Shehu Transform for Solving Abel's Integral Equation

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ABSTRACT: Abel's integral equation is an important singular integral equation and generally appears in many branches of sciences such as atomic scattering, mechanics, radio astronomy, physics, electron emission, X-ray radiography and seismology. In this paper, we use Shehu transform for solving Abel's integral equation and some numerical applications are given to demonstrate the effectiveness of Shehu transform for solving Abel's integral equation.

KEYWORDS: Abel's integral equation, Shehu transforms, Inverse Shehu transform, Convolution theorem.

AMS SUBJECT CLASSIFICATION 2010: 44A05, 44A35, 45E10.

I. INTRODUCTION: In 1823, Niels Henrik Abel discussed the motion of particle on smooth curve lying on a vertical plane using Abel's integral equation in mathematical form as [1-2]

$$f(x) = \int_0^x \frac{1}{\sqrt{x-t}} g(t)\, dt \quad (1)$$

In equation (1), the kernel of integral equation, $K(x,t) = \frac{1}{\sqrt{x-t}}$ becomes ∞ at $t = x$, the function $f(x)$ is known function and the function $g(t)$ is unknown function.

Integral transforms are widely used mathematical techniques for solving advanced problems of science and engineering which mathematically express in terms of differential equations, partial differential equations, integral equations, partial integro-differential equations, integro-differential equations etc. Many researchers used different integral transforms (Laplace transform [3-4], Fourier transform [3], Hankel transform [3], Kamal transfom [5, 16-19, 38], Mahgoub transform [8-12, 24], Elzaki transform [6-7, 30-31], Mohand transform [20-22, 36-37, 39-40], Aboodh transform [13-15, 23, 32-35], Sumudu transform [41-42], Wavelet transform [3]) for solving many problems of science and engineering. Aggarwal and others [25-29] discussed the comparative study between these transforms. Aggarwal and Sharma [43] gave the solution of Abel's integral equation by Aboodh transform method.

The Shehu transform of the function $F(t)$ for all $t \geq 0$ is defined as [44]:

$$S\{F(t)\} = \int_0^\infty F(t) e^{-\frac{vt}{u}} dt = H(v,u), v > 0, u > 0, \quad (2)$$

where the operator S is called the Shehu transform operator.

The Shehu transform of the function $F(t)$ for $t \geq 0$ exist if $F(t)$ is piecewise continuous and of exponential order. These conditions are only sufficient conditions for the existence of Shehu transforms of the function $F(t)$. Aggarwal and Gupta [45] discussed Sumudu transform for the solution of Abel's integral equation. Application of Kamal transform for solving Abel's integral equation was given by Aggarwal and Sharma [46]. Aggarwal et al. [47] gave a new application of Mohand transform for handling Abel's integral equation. Application of Shehu transform for handling growth and decay problems was given by Aggarwal et al. [48]. Aggarwal et al. [49] gave a new application of Shehu transform for handling Volterra integral equations of first kind.

In this paper, we are giving the solution of Abel's integral equation using Shehu transform and explain all procedure by giving some numerical applications in application section.

II. SOME USEFUL PROPERTIES OF SHEHU TRANSFORM:

2.1 Linearity property of Shehu transforms [44, 48-49]:

If Shehu transform of functions $F_1(t)$ and $F_2(t)$ are $H_1(v,u)$ and $H_2(v,u)$ respectively then Shehu transform of $[aF_1(t) + bF_2(t)]$ is given by $[a H_1(v,u) + bH_2(v,u)]$, where a, b are arbitrary constants.

2.2Change of scale property of Shehu transform [44]:

If Shehu transform of function $F(t)$ is $H(v,u)$ then Shehu transform of function $F(at)$ is given by $\frac{1}{a}H\left(\frac{v}{a},u\right)$.

2.3 Shifting property of Shehu transform:

If Shehu transform of function $F(t)$ is $H(v,u)$ then Shehu transform of function $e^{at}F(t)$ is given by $H(v-au,u)$.

