Nonlinear Dispersion Characteristics of an InSb-Nd:YAG System: Magnetic Field Effects

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Abstract: Hydrodynamic model and coupled mode theory has been used to study the dispersion characteristics of laser-plasma interaction incorporating relativistic effects. Expressions for the nonlinear second-order optical susceptibility and nonlinear absorption coefficient of the semiconductor plasma medium have been derived. Effect of cyclotron frequency as a function of magnetic field on threshold field intensity and dispersion coefficient has been plotted for relativistic case. Numerical estimations were made for n-InSb duly irradiated by 1.06 μm Nd:YAG laser at 77K. Influence of varying doping concentrations on both characteristics also observed. Proposed mathematical model successfully examines the nonlinear operational characteristics of the semiconductor medium. Reported magnitude of threshold field intensity and second order optical susceptibility are found in well agreement with other investigations in the field. Heavily doped semiconductors are the appropriate hosts to study the relativistic effects on second order optical nonlinearities of semiconductor plasma medium. Presence of magnetic field increases optical nonlinearity in the relativistic laser plasma medium. Inclusion of magnetic field is found to have favourable threshold and dispersion characteristics. Nd:YAG laser system utilized in this study is capable to achieve the reported amplitude of threshold field intensity. This fact could drastically reduce the operating cost of parametric amplifiers and other related nonlinear devices based on this interaction. The findings of the paper may be utilized for the construction of optical switch and other optical manipulation devices.

IndexTerms - **Semiconductor Plasma, Parametric Interaction, Nonlinear Absorption Coefficient.**

I. Introduction

Study of relativistic phenomenon in various perspectives consistently attracted researchers. Nonlinear effects are essentially significant when the intensity of the incident laser radiation is adequately huge enough (Yousef et al., 2006,Skoric, 2009, De Carvalho et al., 2018,Mishra et al., 2018, Hafez and Talukder, 2015, Lutz, 1999, Das et al., 1976, Yousef I., et al., 2006).With the enormous advancements of laser innovation, relativistic light intensities have opened up, driving electrons nearly at light speed.It is acclaimed that in the relativistic regime when the amplitude of pump wave is large, and the plasma electrons oscillate at relativistic velocities and the electric and magnetic field’s combined action pushes the electron forward. In this case, the liberated electron’s velocity found to approach light velocity. This results in relativistic mass variation (RMV) of electrons. Hence it becomes crucial to consider the relativistic effects in plasmas (Ghosh and Dixit, 1985). Inclusion of magnetic field in the laser-plasma interaction provides tremendous insights. This mass variation plays a crucial role in relativistic nonlinear optics in the plasmas and finds numerous plasmas-based applications, such as fast ignitor fusion, laser-driven accelerators, harmonic generation, X-ray lasers, and so forth.

Due to the quick pertinence to acoustic gain or loss problems, parametric amplification or attenuation of waves in semiconductor plasma are significant in nonlinear acoustics. Parametric interactions end up being a powerful technique and adequately treating optical nonlinearities.

The preliminary study of parametric excitation of low-frequency acoustic waves in unmagnetized piezo-semiconductors by applying a high-frequency electric field was first reported by P.K. Kaw (Kaw, 1973) in the hydrodynamic domain. In general, the detailed description of processes involved in relativistic plasma requires a numerical approach. S. Ghosh (Ghosh and Dixit, 1985) has first utilised the hydrodynamic approach to study the effect of RMV of the electron on parametric excitation of acoustic waves. These investigations were confined to the propagation characteristics in piezoelectric magnetized semiconducting plasma using a dispersion relation.

A distinct survey of available literature in different fields (Hafez and Talukder, 2015, Hafez et al., 2016,Das and Kaw, 1998, Kaplan, 1988, Sodha et al., 1976) reveals that theoretical and experimental investigations of optical properties of relativistic effects within the host materials have been pursued in great numbers. Various relativistic phenomena developed a diverse field of research linking together widespread activities ranging from high-energy heavy-ion collision physics, atomic or molecular physics and chemistry of heavy elements to solid state physics. While some prior work (Ghosh et al., 2010,Guha et al., 1979, Ghosh et al., 2018, Sharma and Ghosh, 2000) explored the gain profile of parametrically generated signal/idler wave in non-relativistic regime. Although some earlier work (Ghosh et al., 2010, Sharma and Ghosh 2000) investigated the gain profile of parametrically generated signal/idler wave in non-relativistic regime. Recently, Present authors have reported the effect of relativistic mass variation of electron on nonlinear absorption and dispersion characteristics in magnetized semiconductor plasma. However a systematic investigation on impact of external magnetic field on dispersion characteristics incorporating relativistic effects is yet to be explored.

Motivated by emerging status of the field, authors have focused attention to develop a theoretical model to study influence of magnetic field on relativistic semiconductor plasma medium. Study of Second-order nonlinearities via a three-wave interaction in infinite plasma is our main concern.

