

Porosity variations on groundwater with and without chemical reaction

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Abstract: A two-dimensional particle size variations has been developed for the purpose of studying variably saturated water flow and solute transport in groundwater, with and without the effect of chemical reaction. The formulation of differential equations are non-dimensionalized and then solved analytically under appropriate initial and boundary conditions using the perturbation method. Effects of Schmidt number, Darcy number, reaction rate parameter and particle parameter on the velocity and concentration profiles are discussed and presented graphically.

Key words: Groundwater, Particle size, Contaminants.

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1 Introduction

The increasing pollution of groundwater aquifers due to a variety of surficial contaminant discharges has become a major environmental issue in recent years. Concerns about health risks due to polluted aquifers have spawned a number of emergency programs to study the movement and the fate of pollutants leaking from various waste disposal sites and from septic tanks into groundwater (Canter and Knox, (1985)). Most contaminant transport models assume that the migration of the miscible phase is mainly due to forced advection by hydraulic gradients, dispersion and possible non conservative reactions (Bear and Verrujit (1987)).

Groundwater contains solids and biomass (McDowell-Boyer et al. 1986), which has been linked with transport of pollutants (McCarthy and Zachara 1989) like pesticides and radio active elements, and of pathogenic bacteria and viruses (Schijven 2001, Foppen 2007).

Particle(grain) size is an important parameter in groundwater flow. Particle size and porosity are interrelated since particle shape affects the pore size distribution and pore shapes. The shape of individual particles affects the pore size distribution and hence the flow characteristics and the particle composition is concerned with the pore size, shape and the porosity. In addition to affecting the pore size distribution, particle shape determines the total surface area available for sorption and

other surface reactions affecting the fate and transport of groundwater contaminants. For filters used in water and waste-water treatment, particle size determines the total surface area and pore geometry for contaminant removal by straining, sedimentation and diffusion. The pore size distribution of a contaminant is often inferred from its particle size distribution (Sperry and Pierce, 1995).

The pore size distribution affects the solute dispersion, which is greater for a broader pore-size distribution. It affects the sorption of solutes in a complex way. The smaller pores are associated with longer residence times and greater relative surface area, but most solutes go quickly through the large pores with minimal opportunity to react. Interchange between fast transporting and slow transporting portions of the pore space is a vital aspect of contaminant transport in groundwater (Nimmo, 2004).

Droppo et al., (2000) examined the influence of floc size, density and porosity on the transport of sediment and associated contaminants. Robinson and Friedman (2002) observed the effects of particle shape and particle size distribution on avalanching of granular media.

Many studies have been devoted to understand the transport and retention of particle transport in saturated porous media (Lecoanet et al., (2004); Moshe et al., (2010) and Wang et al., (2012)). Urumovic and Urumovic Sr (2014) analyzed the effective porosity and grain size relations in permeability functions. Most of the studies have focused on spherical particles in model porous media such as clean quartz sand or spherical glass beads. In reality, nanoparticles can be found in many shapes such as rod-shape and aggregates may form non-spherical colloids. The differences in the shape could greatly change the predicted transport and retention of these nanoparticles in the groundwater flow. These studies have helped to gain some understanding of various factors influencing transport and attachment including changes in ionic strength, pH value and particle size.

The aim of this paper is to study the role of particle size variations on groundwater contaminants with and without the effects of chemical reaction. The variations in particle size is described by the exponential empirical law. The results of our research are not only important for understanding the transport of particles as carriers of pollutants and pathogenic micro organisms, but also for related fields, such as for drinking water production, but also for irrigation, dewatering of building excavations (Reddi 1997, Powers 1992). The description of the physical model as well as the conservation equations which govern the phenomenon are presented. These equations are solved by perturbation technique to obtain velocity and concentration profiles for various physical parameters. The effects of different pertinent parameters are presented graphically.

2 Mathematical Formulation

The physical model and co-ordinate system are depicted in figure 1. We consider the balance of mass, momentum and concentration for unsteady two-dimensional flow in a fluid (groundwater) saturated medium of particle size variations. The groundwater is assumed to be contaminated. In cartesian coordinate system, we consider laminar flow of a viscous, incompressible fluid(groundwater) of uniform cross section h , lying below the porous layer and above the impermeable layer.

