r-fuzzy δ interior of λ, denoted by $\delta-I_\tau(\lambda,r)$, and is defined by $\delta-I_\tau(\lambda,r)=\bigvee\{\mu\in I^X \mid \mu\le\lambda,\mu$ is r-feo $\}$.

Definition 2.4 *[10] Let (X,τ) be a fuzzy topological space, $\lambda\in I^X$ and $r\in I_0$. Then*

(1) a fuzzy set λ is called r-fuzzy e open (for short, r-feo) if $\lambda\le I_\tau(\delta-C_\tau(\lambda,r),r)\vee C_\tau(\delta-I_\tau(\lambda,r),r)$.

(2) A fuzzy set λ is called r-fuzzy regular closed (for short, r-frc) if $\lambda\ge I_\tau(\delta-C_\tau(\lambda,r),r)\wedge C_\tau(\delta-I_\tau(\lambda,r),r)$.

Definition 2.5 *[10] Let (X,τ) be a fts. For $\lambda,\mu\in I^X$ and $r\in I_0$.*

(1) The r-fuzzy e closure of λ, denoted by fe-$C_\tau(\lambda,r)$, and is defined by fe-$C_\tau(\lambda,r)=\bigwedge\{\mu\in I^X \mid \mu\ge\lambda,\mu$ is r-fec $\}$.

(2) The r-fuzzy e interior of λ, denoted by fe-$I_\tau(\lambda,r)$, and is defined by fe-$I_\tau(\lambda,r)=\bigvee\{\mu\in I^X \mid \mu\le\lambda,\mu$ is r-feo $\}$.

Lemma 2.1 *[10] In a fuzzy topological space X,*

1. Any union of r-fuzzy e-open sets is a r-fuzzy e-open set.

2. Any intersection of r-fuzzy e-closed sets is a r-fuzzy e-closed set.

Definition 2.6 *[1]Let (X,T) be a smooth topological space. For $\lambda,\mu\in I^X$ and $r\in I_0$.*

(1) λ is called r-fuzzy $\tilde{e}$-open (briefly r-f$\tilde{e}$o) if fe-$I_\tau(\lambda,r)\geq\mu$, whenever $\lambda\geq\mu$ and $\mu\in I^X$ is r-fec.

(2) λ is called r-fuzzy $\tilde{e}$-closed (briefly r-f$\tilde{e}$c) if fe-$C_\tau(\lambda,r)\leq\mu$, whenever $\lambda\leq\mu$ and $\mu\in I^X$ is r-feo.

(3) The r-fuzzy $\tilde{e}$-interior of λ, denoted by f$\tilde{e}$-$I_T(\lambda,r)$ is defined as f$\tilde{e}$-$I_T(\lambda,r)=\vee\{\mu:\mu\leq\lambda,\mu$ is $r-$f$\tilde{e}o\}$.

(4) The r-fuzzy $\tilde{e}$-closure of λ, denoted by f$\tilde{e}$-$C_T(\lambda,r)$ is defined as f$\tilde{e}$-$C_T(\lambda,r)=\wedge\{\mu:\mu\geq\lambda,\mu$ is $r-$f$\tilde{e}c\}$.

Definition 2.7 *[1] Let (X,T) be a smooth topological space. For each $\lambda\in I^X$ and $r\in I_0$, the r-$f\tilde{e}$-border of λ, denoted by $f\tilde{e}$-$b_T(\lambda,r)$ is defined as $f\tilde{e}$-$b_T(\lambda,r)=\lambda$-$f\tilde{e}$-$I_T(\lambda,r)$.*

Definition 2.8 *[1] Let (X,T) be a smooth topological space. For $\lambda\in I^X$ and $r\in I_0$, the r-fuzzy $\tilde{e}$-frontier of λ, denoted by $f\tilde{e}$-$Fr_T(\lambda,r)$ is defined as $f\tilde{e}$-$Fr_T(\lambda,r)=f\tilde{e}$-$C_T(\lambda,r)-f\tilde{e}$-$I_T(\lambda,r)$.*

Definition 2.9 *[1] Let (X,T) be a smooth topological space. For $\lambda,\mu\in I^X$ and $r\in I_0$, the r-fuzzy $\tilde{e}$-exterior of λ, denoted by $f\tilde{e}$-$Ext_T(\lambda,r)$ is defined as $f\tilde{e}$-$Ext_T(\lambda,r)=f\tilde{e}$-$I_T(\bar{1}-\lambda,r)$.*

3. Properties of fuzzy $\tilde{e}$-continuous and fuzzy $\tilde{e}$-irresolute mappings

In this section, the properties of fuzzy $\tilde{e}$-irresolute and fuzzy $\tilde{e}$-continuous mappings are established.

