Inverse and Disjoint Secure Complementary Tree Domination Number of a Graph.

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Abstract:

The concept of Inverse secure complementary tree domination number and the disjoint secure complementary tree domination number of a graph has been discussed. Bounds for these parameters and their relation with other graph theoretical parameters are attained.

Keywords:

Domination number, Secure domination number, Inverse Complementary tree domination number, Inverse Secure Complementary tree dominating set, Inverse Secure Complementary tree domination number, Disjoint Secure Complementary tree domination number.

Mathematics Subject classification: 05C69.

1. Introduction:

By a ***graph*** we mean a finite, simple, connected and undirected graph G = (V,E), where $V(G)$ denotes its vertex set and $E(G)$ its edge set. Unless otherwise stated, the graph G has p vertices and q edges[1,2]. ***Degree*** of a vertex v is denoted by $d(v)$, the ***maximum degree*** of a graph G is denoted by$\Delta(G)$. We denoted a cycle on p vertices by C_p, a ***path*** on p vertices by P_p, and a ***complete graph*** on p vertices by K_p. A graph G is ***connected*** if any two vertices of G are connected by a path. A maximal connected subgraph of a graph G is called a ***component*** of G. The number of component of G is denoted by $\omega(G)$. The ***complement*** $\overline{G}$ of G is the graph with vertex set $V(G)$ in which two vertices are adjacent if and only if they are not adjacent in G. A ***tree*** is a connected acyclic graph. A ***bipartite graph*** (or ***bigraph***) is a graph whose vertex set can be divided into two disjoint sets V_1 and V_2 such that every edge has one end in V_1 and another end in V_2. A ***complete bipartite graph*** is a bipartite graph where every vertex of V_1 is adjacent to every vertex in V_2. The complete bipartite graph with partitions of order $|V_1| = r$ and $|V_2| = s$ is denoted by $K_{r,s}$. A star graph denoted by $K_{1,p-1}$, is a tree with one root vertex and $(p-1)$ pendant vertices. A bistar$B_{r,s}$ is the graph obtained by joining the root vertices of the the stars $K_{1,r}$and$K_{1,s}$. The ***friendship graph***, denoted by F_p can be constructed by identifying p copies of the cycle at a common vertex. A ***wheel graph***, denoted by W_p is a graph with p vertices, formed by connecting a single vertex to all vertices of

C_{p-1}. A ***helm graph***, denoted by H_p is a graph obtained from the wheel W_p by attaching a pendant vertex in the outer cycle of W_p . ***corona*** of two graphs G_1 and G_2 denoted by $G_1 \circ G_2$ is the graph obtained by taking one copy of $G_1$and $|V_1|$ copies of G_2 ($|V_1|$ is the order G_1) in which i^{th} vertex of G_1 is joined to every vertex in the i^{th} copy of G_2 . If D is a subset of $V(G)$, then $< D >$denotes the vertex induced subgraph of G induced by D. The ***open neighbourhood*** of a set D of vertices of a graph G, denoted by *N(D)* is the set of all vertices adjacent to some vertex in D and $\mathrm{N}(D) \cup D$ is called the ***closed neighbourhood*** of D denoted by $\mathrm{N}[D]$. The ***diameter*** of a connected graph is the maximum distance between two vertices in G and is denoted by *diam(G)*. A ***cut- vertex (cut edge)*** of a graph G is a vertex (edge) whose removal increases the numbers of components. A ***vertex cut***, or ***separating*** set of a connected graph G is a set of vertices whose removal results in a disconnected graph.

The ***connectivity*** or ***vertex connectivity*** of a graph G, denoted by $\mathrm{K}(G)$ (where G is not complete) is the size of a smallest vertex cut. A connected subgraph H of a connected graph G is called a **H-cut** if $\omega(G - H) \geq 2$. The ***chromatic number*** of a graph G , denoted by $\chi(G)$ is the smallest number of colors needed to colour all the vertices of a graph G in which adjacent vertices receive different colours. For any real number x, $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

A ***Nordhaus- Gaddum-type*** result is a (tight) lower or upper bound on the sum or product of parameter of a graph and its complement. Terms not defined here are used in the sense of [2]. A subset D of $V(G)$ is called a ***dominating set*** of G if every vertex in $V(G) - D$ is adjacent to atleast one vertex in G. The ***domination number*** $\gamma(G)$ of G is the mimimum cardinality taken over all dominating sets in G [1,2].