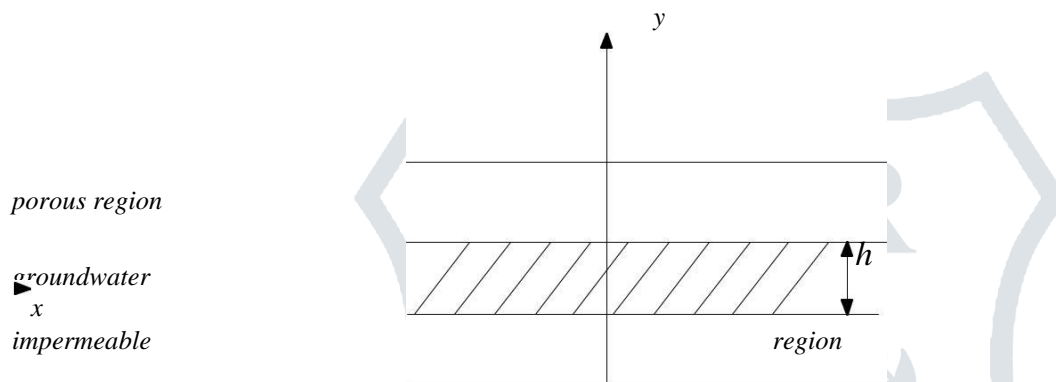


Figure 1. Physical Configuration

Using standard symbols, the governing equations describing the contaminant transport are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\theta \frac{\partial u}{\partial t} + \theta \frac{\partial u}{\partial x} + \theta \frac{\partial u}{\partial y} + \theta \frac{\partial u}{\partial c} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\theta}{v} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \frac{k_p}{v} u, \tag{2}$$

$$\theta \frac{\partial v}{\partial t} + \theta \frac{\partial v}{\partial x} + \theta \frac{\partial v}{\partial y} + \theta \frac{\partial v}{\partial c} = - \frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\theta}{v} \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} - \frac{k_p}{v} v, \tag{3}$$

$$\theta \frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = \theta D \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right) - kc. \tag{4}$$

where u and v are the velocities of the fluid particle along the x and y directions respectively, ρ is the density of the fluid, v is the kinematic viscosity of fluid, k_p represents the medium permeability, c is the solutal concentration, k is the chemical reaction coefficient and D is the mass diffusivity and $\theta = \theta_s a_1 e^{-a_2 y}$ is the porosity where, θ_s is the mean porosity, a_1 and a_2 are empirical constants, which depend on the packing of particles and d_p is the particle diameter.

The complete set of governing equations representing the mathematical model is given by equations (1) to (4). In accordance with the problem description, the initial and boundary conditions are

presented as

$$u = v = 0, \quad c = 0, \quad \text{for } t = 0 \tag{5}$$

$$u = 0, \quad v = 0 \quad \text{at } y = 0 \tag{6}$$

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$$\frac{d}{dy} \left(\frac{u}{\alpha} \right) = p \quad k_p u, \quad v = v_a \rho e^{(\lambda x + \omega t)} \quad \text{at } y = h \tag{7}$$

$$\frac{\partial c}{\partial y} = 0 \quad \text{at } y = 0 \tag{8}$$

$$c = c_a \left(1 + \rho e^{(\lambda x + \omega t)} \right) \quad \text{at } y = h \tag{9}$$

where α is the slip parameter, λ is the stream-wise wave number, ω is the frequency parameter, ρ is the perturbation parameter, v_a and c_a are constants.

The variables are transformed into the dimensionless quantities defined as,

$$u^* = \frac{u}{v_0}, \quad v^* = \frac{v}{v_0}, \quad x^* = \frac{x}{v}, \quad y^* = \frac{y v_0}{v}, \quad p^* = \frac{p}{\rho v_0^2}, \quad t^* = \frac{t v_0^2}{v}, \quad C^* = \frac{c}{c_0} \tag{10}$$

where v_0 and c_0 are the characteristic velocity and characteristic concentration respectively.

The dimensionless form of the governing equations and boundary conditions are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{10}$$

$$\frac{\theta}{1} \frac{\partial u}{\partial t} + \frac{u}{\theta} \frac{\partial u}{\partial x} + \frac{v}{\theta^3} \frac{\partial u}{\partial y} - u \frac{\partial \theta}{\partial y} = - \frac{\partial p}{\partial x} + \frac{1}{\theta} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \sigma^2 u, \tag{11}$$

$$\theta \frac{\partial t}{\partial t} + \frac{\partial x}{\partial x} + \frac{\theta^3}{\partial c} \frac{\partial y}{\partial c} - v \frac{\partial y}{\partial c} = - \frac{\partial y}{\theta} + \frac{\partial x^2}{\partial^2 c} + \frac{\partial y^2}{\partial^2 c} - \sigma^2 v, \tag{12}$$