Definition 3.1 *Let (X,T) and (Y,S) be any two smooth topological spaces. Let $f:(X,T)\to(Y,S)$ be any mapping. Then*

(1) f is called fuzzy $\tilde{e}$-open if $f(\mu)$ is a r-f$\tilde{e}$o set for each r-f$\tilde{e}$o set $\mu\in I^X, r\in I_0$.

(2) f is called fuzzy $\tilde{e}$-closed if $f(\mu)$ is a r-f$\tilde{e}$c set for each r-f$\tilde{e}$c set $\mu\in I^X, r\in I_0$.

(3) f is called fuzzy $\tilde{e}$-continuous if $f^{-1}(\mu)$ is a r-f$\tilde{e}$c for every r-f$\tilde{e}$c set $\mu\in I^Y, r\in I_0$.

(4) f is called fuzzy $\tilde{e}$-irresolute if $f^{-1}(\mu)$ is a r-f$\tilde{e}$c for each r-f$\tilde{e}$c set $\mu\in I^Y, r\in I_0$.

Proposition 3.1 *Let (X,T) and (Y,S) be any two smooth topological spaces. Let $f:(X,T)\to(Y,S)$ be a function. Then the following statements are equivalent.*

(1) f is a fuzzy $\tilde{e}$-irresolute function.

(2) $f(f\tilde{e}$-$C_T(\lambda,r))\leq f\tilde{e}$-$C_S(f(\lambda),r)$, for every $\lambda\in I^X, r\in I_0$.

(3) $f\tilde{e}$-$C_T(f^{-1}(\mu),r)\leq f^{-1}(f\tilde{e}$-$C_S(\mu,r))$, for every $\lambda\in I^Y, r\in I_0$.

Proof. (1) $\Rightarrow$ (2): Let f be a fuzzy $\tilde{e}$-irresolute function and let $\lambda\in I^X$. Then f$\tilde{e}$-$C_S(f(\lambda),r)$

is a r-f $\tilde{e}$ c set. By (1), $f^{-1}(f\tilde{e}\text{-}C_S(f(\lambda),r))$ is a r-f $\tilde{e}$-closed set. Thus f $\tilde{e}$ - $C_T(f^{-1}(f\tilde{e}\text{-}C_S(f(\lambda),r),r)) = (f^{-1}(f\tilde{e}\text{-}C_S(f(\lambda),r)))$. Now, $\lambda \leq f^{-1}(f(\lambda))$. Therefore, f $\tilde{e}$ - $C_T(\lambda,r) \leq f\tilde{e}\text{-}C_T(f^{-1}(f(\lambda)),r) \leq f\tilde{e}\text{-}C_T(f^{-1}(f\tilde{e}\text{-}C_S(f(\lambda),r)),r) = f^{-1}(f\tilde{e}\text{-}C_S(f(\lambda),r))$. Hence, $f(f\tilde{e}\text{-}C_T(\lambda,r)) \leq f\tilde{e}\text{-}C_S(f(\lambda),r)$.

(2) $\Rightarrow$ (3): Let $\mu \in I^Y$, then $f^{-1}(\mu) \in I^X$. By (2), $f(f\tilde{e}\text{-}C_T(f^{-1}(\mu),r)) \leq f\tilde{e}\text{-}C_S(f(f^{-1}(\mu)),r) \leq f\tilde{e}\text{-}C_S(\mu,r)$. Hence $f\tilde{e}\text{-}C_T(f^{-1}(\mu),r) \leq f^{-1}(f\tilde{e}\text{-}C_S(\mu,r))$.

(3) $\Rightarrow$ (1): Let $\gamma \in I^Y$, be a r - $f\tilde{e}$-closed set. Then $f\tilde{e}\text{-}C_S(\gamma,r) = \gamma$. By (3) f $\tilde{e}$ - $C_S(f^{-1}(\gamma),r) \leq f^{-1}(f\tilde{e}\text{-}C_S(\gamma,r)) = f^{-1}(\gamma)$. But $f^{-1}(\gamma) \leq f\tilde{e}\text{-}C_T(f^{-1}(\gamma),r)$. Therefore, $f^{-1}(\gamma) = f\tilde{e}\text{-}C_T(f^{-1}(\gamma),r)$. Hence $f^{-1}(\gamma)$ is a r - $f\tilde{e}$-closed set. Thus f is fuzzy $\tilde{e}$-irresolute function.