A dominating set D of a connected graph G is said to be a ***connected dominating set*** of G if the induced sub graph $< D >$ is connected. The minimum cardinality taken over all connected dominating sets is the ***connected domination*** number and is denoted by $\gamma_c(G)$[9].

Restrictions on the complement set $[V(G)) - D]$ of a graph defines many parameters. A dominating set D'contained in$[V(G) - D]$ is known as the **inverse dominating** set of G with respect to the dominating set D.The inverse domination number of a graph is denoted by, $\gamma^{-1}(G)$[3].

A Subset D of $V(G)$ is a ***secure dominating set*** if for each $u \in [V(G) - D]$, there exist a vertex v such that $v \in N(u) \cap D$ and $(D - \{v\}) \cup \{u\}$ is a dominating set of G. The minimum cardinality of a secure dominating set is the secure domination number of G, denoted by $\gamma_s(G)$. A secure dominating set of G of cardinality $\gamma_s(G)$ is called $\gamma_s(G)$-set [5,6,7].

R.Kulli and B.Janakiram[10], introduced the concept of non-split dominating set in graphs.

A dominating set D of a connected graph G is a ***non-split dominating set***, if the induced subgraph $< V(G) - D >$ is connected. The non-split domination number $\gamma_{ns}(G)$ of G is the minimum cardinality of a non-split dominating set[11].

In [10], S.Muthammai et. al., introduced complementary tree domination number of a graph and found many results on them.

A subset D of $V(G)$ of a non-trivial connected graph G is said to be ***complementary tree dominating set,*** if the induced subgraph $< V(G) - D >$ is a tree. The minimum Cardinality of a complementary tree dominating set is called the complementary tree domination number of G and is denoted by $\gamma_{ctd}(G)$[8,10,12,14].

In[4], Annie Jasmine S.E. and K. Ameenal Bibi introduced the concept of secure complementary tree domination number of a graph and found man results on them.

A subset D of $V(G)$ of a non-trivial connected graph G is called a **secure complementary tree dominating set (sct- set)**, if D is a secure dominating set and the induced subgraph $< V(G) - D >$ is a tree. The minimum cardinality taken over all secure complementary tree dominating set of G is the secure complementary tree domination number of G denoted by $\gamma_{sct}(G)$. Any set with $\gamma_{sct}(G)$ vertices is called γ_{sct}-set of G.

B.H. Hedetniemi et al. [13,15,16] introduced the concept of Disjoint domination number of graph.

A disjoint domination number of a graph is defined as the minimum cardinality of the union of any two disjoint dominating set of a graph. This parameter is denoted by $\gamma\gamma(G)$.

2. Inverse Secure Complementary Tree Domination Number of a Graph:

Definition: 2.1

Let D be the secure complementary tree dominating set of a non-trivial connected graph $G(V, E)$. A dominating set D' contained in $< V(G) - D >$ is said to be an **inverse secure complementary tree dominating set** with respect to D if for every $u \in D'$ there exist a neighbor vertex $v \in V(G) - D'$ such that $[D' - \{u\} U \{v\}]$ is a dominating set of G and $< V(G) - D' >$ is a tree.

A minimal inverse secure complementary tree dominating set having the minimum cardinality is called the inverse secure complementary tree domination number of G. The inverse secure complementary tree domination number is denoted by, $\gamma_{sct}^{-1}(G)$.

Example: 2.2

For the graph G_4, in Figure 2.2

$$\gamma^{-1} = \gamma_s^{-1} = 1, \qquad \gamma_{ct}^{-1} = 2, \qquad \gamma_{sct}^{-1} = 2$$

G4:

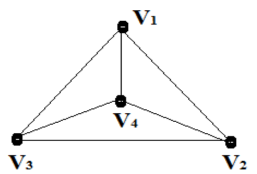

Figure: 2.1. Graph with $\gamma_{ct}^{-1} = \gamma_{sct}^{-1}$.