2.4Shehu transform of the derivatives of the function $F(t)$[44, 49]:

If $S\{F(t)\}=H(v,u)$ then

a) $S\{F'(t)\}=\frac{v}{u}H(v,u)-F(0)$

b) $S\{F''(t)\}=\frac{v^2}{u^2}H(v,u)-\frac{v}{u}F(0)-F'(0)$

c) $S\{F^{(n)}(t)\}=\frac{v^n}{u^n}H(v,u)-\sum_{k=0}^{n-1}\left(\frac{v}{u}\right)^{n-(k+1)}F^{(k)}(0)$

2.5 Convolution theorem for Shehu transforms [49]:

If Shehu transform of functions $F_1(t)$ and $F_2(t)$ are $H_1(v,u)$ and $H_2(v,u)$ respectively then Shehu transform of their convolution $F_1(t)*F_2(t)$ is given by $S\{F_1(t)*F_2(t)\}=S\{F_1(t)\}S\{F_2(t)\}$

$\Rightarrow S\{F_1(t)*F_2(t)\}=H_1(v,u)H_2(v,u)$, where $F_1(t)*F_2(t)$ is defined by

$F_1(t)*F_2(t)=\int_0^t F_1(t-x)F_2(x)dx=\int_0^t F_1(x)F_2(t-x)dx.$

Proof: By the definition of Shehu transform, we have

$$S\{F_1(t)*F_2(t)\}=\int_0^\infty e^{-\frac{vt}{u}}[F_1(t)*F_2(t)]dt$$

$$\Rightarrow S\{F_1(t)*F_2(t)\}=\int_0^\infty e^{-\frac{vt}{u}}\left[\int_0^t F_1(t-x)F_2(x)dx\right]dt$$

By changing the order of integration, we have

$$S\{F_1(t)*F_2(t)\}=\int_0^\infty F_2(x)\left[\int_x^\infty e^{-\frac{vt}{u}}F_1(t-x)\,dt\right]dx$$

Put $t-x=p$ so that $dt=dp$ in above equation, we have

$$S\{F_1(t)*F_2(t)\}=\int_0^\infty F_2(x)\left[\int_0^\infty e^{-\frac{v(p+x)}{u}}F_1(p)\,dp\right]dx$$

$$\Rightarrow S\{F_1(t)*F_2(t)\}=\int_0^\infty F_2(x)e^{-\frac{xv}{u}}\left[\int_0^\infty e^{-\frac{pv}{u}}F_1(p)\,dp\right]dx$$

$$\Rightarrow S\{F_1(t)*F_2(t)\}=H_1(v,u)H_2(v,u).$$

III. SHEHU TRANSFORM OF FREQUENTLY ENCOUNTERED FUNCTIONS [44, 48-49]:

Table: 1

S.N.	$F(t)$	$S\{F(t)\}=H(v,u)$
1.	1	$\frac{u}{v}$
2.	t	$\left(\frac{u}{v}\right)^2$

3.	t^2	$2!\left(\frac{u}{v}\right)^3$
4.	$t^n, n \in N$	$n!\left(\frac{u}{v}\right)^{n+1}$
5.	$t^n, n > -1$	$\Gamma(n+1)\left(\frac{u}{v}\right)^{n+1}$
6.	e^{at}	$\frac{u}{v - au}$
7.	$sinat$	$\frac{au^2}{(v^2 + a^2u^2)}$
8.	$cosat$	$\frac{uv}{(v^2 + a^2u^2)}$
9.	$sinhat$	$\frac{au^2}{(v^2 - a^2u^2)}$
10.	$coshat$	$\frac{uv}{(v^2 - a^2u^2)}$
11	$J_0(at)$	$\frac{u}{\sqrt{(v^2 + a^2u^2)}}$

IV. INVERSE SHEHU TRANSFORM [44, 48-49]:

If $H(v,u)$ is the Shehu transforms of $F(t)$ then $F(t)$ is called the inverse Shehu transform of $H(v,u)$ and in mathematical terms, it can be expressed as $F(t) = S^{-1}\{H(v,u)\}$, where S^{-1} is an operator and it is called as inverse Shehu transform operator.