Propoition 3.2 *Let (X,T) and (Y,S) be any two smooth topological spaces. A mapping $f:(X,T) \to (Y,S)$ is a $f\tilde{e}$-closed iff $f\tilde{e}\text{-}C_S(f(\lambda),r) \leq f(f\tilde{e}\text{-}C_T(\lambda,r))$ for each $\lambda \in I^X$ and $r \in I_0$.*

Proof. Let $\lambda \in I^X$ be a r - $f\tilde{e}$-closed set. Suppose that $f\tilde{e}\text{-}C_S(f(\lambda),r) \leq f(f\tilde{e}\text{-}C_T(\lambda,r))$. Now $f\tilde{e}\text{-}C_T(\lambda,r) = \lambda$. This implies $f\tilde{e}\text{-}C_S(f(\lambda),r) \leq f(f\tilde{e}\text{-}C_T(\lambda,r)) \leq f(\lambda)$. But $f(\lambda) \leq f\tilde{e}\text{-}C_S(f(\lambda),r)$. Hence, $f\tilde{e}\text{-}C_S(f(\lambda),r) = f(\lambda)$. Therefore f is $f\tilde{e}$-closed.

Conversely, let f be an $f\tilde{e}$-closed function. Let $\lambda \in I^X$. Then $f\tilde{e}\text{-}C_T(\lambda,r)$ is r - $f\tilde{e}$-closed. Therefore, $f(f\tilde{e}\text{-}C_T(\lambda,r))$ is r - $f\tilde{e}$-closed. Now $\lambda \leq f\tilde{e}\text{-}C_T(\lambda,r)$. This implies $f(\lambda) \leq f(f\tilde{e}\text{-}C_T(\lambda,r))$. Hence $f\tilde{e}\text{-}C_S(f(\lambda),r) \leq f\tilde{e}\text{-}C_S(f(f\tilde{e}\text{-}C_S(\lambda,r)),r) = f(f\tilde{e}\text{-}C_T(\lambda,r))$. Therefore, $f\tilde{e}\text{-}C_S(f(\lambda),r) \leq f(f\tilde{e}\text{-}C_T(\lambda,r))$.

Proposition 3.3 *Let (X,T) and (Y,S) be any two smooth topological spaces. Let $f:(X,T) \to (Y,S)$ be a bijective function. Then the following statements are equivalent:*

(1) f and f^{-1} are fuzzy $\tilde{e}$-irresolute functions.

(2) f is $f\tilde{e}$-continuous and $f\tilde{e}$-open.

(3) f is $f\tilde{e}$-continuous and $f\tilde{e}$-closed.

(4) $f\tilde{e}\text{-}C_S(f(\lambda),r) = f(f\tilde{e}\text{-}C_T(\lambda,r))$, for each $\lambda \in I^X$, $r \in I_0$.

Proof. (1) $\Rightarrow$ (2): Let $\lambda \in I^Y$, be a r-fuzzy $\tilde{e}$-closed set and hence r - $f\tilde{e}$-closed. Since f is fuzzy $\tilde{e}$-irresolute, $f^{-1}(\lambda)$ is r - $f\tilde{e}$-closed. Hence f is $f\tilde{e}$-continuous. Let $\mu \in I^Y$, be a r - $f\tilde{e}$-open set. Since f^{-1} is fuzzy $\tilde{e}$-irresolute, $(f^{-1})^{-1}(\mu) = f(\mu)$ is r - $f\tilde{e}$-open. Hence f is $f\tilde{e}$-open.

(2) $\Rightarrow$ (3): Let $\mu \in I^X$, be a r - $f\tilde{e}$-closed set. Then $\bar{1} - \mu$ is r - $f\tilde{e}$-open. Since f is $f\tilde{e}$-open, $f(\bar{1} - \mu)$ is r - $f\tilde{e}$-open. But $f(\bar{1} - \mu) = \bar{1} - f(\mu)$. This implies that $f(\mu)$ is r - $f\tilde{e}$-closed. Hence f is $f\tilde{e}$-closed.