Example:2.3

For the complete bipartite graph $K_{3,3}$.

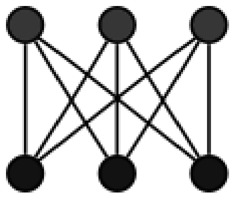

Figure: 2.2. Graph with $\gamma_{ct}^{-1} < \gamma_{sct}^{-1}$.

Example: 2.4

For the graph G_4, in Figure 2.3

$$\boldsymbol{\gamma^{-1} = \gamma_s^{-1} = 1, \gamma_{ct}^{-1} = \gamma_{sct}^{-1} = 2, \quad \gamma_{ns}^{-1} = 1}$$

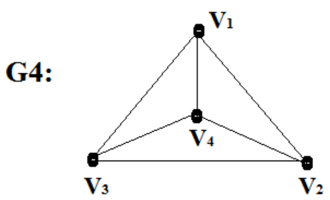

Figure: 2.3. Graph with $\gamma_{ns}^{-1} < \gamma_{sct}^{-1}$.

Result: 2.5[4]

For any path P_p with p$\geq 4, \gamma_{sct}(P_p) = p - 2$.

Theorem:2.6

The inverse secure complementary tree domination number does not exists for the path graph P_p .

Proof:

Let ***G*** be any path graph, then by observation 2.5, $\gamma_{sct}(P_p) = p - 2$. Hence the theorem.

Result:2.7[4]

For any Cycle graph C_p with p$\geq 3, \gamma_{sct}(C_p) = p - 2$.

Theorem:2.8

The inverse secure complementary tree domination number does not exists for the Cycle graph C_p , $p > 4$.

Proof:

Let G be any Cycle graph. Then by observation 2.7, $\gamma_{sct}(C_p) = p - 2, p \geq 3$.

Then for $G = C_3$, $\gamma(G) = \gamma_{sct}(G) = \gamma^{-1}(G) = \gamma_s^{-1}(G) = \gamma_{ct}^{-1}(G) = \gamma_{sct}^{-1}(G) = 1$.

Let $G = C_4$, then $\gamma(G) = \gamma_{sct}(G) = \gamma^{-1}(G) = \gamma_s^{-1}(G) = \gamma_{ct}^{-1}(G) = \gamma_{sct}^{-1}(G) = 2$.

Whereas, when $G = C_p, p \geq 5$, by observation 2.5, $\gamma_{sct}(C_p) = p - 2$. Hence Cycle graph does not contain inverse secure complementary tree domination number.

Result: 2.9[4]

For any Complete graph K_p with, $p \geq 3, \gamma_{sct}(K_p) = p - 2$.

Theorem:2.10

Let G be a Complete graph K_p , $p > 4$. Then, the inverse secure complementary tree domination number does not exists for G.

Proof:

Consider G be to a Complete graph.

When, $G = K_3$, then $\gamma(G) = \gamma_{sct}(G) = \gamma^{-1}(G) = \gamma_s^{-1}(G) = \gamma_{ct}^{-1}(G) = \gamma_{sct}^{-1}(G) = 1$.

When, $G = K_4$, then $\gamma(G) = \gamma_{sct}(G) = \gamma^{-1}(G) = \gamma_s^{-1}(G) = \gamma_{ct}^{-1}(G) = \gamma_{sct}^{-1}(G) = 2$.

For $G = K_p, p \geq 5$, by observation 2.9, $\gamma_{sct}(K_p) = p - 2$. Hence inverse secure complementary tree domination number does not exists for G.

Result:2.11[4]

Let G be a Complete Bipartite graph K_{p_1,p_2} , p = $p_1 + p_2$, then

$$\gamma_{sct}(K_{p_1,p_2}) = \begin{cases} \max(p_1, p_2) & p_1 = 2,3 \; and p_2 \geq 2 \\ 4 & Otherwise \end{cases}$$

Theorem:2.12

For a Complete bipartite graph, $K_{3,3}$

$$\gamma_{sct}^{-1}(G) = 4, \quad \text{for } p_1, p_2 \geq 4.$$

Proof:

Let $G \cong K_{p_1,p_2}$ with the partition sets V_1 and V_2. Let D be the secure complementary tree dominating set of the graph $G \cong K_{p_1,p_2}$. From Result 2.11, secure complementary tree domination number does not exists for $p_1, p_2 < 4$ and when $p_1, p_2 \geq 4$, $\gamma_{sct}\left(K_{p_1,p_2}\right) = 4$.That is, there exists vertices $v_1, v_2 \in V_1$ and $u_1, u_2 \in V_2$ such that $D = \{v_1, v_2, u_1, u_2\}$.Then in $[V(G) - D]$ there exists vertices $v_3, v_4 \in V_1$and $u_3, u_4 \in V_2$ such that $D' = \{v_3, v_4, u_3, u_4\}$ is a minimal secure complementary tree dominating set of G. Thus, $\gamma_{sct}^{-1}(G) = 4$.

Theorem:2.13

The inverse secure complementary tree domination number does not exists for a graph with $|V(G)| > 2$, if it contains vertices of degree one.

Proof:

Let D be the secure complementary tree domination number of a graph G. Then, $< V(G) - D >$ is connected. This means that if there exist a vertex $v \in G$ such that $\deg(v) = 1$, then v is a member of D, otherwise, the vertex adjacent to v will be the member of D. Which implies that $< V(G) - D >$ is not connected. Thus every vertex with degree one is a member of the secure complementary tree domination number of the graph G. Hence G does not contain the inverse secure complementary tree domination number if there exists a vertex with degree one.

Observation:2.14

The inverse secure complementary tree domination number does not exist for all graphs.

Theorem:2.15

Let the graph G contains the inverse secure complementary tree domination number then, G contains the non-split domination number.

Proof:

Let D' be the inverse secure complementary tree domination number of G. Then, by the definition of secure complementary tree dominating set, $< V(G) - D' >$ is a tree. (i.e) $< V(G) - D' >$ is connected. Thus D' is the non-split dominating set of G.

Theorem:2.16

Let $G \cong P_2 \times P_p, p \geq 2$, then

$$\gamma_{sct}^{-1}(G) = p.$$

Proof:

Let G be the Cartesian product graph of $P_2 \times P_p$. Then the vertex set of G will have $2p$ vertices. Thus $|V(P_2 \times P_p)| = 2p$. Let these $2p$ vertices be denoted by $\{v_{11}, v_{12}, \dots v_{1p}, v_{21}, v_{22}, \dots v_{2p}\}$. It is observed that the vertex set $D = \{v_{11}, v_{12}, \dots v_{1p}\}$ forms the secure complementary tree dominating set of G. Then, the vertices $\{v_{21}, v_{22}, \dots v_{2p}\}$ in $[V(G) - D]$ forms a dominating set D'. Moreover, for every vertex v_{2i} in D' there exists a vertex v_{1i} in D such that $[D' - \{v_{2i}\}] \cup \{v_{1i}\}$ is a dominating set of G and $< V(G) - D' >$ is a tree. Thus D' is the inverse secure complementary tree dominating set of G.

3. Bounds for the Inverse Secure Complementary Tree Domination Number of a Graph:

Theorem: 3.1

For any connected graph,

$$\gamma^{-1}(G) \le \gamma_s^{-1}(G) \le \gamma_{sct}^{-1}(G).$$

Bound is attained for P_2, C_3.

Proof:

Let the inverse secure complementary tree domination number of the graph G be denoted by D'. This means that for every vertex $v \in V(G) - D'$ there exists a vertex $u \in D'$ such that $[D' - \{u\}] \cup \{v\}$ is a dominating set of G.Thus D' is an inverse secure dominating set. And every inverse secure dominating set is an inverse dominating set.

Theorem:3.2

Let be any connected graph, then

$$\gamma^{-1}(G) \le \gamma_{ct}^{-1}(G) \le \gamma_{sct}^{-1}(G)$$

Bound is attained for P_2, C_3.

Proof:

Every inverse secure complementary tree domination number is the inverse complementary tree domination number of the graph and every inverse complementary tree domination number is the inverse domination number of the graph.

Observation:3.3

For any connected graph G,

$$\gamma^{-1}(G) \le \gamma_{ct}^{-1}(G) \le \gamma_s^{-1}(G) \le \gamma_{sct}^{-1}(G).$$

Proof:

The proof of the statement follows from the previous two Theorems.

