A STUDY ON COMBINATION OF SHELL GRAPH AND PATH P_2 GRAPH

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Abstract: A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. The main aim of this paper is to introduce the new kind of graph which we call ice – cream graph. Moreover, we proved that ice – cream graphs are 7- cordial for all n.

Keywords: **Graph labeling, 7- cordial, Shell graph, Path graph, Ice - cream graph.**

1. INTRODUCTION:

Graph Theory is one of the Expanding Branches of Discrete Mathematics with applications in almost all the disciplines of Science and Technology. There are many potential fields of research in Graph Theory. Some of them are Enumeration of Graphs, Domination in Graphs, Algorithmic Graph Theory, Topological Graph Theory, Fuzzy Graph Theory, Labeling of Graph etc.

Graph Labeling is an Integral part of Graph Theory which assigns numeral values to the vertices or edges or both, subject to certain conditions. The concept of Graph Labeling was introduced by A. Rosa in 1967[3]. The present authors are motivated by the Research Article" A-cordial graphs" by Hovey [4]. In the intervening years various labeling of graphs have been investigated in over 2000 papers. The concept of cordial labeling was introduced by I. Cahit [2].

We begin with simple, finite and undirected graph G=(V, E) with p vertices and q edges. For standard terminology and notation related to graph theory we refer to J.A. Gallian [1]. We will provide brief summary of definition.

In this research paper we consider finite, connected, undirected and simple graphs. In the graph G=(V(G), E(G)) the cardinality of the vertex set is called order of G and the cardinality of the edge set is called the size of G. They are denoted by |V (G)| and |E(G)| respectively. In Graph Labeling we assign numerical values to vertices or edges or both subject to certain conditions.

2. PRELIMINARIES:

Definition 2.1:

A ***graph labeling*** is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices(edges) then the labeling is called a vertex labeling (an edge labeling).

Definition 2.2:

Let $\langle A, *\rangle$ be any Abelian group. A graph G = (V (G), E(G)) is said to be ***A-cordial*** if there is a mapping f: V (G) → A which satisfies the following two conditions when the edge e = uv is labeled as f(u) $*$ f(v).

(i) $|v_f(a) - v_f(b)| \leq 1$; for all a, b ∈ A,

(ii) $|e_f(a) - e_f(b)| \leq 1$; for all a, b ∈ A.

Where,

$v_f(a)$ = the number of vertices with label a;
$v_f(b)$ = the number of vertices with label b;
$e_f(a)$ = the number of edges with label a;
$e_f(b)$ = the number of edges with label b.

We note that if A = $< Z_k, +_k >$, that is additive group of modulo k then the labeling is known as *k-cordial labeling.*

Here we consider A =$< Z_7, +_7 >$, that is additive group of modulo 7.

Definition 2.3:

A ***shell graph*** is defined as a cycle C_n with $(n-3)$ chords sharing a common end point called the apex. Shell graph are denoted as $C_{(n,n-3)}$. A shell S_n is also called fan f_{n-1}.

Definition 2.4:

A ***path graph*** or linear graph is a graph whose vertices can be listed in the order $v_1, v_2, v_3, \dots, v_n$ ∋ edges are $\{v_i, v_{i+1}\}$ where $i = 1,2,3, \dots, n-1$.

Definition 2.5:

An ***Ice - cream graph*** is obtained by combining a shell graph and a path P_2 graph keeping v_1 and v_{n-1} common where $n > 3$ sharing common end point called the apex vertex v_0. It is denoted by $\boldsymbol{I_n}$.

Example:

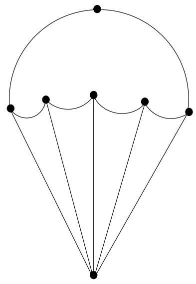

Fig.2.5.1. Ice – cream graph I_6

3. MAIN RESULT:

Theorem : All ice – cream graphs I_n are 7 – cordial.

Proof :

Let $G = I_n$ be the ice – cream graph. Let $v_1, v_2, v_3, \ldots, v_n$ be the path vertices of I_n and v_0 be the apex vertex.
We note that $|V(G)| = n + 1$ and $|E(G)| = 2n - 1$.

Define 7 - cordial labeling $f : V(G) \to \langle Z_7, +_7 \rangle$ as follows:

$f(v_0) = 0$

$f(v_i) = 0$ if $i \equiv 0 (mod\ 7)$

$f(v_i) = 1$ if $i \equiv 4 (mod\ 7)$

$f(v_i) = 2$ if $i \equiv 1 (mod\ 7)$

$f(v_i) = 3$ if $i \equiv 5 (mod\ 7)$

$f(v_i) = 4$ if $i \equiv 2 (mod\ 7)$

$f(v_i) = 5$ if $i \equiv 6 (mod\ 7)$

$f(v_i) = 6$ if $i \equiv 3 (mod\ 7)$; $1 \le i \le n$

The labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for 7 – cordial labeling as shown in Table 3.1. That is ice – cream graph admits 7 - cordial labeling.

Let $n = 7a + b$, $a, b \in N \cup \{0\}$

Table 3.1

B	Vertex conditions	Edge conditions
0,1,2,3, 4,5,6	$v_f(0) = v_f(1) + 1 = v_f(2) = v_f(3) + 1 = v_f(4) = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3) + 1 = e_f(4) = e_f(5) + 1 = e_f(6) + 1$

Illustration 3.1: The ice – cream graph I_{10} and its 7 – cordial labeling is shown in Figure 3.1.

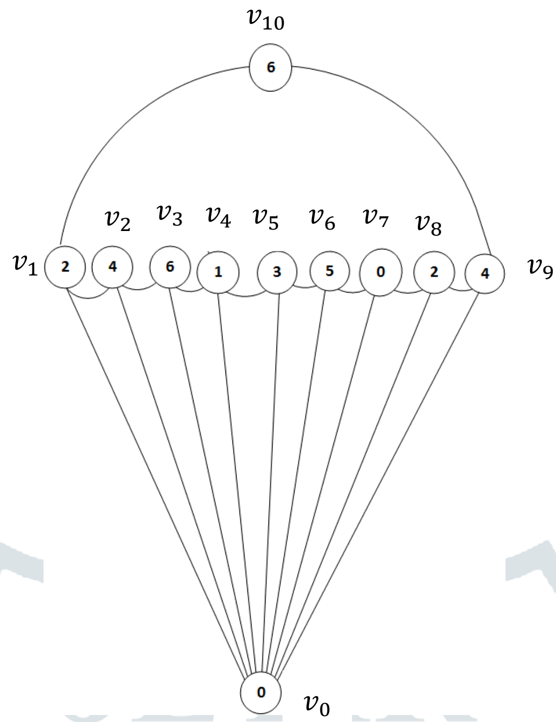

Fig. 3.1: 7-cordial labeling of ice cream graph I_{10}.

4. CONCLUSION:

Graph labeling technique is a wide area of research. As all the shell graphs are cordial graphs it is very interesting and challenging one. Here we have contributed some new definition and its result is by investigating 7 – cordial labeling for all ice – cream graphs. For better understanding of labeling pattern, we have given some illustration. To investigate more graph families which admit k-cordial labeling is an open area of research.

5. ACKNOWLEDGMENT:

I am thankful to God for His grace and blessings without which this attempt would have been impossible. It is my privilege to acknowledge my guide Ms. Vanithashree, Assistant Professor, Department of Mathematics, St. Peter's Research Institute and Technology for her valuable guidance and motivation throughout this paper.

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