V. LINEARITY PROPERTY OF INVERSE SHEHU TRANSFORMS [48-49]:

If $S^{-1}\{H_1(v,u)\} = F(t)$ and $S^{-1}\{H_2(v,u)\} = G(t)$ then $S^{-1}\{a\,H_1(v,u) + b\,H_2(v,u)\} = aS^{-1}\{H_1(v,u)\} + bS^{-1}\{H_2(v,u)\}$

$\Rightarrow S^{-1}\{a\,H_1(v,u) + b\,H_2(v,u)\} = aF(t) + bG(t)$, where a, b are arbitrary constants.

VI. INVERSE SHEHU TRANSFORM OF FREQUENTLY ENCOUNTERED FUNCTIONS [48-49]:

Table: 2

S.N.	$H(v,u)$	$F(t) = S^{-1}\{H(v,u)\}$
1.	$\frac{u}{v}$	1
2.	$\left(\frac{u}{v}\right)^2$	t
3.	$\left(\frac{u}{v}\right)^3$	$\frac{t^2}{2!}$
4.	$\left(\frac{u}{v}\right)^{n+1}, n \in N$	$\frac{t^n}{n!}$
5.	$\left(\frac{u}{v}\right)^{n+1}, n > -1$	$\frac{t^n}{\Gamma(n+1)}$

6.	$\frac{u}{v - au}$	e^{at}
7.	$\frac{u^2}{(v^2 + a^2u^2)}$	$\frac{sinat}{a}$
8.	$\frac{uv}{(v^2 + a^2u^2)}$	$cosat$
9.	$\frac{u^2}{(v^2 - a^2u^2)}$	$\frac{sinhat}{a}$
10.	$\frac{uv}{(v^2 - a^2u^2)}$	$coshat$
11.	$\frac{u}{\sqrt{(v^2 + a^2u^2)}}$	$J_0(at)$

VII. SHEHU TRANSFORM FOR SOLVING ABEL'S INTEGRAL EQUATION: In this section, we present Shehu transform for the solution of Abel's integral equation.

Taking Shehu transform of both sides of (1), we have

$$S\{f(x)\} = S\left\{\int_0^x \frac{1}{\sqrt{x-t}} g(t)\, dt\right\}$$

$$\Rightarrow S\{f(x)\} = S\{x^{-1/2} * g(x)\} \qquad (3)$$

Applying convolution theorem of Shehu transform in (3), we have

$$S\{f(x)\} = S\{x^{-1/2}\}S\{g(x)\}$$

$$\Rightarrow S\{f(x)\} = \left[\sqrt{\pi}\left(\frac{u}{v}\right)^{1/2}\right] S\{g(x)\}$$

$$\Rightarrow S\{g(x)\} = \frac{1}{\sqrt{\pi}}\left(\frac{v}{u}\right)^{1/2} S\{f(x)\}$$

$$\Rightarrow S\{g(x)\} = \frac{v}{\pi u}\left[\sqrt{\pi}\left(\frac{u}{v}\right)^{1/2} S\{f(x)\}\right]$$

$$\Rightarrow S\{g(x)\} = \frac{v}{\pi u}\left[S\{x^{-1/2}\}S\{f(x)\}\right]$$

$$\Rightarrow S\{g(x)\} = \frac{v}{\pi u} S\{x^{-1/2} * f(x)\}$$

$$\Rightarrow S\{g(x)\} = \frac{v}{\pi u}\left[S\left\{\int_0^x \frac{1}{\sqrt{x-t}} f(t)\, dt\right\}\right]$$

$$\Rightarrow S\{g(x)\} = \frac{v}{\pi u} S\{F(x)\} \qquad (4)$$

where $F(x) = \int_0^x \frac{1}{\sqrt{x-t}} f(t)\, dt$ (5)