Theorem:3.3

Let G be a connected graph then,

$$\gamma_{ns}^{-1}(G) \leq \gamma_{sct}^{-1}(G)$$

and this bound is for the graph K_3.

Proof:

Let G be a graph and let D' be the inverse secure complementary tree dominating set of G. Then $< V(G) - D' >$ is a tree, that is, $< V(G) - D' >$ is connected. Thus D' is the non-split dominating set of G.

Theorem:3.4

For any connected graph G,

$$\gamma_{sct}(G) \leq \gamma_{sct}^{-1}(G)$$

Proof:

Let D be the secure complementary tree domination number of a connected graph G. That is, D is the minimum cardinality minimal secure complementary tree dominating set of G. Thus if $< V(G) - D >$ contain a secure complementary tree dominating set D'. Then $|D'| \geq |D|$.

Theorem:3.5

For any connected graph G,

$$\gamma_{sct}(G) + \gamma_{sct}^{-1}(G) \leq p .$$

Equality holds for the graph P_2 ,C_4, K_4, $P_2 \times P_p$.

Proof:

Let D be a secure complementary tree domination number and let D' be the inverse secure complementary tree dominating set of G. Then $< V(G) - D' >$ is a tree, that is, $< V(G) - D' >$ is non-empty.

Observation:3.6

For any connected graph G,

$$\gamma^{-1}(G) + \gamma_{sct}^{-1}(G) \leq p .$$

Equality holds for the graph P_2, C_4, K_4, $P_2 \times P_p$.

Theorem:3.7

For any connected graph G,

$$1 \leq \gamma_{sct}^{-1}(G) \leq p - 1.$$

Proof:

Let D' be the inverse secure complementary tree dominating set of G. Since $< V(G) - D' >$ is a tree the lower bound is obtained, moreover the existence of inverse Secure complementary tree dominating set implies that the minimum number of vertices in any graph is greater than or equal to two. Hence the upper bound.

Theorem: 3.8

Let G be any graph, then

$$\gamma(G) \leq mini\{\gamma^{-1}(G), \gamma_{sct}^{-1}(G)\}.$$

Proof:

Let D' be the inverse secure complementary tree dominating set of G. Then by observation 3.3, D' is the inverse dominating set of the graph G. That is, there exists a dominating set D in G and $| D | \leq | D'|$.

Theorem: 3.9

Let T be a Tree which is not a star and diam$(T) \geq 2$, then T does not contain inverse complementary tree dominating set.

Proof:

For a Tree T which is not a Star graph, $\gamma_{sct}(G) \leq p - 2$. By Theorem 2.13, the proof of the theorem follows.

4. Nordhaus- Gaddum type results:

Let G be a connected graph and $\bar{G}$ be the complement of G with no isolated vertex, then

$$\gamma_{sct}^{-1}(G) + \gamma_{sct}^{-1}(\bar{G}) \leq 2p \text{ and}$$

$$\gamma_{sct}^{-1}(G) . \gamma_{sct}^{-1}(\bar{G}) \leq p^2.$$

The bound is attained when $G \cong P_2 \times P_p$.

Proof:

Let G and $\bar{G}$ be connected. From Theorem 3.7, $\gamma_{sct}^{-1}(G) \leq p - 1$ and also $\gamma_{sct}^{-1}(\bar{G}) \leq p - 1$. Thus, $\gamma_{sct}^{-1}(G) + \gamma_{sct}^{-1}(\bar{G}) \leq p - 1 + p - 1 = 2p - 2 \leq 2p$. The sharpness of the bound can be observed from the graph of $P_2 \times P_p$.

5. Disjoint Secure Complementary Tree Domination Number of a Graph:

Definition: 5.1 [13]

Let $D_1, D_2, D_3, \ldots\ldots D_n$ be n disjoint secure complementary tree dominating set of a graph G.The minimum cardinality of the union of two disjoint secure complementary tree dominating set is called as disjoint secure complementary tree domination number of the graph G, denoted as $\gamma_{sct}\gamma_{sct}(G)$.

$$\text{(i.e) } \gamma_{sct}\gamma_{sct}[G] = min\,\{|D_1| + |D_2|\}.$$

Definition: 7.2 [13]

Two disjoint secure complementary tree dominating set (D_1, D_2) is called $\gamma_{sct}\gamma_{sct}[G]$ – pair if their union has the cardinality $\gamma_{sct}\gamma_{sct}[G]$.