$$\theta \frac{\partial t}{\partial t} + u \frac{\partial x}{\partial x} + v \frac{\partial y}{\partial y} = Sc \left(\frac{\partial x^2}{\partial x^2} + \frac{\partial y^2}{\partial y^2} \right) - KC, \tag{13}$$

where $\sigma = \sqrt{\frac{v}{k_p v_0}}$ is the porous parameter, $Sc = \frac{v}{D}$ is the Schmidt number and $K = \frac{vk}{v_0}$ is the chemical reaction parameter. The asterisks have been

omitted from the dimensionless governing equations (10) - (13) for the sake of convenience.

Accordingly the non-dimensional boundary conditions are:

$$u = 0, \quad v = 0, \quad C = 0 \quad \text{at } t = 0$$

$$u = 0, v = 0 \text{ at } y = 0 \tag{14}$$

$$\frac{du}{dy} = \alpha\sigma u, v = v_a\varrho e^{(\lambda x + \omega t)} \text{ at } y = H, \tag{15}$$

$$\frac{dC}{dy} = 0 \text{ at } y = 0, \tag{16}$$

$$C = C_a(1 + \varrho e^{\lambda x + \omega t}) \text{ at } y = H, \tag{17}$$

where $H = \frac{h\nu_0}{v}$ is the non-dimensionalized height.

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3 Solution Method

The governing equations of momentum (11) and (12), along with the species equation (13) are solved subject to the boundary conditions (14) to (17) for the velocity and concentration distributions. Since the equations are two-dimensional and nonlinear, the solutions are expressed through the perturbation method. Adopting this technique, solutions for velocity and concentration are assumed in the form (assuming perturbation parameter ϱ usually small)

$$\begin{aligned} u(x, y, t) &= u_0(y) + \varrho e^{(\lambda x + \omega t)} u_1(y) + o(\varrho^2), \\ v(x, y, t) &= \varrho e^{(\lambda x + \omega t)} v_1(y) + o(\varrho^2), \\ p(x, y, t) &= p_0(x) + \varrho e^{(\lambda x + \omega t)} p_1(y) + o(\varrho^2), \\ C(x, y, t) &= C_0(y) + \varrho e^{(\lambda x + \omega t)} C_1(y) + o(\varrho^2), \end{aligned} \tag{18}$$

where the perturbations u_1, v_1, p_1 and C_1 are small compared with the zeroth order quantities. Equations (10) to (13) yield the following non-dimensional equations.

Zeroth-order equations

$$\frac{1}{\theta} \frac{d^2 u_0}{dy^2} - \sigma^2 u_0 + g_1 = 0 \tag{19}$$

$$\frac{\theta}{Sc} \frac{d^2 C_0}{dy^2} - KC_0 = 0 \tag{20}$$

First-order equations

$$\frac{1}{\theta} \frac{\partial^2 u}{\partial y^2} - \frac{v_1}{\theta_2} \frac{\partial u_0}{\partial y} + \frac{u_0 v_1}{\theta_3} \frac{\partial \theta}{\partial y} - \frac{\lambda u_0 u_1}{\theta_2} - \frac{w}{\theta} \frac{\lambda^2}{\theta} + \sigma^2 u_1 - \lambda p_1 = 0, \tag{21}$$

$$\frac{1}{\theta} \frac{\partial^2 v_1}{\partial y^2} - \frac{\omega}{\theta} \frac{\lambda^2}{\theta} + u_0 \lambda + \frac{1}{\sigma} v_1 - p_1 = 0, \tag{22}$$

$$\frac{\theta \partial^2 C}{Sc \partial y^2} - v_1 \frac{\partial C}{\partial y} + \frac{\theta}{Sc \lambda^2} - \theta \omega + u_0 \lambda - K C_1 = 0. \tag{23}$$

In view of equation(18) the relevant boundary conditions are also splitted into the following two parts.