Now applying the property, Shehu transform of derivative of the function, on (5), we have

$$S\{F'(x)\} = \frac{v}{u}S\{F(x)\} - F(0)$$

$$\Rightarrow S\{F'(x)\} = \frac{v}{u}S\{F(x)\}$$

$$\Rightarrow S\{F(x)\} = \frac{u}{v}S\{F'(x)\} \quad (6)$$

Now from (4) and (6), we have

$$S\{g(x)\} = \frac{1}{\pi}S\{F'(x)\} \quad (7)$$

Taking inverse Shehu transform on both sides of (7), we get

$$u(x) = \frac{1}{\pi}F'(x) = \frac{1}{\pi}\frac{d}{dx}F(x) \quad (8)$$

Using (5) in (8), we have

$$g(x) = \frac{1}{\pi}\left[\frac{d}{dx}\int_0^x \frac{1}{\sqrt{x-t}}f(t)\,dt\right] \quad (9)$$

which is the required solution of (1).

VIII. APPLICATIONS: In this section, we present some numerical applications to demonstrate the effectiveness of Shehu transform to solve Abel's integral equation**.**

8.1 Consider the Abel's integral equation:

$$x = \int_0^x \frac{1}{\sqrt{x-t}}g(t)\,dt \quad (10)$$

Taking Shehu transform of both sides of (10), we have

$$S\{x\} = S\left\{\int_0^x \frac{1}{\sqrt{x-t}}g(t)\,dt\right\}$$

$$\Rightarrow \left(\frac{u}{v}\right)^2 = S\{x^{-1/2} * g(x)\} \quad (11)$$

Applying convolution theorem of Shehu transform in (11), we have

$$\left(\frac{u}{v}\right)^2 = S\{x^{-1/2}\}S\{g(x)\}$$

$$\Rightarrow \left(\frac{u}{v}\right)^2 = \left[\sqrt{\pi}\left(\frac{u}{v}\right)^{1/2}\right]S\{g(x)\}$$

$$\Rightarrow S\{g(x)\} = \frac{1}{\sqrt{\pi}}\left(\frac{u}{v}\right)^{3/2} \quad (12)$$

Applying inverse Shehu transform on both sides of (12), we get

$$g(x) = \frac{1}{\sqrt{\pi}}S^{-1}\left\{\left(\frac{u}{v}\right)^{3/2}\right\}$$

$$\Rightarrow g(x) = \frac{2}{\pi}x^{1/2} \quad (13)$$

which is the required solution of (10).

8.2 Consider the Abel's integral equation:

$$1 + x + x^2 = \int_0^x \frac{1}{\sqrt{x-t}} g(t)\, dt \qquad (14)$$

Taking Shehu transform of both sides of (14), we have

$$S\{1\} + S\{x\} + S\{x^2\} = S\left\{\int_0^x \frac{1}{\sqrt{x-t}} g(t)\, dt\right\}$$

$$\Rightarrow \frac{u}{v} + \left(\frac{u}{v}\right)^2 + 2.\left(\frac{u}{v}\right)^3 = S\{x^{-1/2} * g(x)\} \qquad (15)$$

Applying convolution theorem of Shehu transform in (15), we have

$$\frac{u}{v} + \left(\frac{u}{v}\right)^2 + 2.\left(\frac{u}{v}\right)^3 = S\{x^{-1/2}\}S\{g(x)\}$$

$$\Rightarrow \frac{u}{v} + \left(\frac{u}{v}\right)^2 + 2.\left(\frac{u}{v}\right)^3 = \left[\sqrt{\pi}\left(\frac{u}{v}\right)^{1/2}\right] S\{g(x)\}$$

$$\Rightarrow S\{g(x)\} = \frac{1}{\sqrt{\pi}}\left[\left(\frac{u}{v}\right)^{1/2} + \left(\frac{u}{v}\right)^{3/2} + 2.\left(\frac{u}{v}\right)^{5/2}\right] \qquad (16)$$