Result:7.3

Exact values of $\gamma_{sct}\gamma_{sct}[G]$ for some standard graphs:

1. $\gamma_{sct}\gamma_{sct}[P_2] = 2$
2. $\gamma_{sct}\gamma_{sct}[C_p] = 2p - 4,$ $\quad p = 3, 4.$
3. $\gamma_{sct}\gamma_{sct}[K_3] = 2$
4. $\gamma_{sct}\gamma_{sct}[K_{p_1, p_2}] = 8,$ $\quad p_1, p_2 > 3,$
5. $\gamma_{sct}\gamma_{sct}[p_2 \times p_p] = 2p$ $\quad for\ \ p \geq 2.$

Theorem:7.4

Let G be a graph, then

$$\gamma_{sct}\gamma_{sct}[G] \leq p.$$

The bounds are sharp for the graphs $P_2, C_4, K_{4,4}$ and $p_2 \times p_p$.

Proof:

Let D_1 be the secure complementary tree dominating set, then $|D_1| \leq p - 1$. Any disjoint secure complementary tree dominating set D_2 will contain maximum of $(p - 1)$ vertices.

Sharpness of the bound is observed from Result 7.3.

Theorem: 7.5

For any connected graph G,

$$2\,\gamma_{sct}[G] \leq \gamma_{sct}\gamma_{sct}[G].$$

Equality holds for the standard graph of $P_2, C_3, C_4, K_{4,4}$ and $p_2 \times p_p$

Proof:

Let D_1 and D_2 be any two disjoint secure complementary tree dominating sets of G.Then by the definition of the secure complementary tree domination number of a graph, $|D_1| \leq |D_2|$.

Result 7.3 proves the sharpness of the bound.

Theorem:7. 6

For any graph G, with inverse secure complementary tree dominating set,

$$2\,\gamma_{sct}(G) \leq \gamma_{sct}(G) + \gamma_{sct}^{-1}(G)$$

Equality holds for $P_2, C_3, C_4, K_3, K_{4,p_2}, p_2 \times p_p$.

Proof:

Let D_1 and D_2 be any two disjoint secure complementary tree dominating sets of G. Then,$|D_1| \leq |D_2|$.

Result 7.3 proves the equality of the bound.

Theorem: 7.7[3]

For the connected graph $G(p, q)$,

$$\gamma\gamma[J(G)] \leq \gamma_{sct}\gamma_{sct}[\,G].$$

Bounds are sharp for the graph of P_2, C_3 , K_3 $K_{4,p}$ and $p_2 \times p_p$

Definition: 7.8 [3]

A connected graph G is said to be $\gamma_{sct}\gamma_{sct}[\,G]$ −minimum if $\gamma_{sct}\gamma_{sct}[\,G] = 2\gamma_{sct}[J(G)]$.

Definition: 7.9[3]

A connected graph G is $\gamma_{sct}\gamma_{sct}[\,G]$ −maximum if $\gamma_{sct}\gamma_{sct}[\,G] = p$.

Definition: 7.10[3]

A connected graph G is $\gamma_{sct}\gamma_{sct}[\,G]$ −strong if

$$\gamma_{sct}\gamma_{sct}[\,G] = 2\,\gamma_{sct}[\,G] = p.$$

Example: 7.11

(i). The graphs P_2, C_4 , $K_{4,4}$ and $p_2 \times p_p$ are $\gamma_{sct}\gamma_{sct}$ −minimum.

(ii).The graphs P_2, C_4 , $K_{4,4}$ and $P_2 \times P_p$ are $\gamma_{sct}\gamma_{sct}[\,G]$ −maximum (iii). When G is P_2, C_4 , $K_{4,4}$ and $P_2 \times P_p$ then G is $\gamma_{sct}\gamma_{sct}[G]$ − strong.

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