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Zeroth-order boundary conditions

$$u_0 = 0 \text{ at } y = 0; \quad \frac{du_0}{dy} = \alpha \sigma u_0 \text{ at } y = H, \tag{24}$$

$$\frac{dC_0}{dy} = 0 \text{ at } y = 0; \quad C_0 = C_a \text{ at } y = H. \tag{25}$$

First-order boundary conditions

$$u_1 = 0 \text{ at } y = 0; \quad \frac{du_1}{dy} = \alpha \sigma u_1 \text{ at } y = H \tag{26}$$

$$\frac{dC_1}{dy} = 0 \text{ at } y = 0; \quad C_1 = C_a \text{ at } y = H \tag{27}$$

Zeroth-order solution

Transforming equation(19) by “Changing the independent variable method ”we have

$$\frac{d^2 y}{dx^2} + P \frac{dy}{dx} + Qy = X \tag{28}$$

where P , Q and X are functions of x .

Changing the independent variable from x to z and defining the new functions

P₁, Q₁ and X₁ , the above equation takes the form

$$\frac{d^2 y}{dz^2} + P_1 \frac{dy}{dz} + Q_1 y = X_1 \tag{29}$$

where,

$$P_1 = \frac{\frac{d^2 z}{dz^2} + P \frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2} \quad Q \quad X$$

$$Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2} \quad X$$

$$X_1 = \frac{X}{\left(\frac{dz}{dx}\right)^2}$$

Choose Q₁ such that Q₁ =constant. This reduces equation(29) to a second order differential equation with constant coefficient whose solution is of the form

$$y = CF + P I \tag{30}$$

Replacing z by x in equation(30) gives the solution to equation(28).

Using this method equation(19) can be written as

$$\frac{d^2u_0}{dy^2} + P \frac{du_0}{dy} + Qy = X \tag{31}$$

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where $P = 0, Q = -\theta\sigma^2, X = -g_1\theta$

Changing the independent variable from y to z, we get

$$\frac{d^2u_0}{dz^2} + Q_1 u_0 = X_1 \tag{32}$$

Choose $Q_1 = A\sigma^2$ as constant, then $z = B^2 - e^{-By}$ and $X_1 = Ag_1$.
 where $A = \theta_s a_1$ and $B = \frac{-a_2}{d_p}$

Solution to equation(32) which is the sum of complementary function and particular integral gives

$$u_0 = g_2 \cos \sqrt{A\sigma^2 z} + g_3 \sin \sqrt{A\sigma^2 z} + \frac{g_1}{\sigma^2} \tag{33}$$

Again transforming z to y the solution of equation(19), subject to the boundary conditions(24) is given by,

$$\frac{2d}{\sqrt{A\sigma^2}} \cos \left(\frac{a_2}{y} \sqrt{2d} \right) + \frac{2d}{\sqrt{A\sigma^2}} \sin \left(\frac{a_2}{y} \sqrt{2d} \right) + \frac{g_1}{\sigma^2} \tag{34}$$

Applying the same method to equation (20) with $P = 0, Q = -\frac{KSc}{\theta}$, the boundary conditions(25) we get

$$\frac{2d_p}{\sqrt{KSc}} \cos \left(\frac{a_2}{-2d} \sqrt{KSc} \right) + \frac{2d_p}{\sqrt{KSc}} \sin \left(\frac{a_2}{-2d} \sqrt{KSc} \right)$$

where,

$$g_2 = \frac{g_1}{\sigma^2 \sqrt{A\sigma^2}}$$

$$g_3 = \frac{-\alpha \sigma g_2 \cos B - A e^{-\alpha \sigma \sqrt{A\sigma^2}} \cos B - A e^{-\alpha \sigma \sqrt{A\sigma^2}} \sin B - A e^{-\alpha \sigma \sqrt{A\sigma^2}}}{\alpha \sigma \sqrt{A\sigma^2} \sin B - \sigma \sqrt{A\sigma^2} \cos B - \sigma \sqrt{A\sigma^2}}$$

$$g_4 = \frac{C_a e^{-2g_6}}{e^{-g_6} e^{-B} - e^{g_6} e^{-B}}$$

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$$g_5 = \frac{e^{2g_6} e^{-g_6}}{\sqrt{\frac{e^{-B} - e^{g_6}}{KSc} \sqrt{\frac{e^{-B}}{A}}}}$$

$$g_6 = 2B$$

To solve the first order equations (21) to (23) we introduce stream function ψ_1 in the following form

$$u_1 = - \frac{\partial \psi_1}{\partial y}, v_1 = \frac{\partial \psi_1}{\partial x} \tag{35}$$

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This reduces the number of dependent variables to be solved and also eliminates pressure term from the list of variables.