Applying inverse Shehu transform on both sides of (16), we get

$$g(x) = \frac{1}{\sqrt{\pi}} S^{-1}\left\{\left(\frac{u}{v}\right)^{1/2} + \left(\frac{u}{v}\right)^{3/2} + 2.\left(\frac{u}{v}\right)^{5/2}\right\}$$

$$\Rightarrow g(x) = \frac{1}{\sqrt{\pi}}\left[S^{-1}\left\{\left(\frac{u}{v}\right)^{1/2}\right\} + S^{-1}\left\{\left(\frac{u}{v}\right)^{3/2}\right\} + 2S^{-1}\left\{\left(\frac{u}{v}\right)^{5/2}\right\}\right]$$

$$\Rightarrow g(x) = \frac{1}{\pi}\left[x^{-1/2} + 2x^{1/2} + \frac{8}{3}x^{3/2}\right] \qquad (17)$$

which is the required solution of (14).

8.3 Consider the Abel's integral equation:

$$3x^2 = \int_0^x \frac{1}{\sqrt{x-t}} g(t)\, dt \qquad (18)$$

Taking Shehu transform of both sides of (18), we have

$$3S\{x^2\} = S\left\{\int_0^x \frac{1}{\sqrt{x-t}} g(t)\, dt\right\}$$

$$\Rightarrow 6.\left(\frac{u}{v}\right)^3 = S\{x^{-1/2} * g(x)\} \qquad (19)$$

Applying convolution theorem of Shehu transform in (19), we have

$$6.\left(\frac{u}{v}\right)^3 = S\{x^{-1/2}\}S\{g(x)\}$$

$$\Rightarrow 6.\left(\frac{u}{v}\right)^3 = \left[\sqrt{\pi}\left(\frac{u}{v}\right)^{1/2}\right] S\{g(x)\}$$

$$\Rightarrow S\{g(x)\} = \frac{6}{\sqrt{\pi}}\left(\frac{u}{v}\right)^{5/2} \qquad (20)$$

Applying inverse Shehu transform on both sides of (20), we get

$$g(x) = \frac{6}{\sqrt{\pi}} S^{-1}\left\{\left(\frac{u}{v}\right)^{5/2}\right\}$$

$$\Rightarrow g(x) = \frac{8}{\pi} x^{3/2} \tag{21}$$

which is the required solution of (18).

8.4 Consider the Abel's integral equation:

$$\frac{4}{3} x^{3/2} = \int_0^x \frac{1}{\sqrt{x-t}} g(t)\, dt \tag{22}$$

Taking Shehu transform of both sides of (22), we have

$$\frac{4}{3} S\{x^{3/2}\} = S\left\{\int_0^x \frac{1}{\sqrt{x-t}} g(t)\, dt\right\}$$

$$\Rightarrow \sqrt{\pi}.\left(\frac{u}{v}\right)^{5/2} = S\{x^{-1/2} * g(x)\} \tag{23}$$

Applying convolution theorem of Shehu transform in (23), we have

$$\sqrt{\pi}.\left(\frac{u}{v}\right)^{5/2} = S\{x^{-1/2}\} S\{g(x)\}$$

$$\Rightarrow \sqrt{\pi}.\left(\frac{u}{v}\right)^{5/2} = \left[\sqrt{\pi}\left(\frac{u}{v}\right)^{1/2}\right] S\{g(x)\}$$

$$\Rightarrow S\{g(x)\} = \left(\frac{u}{v}\right)^2 \tag{24}$$

Applying inverse Shehu transform on both sides of (24), we get

$$g(x) = S^{-1}\left\{\left(\frac{u}{v}\right)^2\right\}$$

$$\Rightarrow g(x) = x \tag{25}$$

which is the required solution of (22).