(3) $\Rightarrow$ (4): Let $\lambda \in I^X$, by Proposition **Error! Reference source not found.**(2), $f(f\tilde{e}\text{-}C_T(\lambda,r)) \leq f\tilde{e}\text{-}C_S(f(\lambda),r)$. By Proposition 3.2, $f\tilde{e}\text{-}C_S(f(\lambda,r)) \leq f(f\tilde{e}\text{-}C_T(\lambda,r))$. Hence $f\tilde{e}\text{-}C_S(f(\lambda),r) = f(f\tilde{e}\text{-}C_T(\lambda,r))$.

(4) $\Rightarrow$ (1): Let $\lambda \in I^X$, by (4), $f\tilde{e}\text{-}C_S(f(\lambda),r) = f(f\tilde{e}\text{-}C_T(\lambda,r))$. Then $f(f\tilde{e}\text{-}C_T(\lambda,r)) \leq f\tilde{e}\text{-}C_S(f(\lambda),r)$, implies f is fuzzy $\tilde{e}$-irresolute function by Proposition 3.1 Let $\mu \in I^X$ be a r - $f\tilde{e}$

-closed. Then $f\tilde{e}\text{-}C_S(\mu,r)=\mu$. Then $f(f\tilde{e}\text{-}C_T(\mu,r))=f(\mu)$. By (4), $f\tilde{e}\text{-}C_S(f(\mu),r)=f(f\tilde{e}\text{-}C_T(\mu,r))=f(\mu)$. Hence $f(\mu)$ is $r\text{-}f\tilde{e}$-closed. Therefore f^{-1} is fuzzy $\tilde{e}$-irresolute.

Proposition 3.4 *Let (X,T) and (Y,S) be any two smooth topological spaces. Let $f:(X,T)\to(Y,S)$ be a fuzzy $\tilde{e}$-irresolute function. Then $f\tilde{e}\text{-}b_T(f^{-1}(\lambda,r))=\bar{0}$, for a $r\text{-}f\tilde{e}$-open set $\lambda\in I^Y$.*

Proof. Let $\lambda\in I^Y$ be a $r\text{-}f\tilde{e}$-open set. Since f is fuzzy $\tilde{e}$-irresolute function, $f^{-1}(\lambda)$ is a $r\text{-}f\tilde{e}$-open set. Then $f\tilde{e}\text{-}I_T(f^{-1}(\lambda,r)=f^{-1}(\lambda)$. Now, $f\tilde{e}\text{-}b_T(f^{-1}(\lambda),r)=f^{-1}(\lambda)-f\tilde{e}\text{-}I_T(f^{-1}(\lambda),r)=f^{-1}(\lambda)-f^{-1}(\lambda)=\bar{0}$.

4. Interrelations

The interrelations among the concepts of r-fuzzy $\tilde{e}$-border, r-fuzzy $\tilde{e}$-exterior, r-fuzzy $\tilde{e}$-frontier are established and studied with necessary examples.

Definition 4.1 *A smooth fuzzy topological space (X,T) is called $f\tilde{e}\text{-}T_{\frac{1}{2}}$ space if every $r\text{-}f\tilde{e}$-closed set $\lambda\in I^X$ is $r\text{-}f\tilde{e}$ closed.*

Proposition 4.1 *Let (X,T) and (Y,S) be any two smooth topological spaces. Let $f:(X,T)\to(Y,S)$ be a $f\tilde{e}$-continuous mapping. Then for any $r\text{-}f\tilde{e}$-closed set $\lambda\in I^Y$, $f\tilde{e}\text{-}b_T(f^{-1}(\lambda),r)=f\tilde{e}\text{-}Fr_T(f^{-1}(\lambda),r)$.*

Proof. Let $\lambda\in I^Y$ be a r-fuzzy $\tilde{e}$-closed set. Since f is a $f\tilde{e}$-continuous, $f^{-1}(\lambda)$ is $r\text{-}f\tilde{e}$-closed set. Then $f\tilde{e}\text{-}C_T(f^{-1}(\lambda),r)=f^{-1}(\lambda)$. Now, $f\tilde{e}\text{-}b_T(f^{-1}(\lambda),r)=(f^{-1}(\lambda))-(f\tilde{e}\text{-}I_T(f^{-1}(\lambda),r))=f\tilde{e}\text{-}C_T(f^{-1}(\lambda),r)-(f\tilde{e}\text{-}I_T(f^{-1}(\lambda),r))=f\tilde{e}\text{-}Fr_T(f^{-1}(\lambda),r)$. Hence, $f\tilde{e}\text{-}b_T(f^{-1}(\lambda),r)=f\tilde{e}\text{-}Fr_T(f^{-1}(\lambda),r)$.