Substituting (35) in (21), (22) and eliminating the pressure we get

$$\frac{\psi_1''}{\theta} - \frac{\psi_1' \theta'}{\theta^2} - \frac{\lambda u_0}{\theta^2} + \frac{\omega}{\theta} - \frac{\lambda^2}{\theta - \sigma^2} + \frac{\lambda}{\theta} \psi_1 - \frac{\lambda u_0' - \omega \theta' + \lambda^2 \theta' + \lambda u_0 \theta'' + \lambda u_0' \theta' - u_0''}{\theta^2 \theta^2 \theta^2 \theta^3 \theta^3 \theta^2} \psi_1 + \frac{\lambda u_0 \theta_0' - u_0 \theta_0'' + \lambda u_0 \theta_0' - \lambda u_0' \theta_0 - \omega \lambda_2 + \lambda_4 - u_0 \lambda_3 - \lambda_2}{\theta^3 \theta^6 \theta^4 \theta^3 \theta \theta^2} \psi_1 = 0 \tag{36}$$

Similarly substituting (35) in (23) we get

$$Sc \frac{C_1'}{\theta} - \lambda \psi_1 C_0 + Sc \lambda^2 - \theta \omega + u_0 \lambda - K C_1 = 0 \tag{37}$$

subject to the boundary conditions

$$\psi_1' = 0, \psi_1 = 0, \text{ at } y = 0 \tag{38}$$

$$\psi_1'' = \alpha \psi_1', \quad \psi_1 = \frac{a}{\lambda} \text{ at } y = H \tag{39}$$

$$\frac{dC_1}{dy} = 0 \text{ at } y = 0, C_1 = C_a \text{ at } y = H \tag{40}$$

where the prime (') denotes differentiation with respect to y.

Without chemical reaction

When $k = 0$, equation(4) becomes

$$\theta \left(\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} \right) = \theta D \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right) \tag{41}$$

where c is the solutal concentration of groundwater without chemical reaction. After non-dimensionalization and applying the perturbation technique using (18), the equation(41) takes the zeroth order and first order equations in the form

$$\frac{d^2C_0}{dy^2} = 0 \tag{42}$$

and

$$\frac{Sc C_1}{\theta} - \lambda \psi_1 C_0 + \frac{Sc \lambda^2}{\theta} - \theta \omega + u_0 \lambda C_1 = 0 \tag{43}$$

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with the boundary conditions (25) and (40).

Solving the first order equations (36), (37) and the equations (42), (43) with the boundary conditions (38) to (40) gives the solution of ψ_1 and C_1 for both the cases of with and without chemical reaction respectively.

Differentiating ψ_1 with respect to y , we get u_1 . The sum of the base part and perturbed part gives the required velocity and concentration of species (neglecting the higher order terms (order of q^2)) respectively as:

$$u = u_0 + q e^{(\lambda x + \omega t)} u_1 \tag{44}$$

$$C = C_0 + q e^{(\lambda x + \omega t)} C_1 \tag{45}$$

4 Results and Discussion

The obtained comprehensive solutions are presented in Figures 2 to 7. The expression for velocity profile u is evaluated using equation (44) and the effect of porous parameter σ on non-dimensional velocity is shown in Figure 2. Generally porous parameter σ represents the relative effect of the permeability of the medium versus its cross-sectional area. Numerous factors affect the magnitude and direction of permeability are particle size, shape of particles and packing of particles of a material. It is seen that the velocity profile enhances, when the porous parameter σ increases. By permeability definition, it is seen that if the bubbles of porous media are very small or if they are poorly connected, the permeability will be high and the fluid flows through easily. Thus, the velocity profile enhances with increase in porous parameter.

The expression for concentration profile C is evaluated using equation (45) is shown in Figure 3 with the effect of chemical reaction. It is obvious that an increase in the chemical reaction parameter results a decreasing in the solutal concentration profile. The distribution of solutal concentration becomes weak in the presence of chemical reaction.

Figure 4 is a graphical representation of three dimensional velocity surface

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plot for the response of particle parameter. It is noticed that the velocity decreases with increase in particle diameter. Also the velocity decreases with bed depth. This is explained as the core layer thickness increases with increase in bed height. The graphical representation of three dimensional surface plot for concentration in response of particle diameter ($2\mu\text{m} \leq d_p \leq 15\mu\text{m}$) (Leilei et al., 2008) is explained by Figure 5. The result shows, as the contaminant is added in the groundwater even in small amount there is a considerable decrease. Hence increase in particle size, declines the concentration simultaneously.