8.5 Consider the Abel's integral equation:

$$2\sqrt{x} + \frac{8}{3} x^{3/2} = \int_0^x \frac{1}{\sqrt{x-t}} g(t)\, dt \tag{26}$$

Taking Shehu transform of both sides of (26), we have

$$2S\{x^{1/2}\} + \frac{8}{3} S\{x^{3/2}\} = S\left\{\int_0^x \frac{1}{\sqrt{x-t}} g(t)\, dt\right\}$$

$$\Rightarrow \sqrt{\pi}.\left(\frac{u}{v}\right)^{3/2} + 2\sqrt{\pi}.\left(\frac{u}{v}\right)^{5/2} = S\{x^{-1/2} * g(x)\} \tag{27}$$

Applying convolution theorem of Shehu transform in (27), we have

$$\sqrt{\pi}.\left(\frac{u}{v}\right)^{3/2} + 2\sqrt{\pi}.\left(\frac{u}{v}\right)^{5/2} = S\{x^{-1/2}\} S\{g(x)\}$$

$$\Rightarrow \sqrt{\pi}.\left(\frac{u}{v}\right)^{3/2} + 2\sqrt{\pi}.\left(\frac{u}{v}\right)^{5/2} = \left[\sqrt{\pi}\left(\frac{u}{v}\right)^{1/2}\right] S\{g(x)\}$$

$$\Rightarrow S\{g(x)\} = \frac{u}{v} + 2.\left(\frac{u}{v}\right)^2 \tag{28}$$

Applying inverse Shehu transform on both sides of (28), we get

$$g(x) = S^{-1}\left\{\frac{u}{v}\right\} + 2S^{-1}\left\{\left(\frac{u}{v}\right)^2\right\}$$

$$\Rightarrow g(x) = 1 + 2x \qquad (29)$$

which is the required solution of (26).

8.6 Consider the Abel's integral equation:

$$\frac{3}{8}\pi x^2 = \int_0^x \frac{1}{\sqrt{x-t}} g(t)\,dt \qquad (30)$$

Taking Shehu transform of both sides of (30), we have

$$\frac{3}{8}\pi S\{x^2\} = S\left\{\int_0^x \frac{1}{\sqrt{x-t}} g(t)\,dt\right\}$$

$$\Rightarrow \frac{3\pi}{4}\left(\frac{u}{v}\right)^3 = S\{x^{-1/2} * g(x)\} \qquad (31)$$

Applying convolution theorem of Shehu transform in (31), we have

$$\frac{3\pi}{4}\left(\frac{u}{v}\right)^3 = S\{x^{-1/2}\}S\{g(x)\}$$

$$\Rightarrow \frac{3\pi}{4}\left(\frac{u}{v}\right)^3 = \left[\sqrt{\pi}\left(\frac{u}{v}\right)^{1/2}\right] S\{g(x)\}$$

$$\Rightarrow S\{g(x)\} = \frac{3}{4}\sqrt{\pi}\left(\frac{u}{v}\right)^{5/2} \qquad (32)$$

Applying inverse Shehu transform on both sides of (32), we get

$$g(x) = \frac{3}{4}\sqrt{\pi} S^{-1}\left\{\left(\frac{u}{v}\right)^{5/2}\right\}$$

$$\Rightarrow g(x) = x^{3/2} \qquad (33)$$

which is the required solution of (30).

IX. CONCLUSION: In this paper, we have successfully discussed Shehu transform for the solution of Abel's integral equation. The given numerical applications in the application section explain the complete procedure for the solution of Abel's integral equation using Shehu transform. The results show that Shehu transform is a powerful integral transform method for the solution of Abel's integral equation. In the future, Shehu transform can be used for solving other singular integral equations.

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