Proposition 4.2 *Let (X,T) and (Y,S) be any two smooth fuzzy topological spaces. Let $f:(X,T)\to(Y,S)$ be a mapping. Then for $\lambda\in I^Y$*

$$f\tilde{e}\text{-}Ext_T(f^{-1}(\lambda),r)\le f\tilde{e}\text{-}C_T(\bar{1}-f^{-1}(\lambda),r).$$

Proof. Let $\lambda\in I^Y$. Now, $f\tilde{e}\text{-}Ext_T(f^{-1}(\lambda),r)=f\tilde{e}\text{-}I_T(\bar{1}-f^{-1}(\lambda),r)\le f\tilde{e}\text{-}C_T(\bar{1}-f^{-1}(\lambda),r)$.

Proposition 4.3 *Let (X,T) be a $f\tilde{e}\text{-}T_{\frac{1}{2}}$ space. Let $\lambda\in I^X$ be a $r\text{-}f\tilde{e}$-closed set. Then the following statements hold:*

(1) $f\tilde{e}\text{-}b_T(\lambda,r)=f\tilde{e}\text{-}Fr_T(\lambda,r)$.

(2) $f\tilde{e}\text{-}Ext_T(f(\lambda),r)=\bar{1}-\lambda$.

Proof. Let $\lambda\in I^X$ be a $r\text{-}f\tilde{e}$-closed set. Since (X,T) is a $f\tilde{e}\text{-}T_{\frac{1}{2}}$ space, λ is $r\text{-}f\tilde{e}$ closed. This implies $\lambda=f\tilde{e}\text{-}C_T(\lambda,r)$. Now, $f\tilde{e}\text{-}b_T(\lambda,r)=\lambda-f\tilde{e}\text{-}I_T(\lambda,r)=f\tilde{e}\text{-}C_T(\lambda,r)-f\tilde{e}\text{-}I_T(\lambda,r)=f\tilde{e}\text{-}Fr_T(\lambda,r)$. $f\tilde{e}\text{-}Ext_T(\lambda,r)=f\tilde{e}\text{-}I_T(\bar{1}-\lambda,r)=\bar{1}-f\tilde{e}\text{-}C_T(\lambda,r)=\bar{1}-\lambda$.

Proposition 4.4 *Let (X,T) and (Y,S) be any two smooth topological spaces. Let $f:(X,T)\to(Y,S)$ be a $f\tilde{e}$-irresolute function and (X,T) is a $f\tilde{e}T_{\frac{1}{2}}$ space. Then for a r-$f\tilde{e}$-closed set $\lambda\in I^Y$ and $r\in I_0$, the following statements hold:*

(1) $f\tilde{e}\text{-}b_T(f^{-1}(\lambda),r)=f\tilde{e}\text{-}Fr_T(f^{-1}(\lambda),r)$.

(2) $f\tilde{e}\text{-}Ext_T(f^{-1}(\lambda),r)=\bar{1}-f^{-1}(\lambda)$.

Proof. Let $\lambda\in I^Y$ be a r-$f\tilde{e}$-closed set. Since f is a $f\tilde{e}$-irresolute, $f^{-1}(\lambda)$ is a r-$f\tilde{e}$-closed. Since (X,T) is a $f\tilde{e}\text{-}T_{\frac{1}{2}}$, $f^{-1}(\lambda)$ is a r-$f\tilde{e}$-closed. This implies $f\tilde{e}\text{-}C_T(f^{-1}(\lambda),r)=f^{-1}(\lambda)$. Now $f\tilde{e}$ - $b_T(f^{-1}(\lambda),r)=f^{-1}(\lambda)-f\tilde{e}$ - $I_T(f^{-1}(\lambda),r)=f\tilde{e}$ - $C_T(f^{-1}(\lambda),r)-f\tilde{e}$ - $I_T(f^{-1}(\lambda),r)=f\tilde{e}$ - $Fr_T(f^{-1}(\lambda),r)$ and $f\tilde{e}\text{-}Ext_T(f^{-1}(\lambda),r)=f\tilde{e}\text{-}I_T(\bar{1}-f^{-1}(\lambda),r)=\bar{1}-f\tilde{e}\text{-}C_T(f^{-1}(\lambda),r)=\bar{1}-f^{-1}(\lambda)$.