Figure 6 is the graphical representation of the concentration profiles for various values of Schmidt number Sc with chemical reaction. The value of the solute profile decreases with increasing values of Sc . As the Schmidt number is inversely proportional to the diffusion coefficient, an increment in Sc produces a reduction of diffusion coefficient and which acts in thinning the concentration boundary layer thickness. For this reason the concentration profile decreases.

The expression for concentration profile C is evaluated using equation (45) without the effect of chemical reaction are plotted in Figure 7 for various values of Schmidt number Sc . As Sc increases, for the non-reactive flow case, concentration values are strongly reduced. Although the influence of Sc on the concentration profiles for the reactive flow case are similar to that of the non-reactive flow case, the profiles are comparatively decreased with a rise in Sc , in the presence of chemical reaction. Therefore we can conclude that the effect of chemical reaction decreases mass transfer significantly throughout the medium.

Table 1 reveals the effect of particle diameter on concentration without the effect of chemical reaction $k = 0$. As d_p values are increased the concentration decreases with very negligible effect.

By comparing the graphs 6 and 7 we conclude that concentration (C) decreases in the domain. For the reactive flow case, concentration values are strongly reduced which is equivalent to a reduction in the chemical molecular diffusivity, i.e. less diffusion takes place by mass transport. It also shows that if

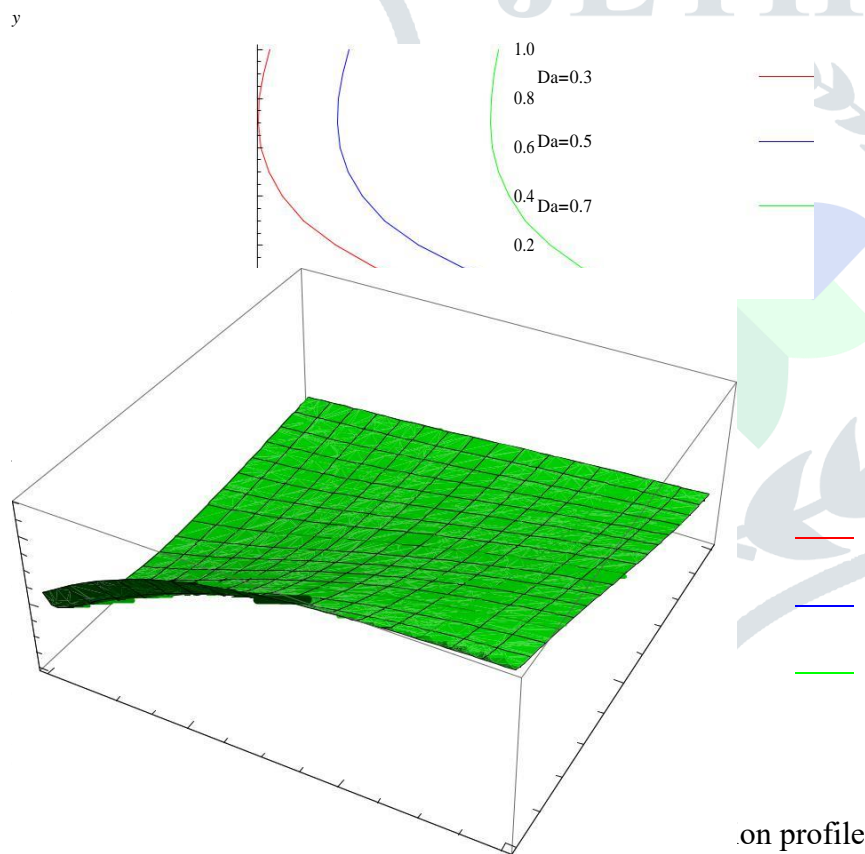
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there is a reaction of a substance of a system, there is a greater chance that molecules will collide and speed up the rate of reaction. If there is no chemical reaction, there will be fewer collisions and the reaction will probably happen at a lower speed. Therefore the rate of reaction depends on the type of molecules they combine.

5 Conclusion

The purpose of this chapter is to address the significance of porosity variations to flow and transport of contaminants in groundwater with and without chemical reaction. A major importance of a particle size distribution is that it relates to other soil properties in a complex way, which indicates complexity of structure in more detail than porosity alone. It is found that the spatial variability of the particle size has a significant influence on the contaminant spreading. For reactive transport in physically and chemically homogeneous media, the particle variations affects the groundwater flow. When the chemical reactions are slow, the porosity variations contaminant evolve for slow change. When the reactions are fast, the porosity variations affects the spreading. Since this is a homogeneous reaction, the effect of porosity variation dominates the contaminant spreading.