Proposition 4.5 *Let (X,T) and (Y,S) be any two smooth topological spaces. Let $f:(X,T)\to(Y,S)$ be a $f\tilde{e}$-closed mapping and (Y,S) be a $f\tilde{e}\text{-}T_{\frac{1}{2}}$ space. Then for a r-$f\tilde{e}$-closed set $\lambda\in I^X$ and $r\in I_0$ the following statements hold:*

(1) $f\tilde{e}\text{-}b_S(f(\lambda),r)=f\tilde{e}\text{-}Fr_S(f(\lambda),r)$.

(2) $f\tilde{e}\text{-}Ext_S(f(\lambda),r)=\bar{1}-f(\lambda)$.

Proof. Let $\lambda\in I^Y$ be a r-$f\tilde{e}$-closed set. Since f is r-$f\tilde{e}$-closed set, $f(\lambda)$ is r-$f\tilde{e}$-closed. Since (Y,S) is $f\tilde{e}\text{-}T_{\frac{1}{2}}$-space, $f(\lambda)$ is r-$f\tilde{e}$-closed. This implies $f\tilde{e}\text{-}C_T(f(\lambda),r)=f(\lambda)$. Now $f\tilde{e}\text{-}b_S(f(\lambda),r)=f(\lambda)-f\tilde{e}\text{-}I_S(f(\lambda),r)=f\tilde{e}\text{-}C_S(f(\lambda),r)-f\tilde{e}\text{-}I_S(f(\lambda),r)=f\tilde{e}\text{-}Fr_S(f(\lambda),r)$ and $f\tilde{e}\text{-}Ext_S(f(\lambda),r)=f\tilde{e}\text{-}I_S(\bar{1}-f(\lambda),r)=\bar{1}-f\tilde{e}\text{-}C_S(f(\lambda),r)=\bar{1}-f(\lambda)$.

Proposition 4.6 *Let (X,T), (Y,S) and (Z,R) be any three smooth topological spaces. Let $f:(X,T)\to(Y,S)$ and $g:(Y,S)\to(Z,R)$ be $f\tilde{e}$-irresolute mappings. If (X,T) is a $f\tilde{e}\text{-}T_{\frac{1}{2}}$ space, then*

(1) $f\tilde{e}\text{-}b_T((g\circ f)^{-1}(\lambda),r)=f\tilde{e}\text{-}Fr_T((g\circ f)^{-1}(\lambda),r)$.

(2) $f\tilde{e}\text{-}Ext_T((g\circ f)^{-1}(\lambda),r)=\bar{1}-(g\circ f)^{-1}(\lambda)$.

Proof. Let $\lambda\in I^Z$ be a r-$f\tilde{e}$-closed set. Since g is a $f\tilde{e}$-irresolute, $g^{-1}(\lambda)$ is r-$f\tilde{e}$-closed. Since (X,T) is $f\tilde{e}\text{-}T_{\frac{1}{2}}$ space, $(g\circ f)(\lambda)=f^{-1}(g^{-1}(\lambda))$ is r-$f\tilde{e}$ closed. This implies $f\tilde{e}$-$C_T((g\circ f)^{-1}(\lambda),r)=(g\circ f)^{-1}(\lambda)$. Now $f\tilde{e}\text{-}b_T((g\circ f)^{-1}(\lambda),r)=((g\circ f)^{-1}(\lambda)-f\tilde{e}\text{-}I_T((g\circ f)^{-1}(\lambda),r)=f\tilde{e}$ - $C_T((g\circ f)^{-1}(\lambda),r)-f\tilde{e}$ - $I_T((g\circ f)^{-1}(\lambda),r)=f\tilde{e}$ - $Fr_T((g\circ f)^{-1}(\lambda),r)$ and $f\tilde{e}$ - $Ext_T((g\circ f)^{-1}(\lambda),r)=\bar{1}-f\tilde{e}\text{-}C_T((g\circ f)^{-1}(\lambda),r)=\bar{1}-((g\circ f)^{-1}(\lambda)$.

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