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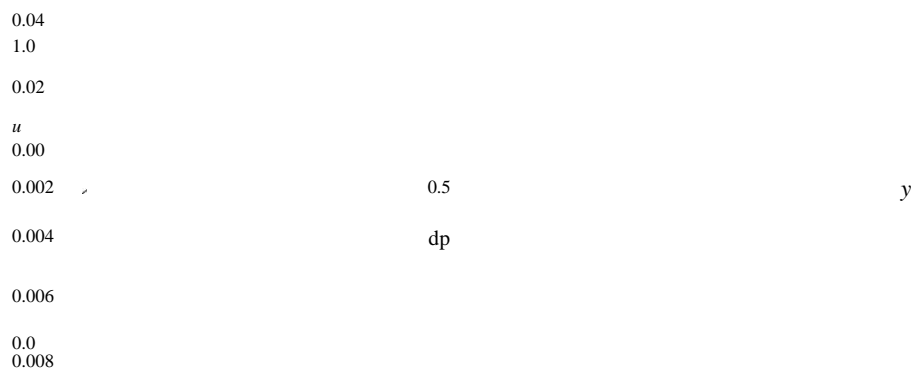


Figure 4: Velocity surface plot for response of particle diameter 14

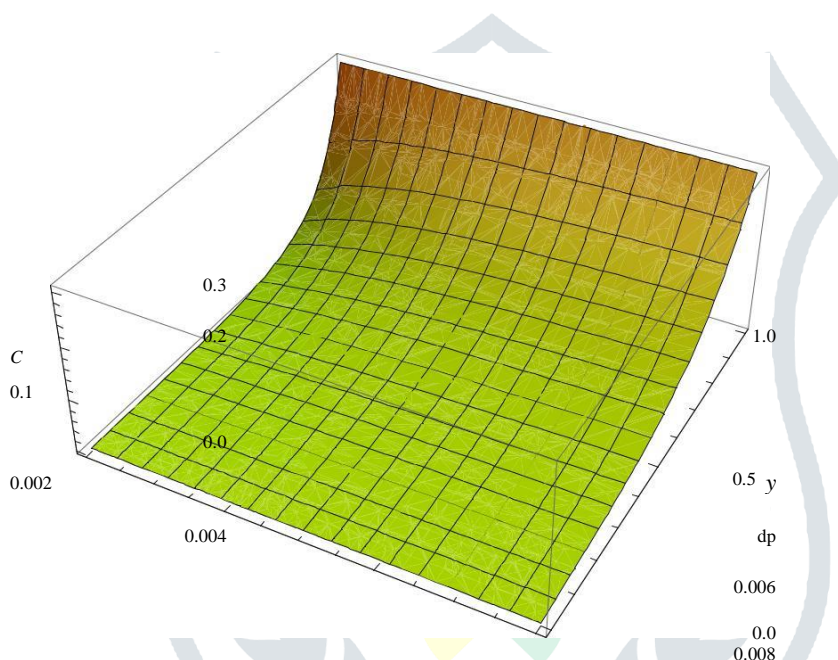


Figure 5: Concentration surface plot for response of particle diameter

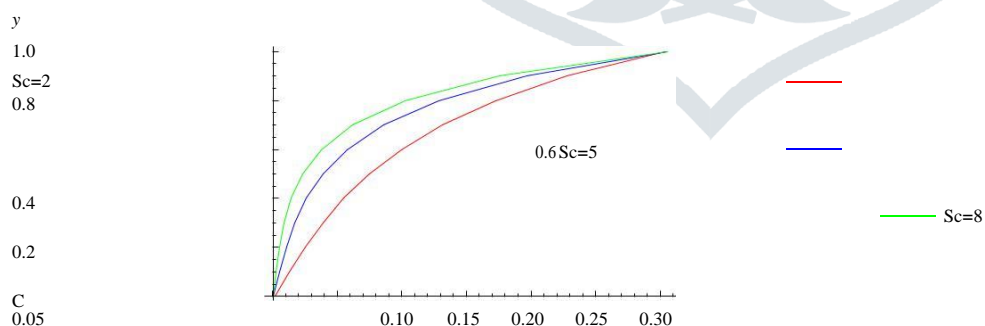


Figure 6: Effect of Schmidt number on concentration profile

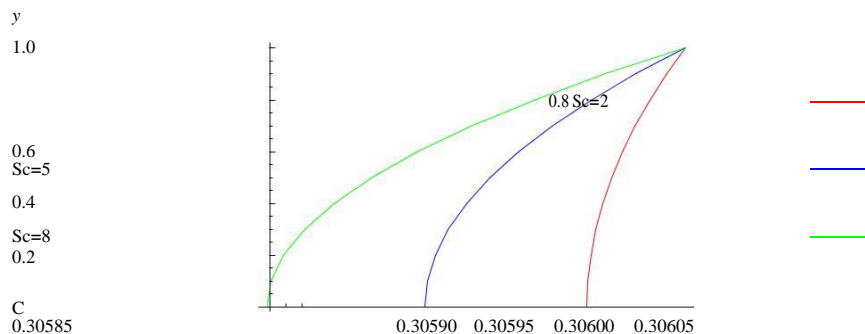


Figure 7: Effect of Schmidt number on concentration profile(without chemical)

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Table 1. Effects of particle parameter on concentration profile(without chemical)

y	dp=0.004	dp=0.006	dp=0.008
0	0.305826	0.305825	0.305825
0.1	0.305829	0.305828	0.305827
0.2	0.305835	0.305835	0.305834
0.3	0.305847	0.305846	0.305846
0.4	0.305863	0.305863	0.305862
0.5	0.305884	0.305884	0.305883
0.6	0.30591	0.30591	0.305909
0.7	0.305941	0.30594	0.30594
0.8	0.305976	0.305975	0.305975
0.9	0.306016	0.306015	0.306015
1.0	0.30606	0.30606	0.30606

References

[1] Bear, J. and A.Verruijt., Modeling Groundwater Flow and Pollution., D.Reidel Publishing

Company, Boston, MA., (1987).

- [2] Canter, L.W., and R.C. Knox., Groundwater Pollution Control, Lewis Publishers Inc. Chelsea, MI., (1985).
- [3] Foppen, J.W.A., Transport of Escherichia coli in Saturated Porous Media. Ph D thesis, VU University Amsterdam, the Netherlands, (2007).
- [4] Lecoanet, H. F., Bottero, J., Wiesner, M. R., Laboratory assessment of the mobility of nanomaterials in porous media, Environ. Sci. Technol., 38(19), 5164- 5169, (2004).
- [5] McCarthy, J. F., Zachara, J.M., Subsurface transport of contaminants, Environmental Science and Technology, 23(5), 496-502, (1989).
- [6] McDowell-Boyer, L.M., Hunt, J.R., Sitar, N., Particle transport through porous media, Water Resources Research 22(13), 1901-1921, (1986).
- [7] Powers, J.P., Construction Dewatering, New Methods and Applications, John Wiley, New York, (1992).
- [8] Reddi, L.N., Particle transport in soils: review of significant processes in infrastructure systems. Journal of Infrastructure Systems ,3(2), 78-86, (1997).
- [9] Schijven, J.F., Virus Removal from Groundwater by Soil Passage, Modeling, Field and Laboratory Experiments, PhD thesis Technical University Delft, the Netherlands, (2001).
- [10] Wang, C., Bobba, A. D., Attinti, R., Shen, C., Lazouskaya, V., Wang, L., Jin, Y., Retention and transport of silica nanoparticles in saturated porous media: effect of concentration and particle size, Environ. Sci. Technol., 46 (13), 7151-7158, (2012).
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- [11] Sperry, J.M. and Peirce, J.J., A model for estimating the hydraulic conductivity of granular material based on grain Shape, grain size and porosity, Ground Water, 33(6), 892-898, (1995).
- [12] Nimmo, J.R., Porosity and pore size distribution, Encyclopedia of Soils in the Environment, 3, 295-303, (2004).
- [13] Droppo, I.G., Walling, D.E., Ongley, E.D, The influence of floc size, density and porosity on sediment and contaminant transport, In: The role of erosion and sediment transport in nutrient and

contaminant transfer, IAHS Press, Wallingford, UK, 141-148, (2000).

[14] Robinson,D.A. and Friedman,S.P, Observations of the effects of particle shape and particle size distribution on avalanching of granular media, *Physica A*, 311, 97-110, (2002).

[15] Moshe,T.B., Dror,I. and Berkowitz,B., Transport of metal oxide nanoparticles in saturated porous media, *Chemosphere*, 81(3), 387-393, (2010).

[16] Urumovic,K. and Urumovic,Sr.K. The effective porosity and grain size relations in permeability functions, *Hydrology and Earth System Sciences*, 11, 6675-6714, (2014).