In the present work, mathematical model is developed for three wave interactions in an infinite plasma medium using well known hydrodynamic model. For the purpose of possible relativistic oscillations of plasma electrons, a laser of 1.06 μm wavelength may enable electrons to drift at relativistic velocities with momentum nearly equals to $m_e c$. These specifications allow us to use Nd:YAG laser-plasma system for the theoretical and numerical formalisms in our model.

This paper is organized in following manner: Section-II contains the basic equations describing the phenomenon and also deals with the complete formulation of the second order nonlinear susceptibility for n-type semiconductor plasma duly shined by a high-power Nd:YAG laser system. In section-III, Numerical analysis of problem under study is done and the consequences of incorporating relativistic effects in the presence of external magnetic field are also discussed. Significant observations and their possible applications have been reported.

II. MATHEMATICAL FORMULATION

In this section, the well-known hydrodynamic model of homogenous n-type semiconductor plasma of infinite extension (i.e., $k_a l << 1$, where k_a is the wave number of acoustic mode and l is the mean free path of charge carriers) has been considered. In this investigation electrons are assumed as carriers subjected to the electromagnetic pump wave $\vec{E}_O = \exp i(k_O\hat{x} - \omega_O t)$ and external magnetic field $B_O(\hat{z})$ across the propagation vector $k_O(\hat{x})$ under thermal equilibrium condition.

Basic equations used for analysis are as follows:

$$\frac{\partial \vec{V}_O}{\partial t} + \left(\vec{V}_O.\nabla\right)\vec{V}_O + \upsilon.\vec{V}_O = \frac{-e}{\Gamma m}\left(\vec{E}_O + \vec{V}_O \times \vec{B}_O\right) \quad (1)$$

$$\frac{\partial \vec{V}_1}{\partial t} + \left(\vec{V}_O.\nabla\right)\vec{V}_1 + \upsilon.\vec{V}_1 = \frac{-e}{\Gamma m}\left(\vec{E}_1 + \vec{V}_1 \times \vec{B}_O\right) - \frac{\vartheta_{th}^2 \nabla n_1}{n_O} \quad (2)$$

$$\frac{\partial^2 u}{\partial t^2} + \beta\frac{\partial E}{\partial x} + 2i\gamma_s\frac{\partial u}{\partial t} = C_{44}\frac{\partial^2 u}{\partial x^2} \quad (3)$$

$$\frac{\partial E}{\partial x} = \frac{-en_1}{\varepsilon} - \frac{\beta}{\varepsilon}\frac{\partial^2 u}{\partial x^2} \quad (4)$$

$$\frac{\partial n_1}{\partial t} + n_O\left(\frac{\partial V_1}{\partial x}\right) + \vec{V}_O\left(\frac{\partial n_1}{\partial x}\right) + n_1\left(\frac{\partial V_O}{\partial x}\right) = O \quad (5)$$

Here $\Gamma = \left(1 - V^2/c^2\right)^{-1/2}$ is the well-known relativistic factor.

The zeroth- and first- order momentum transfer equations are expressed by eqs. (1) and (2). Here Relativistic factor Γ and crude pressure term $\vartheta_{th}^2\nabla n_1/n_0$ with thermal velocities ϑ_{th} of electrons have been added to relativistic momentum term of eq. (2). Eq. (3) represents the equation of motion of the lattice in the piezoelectric crystal. Eqs. (4) and (5) are the Poisson's and the continuity equation respectively. Detailed mathematical procedures and meaning of the symbols are given in our previous publication (Chouhan et al., 2019).

Following standard approach (Neogi and Ghosh, 1989) time evolution of perturbed carrier density may be obtained with the aid of eqs (1) to (5), as

$$\frac{\partial^2 n_1}{\partial t^2} + \upsilon\frac{\partial n_1}{\partial t} + \omega_R^2 n_1 + \overline{\omega}_p^2\frac{k_a^2\beta u}{e} = -i(k_0 + k_1)n_1\overline{E} \quad (6)$$

Here $\overline{E} = \frac{-e}{\Gamma m}\vec{E}_0 + \omega_c\vec{V}_{0x}$,

$\omega_c = \left(\frac{-eB}{\Gamma m}\right)$ Cyclotron frequency,

$\omega_p^2 = \left(\frac{n_0 e^2}{m\varepsilon}\right)$ Plasma frequency,

$\overline{\omega}_p^2 = \omega_p^2\left(\frac{\upsilon^2}{\upsilon^2 + \omega_c^2}\right)$ Magnetic field modified plasma frequency,

$\omega_R^2 = [\overline{\omega}_p^2 + k_a^2\vartheta_{th}^2\frac{\upsilon}{(\upsilon + \omega_c)}]$ Relativistically modified plasma frequency.

2.1Second-Order Nonlinear Optical Susceptibility

Let us now consider the formulations for lowest order optical susceptibility arising due to nonlinear polarization of the medium. Dielectric susceptibility is a key parameter whose magnitude defines the ease with which a material could be polarized. Real part of $\chi^{(2)}$ is relevant to explore the dispersion characteristics of the medium under investigation. Positive $\chi_r^{(2)}$ is significant for self focusing of pump beam, whereas negative $\chi_r^{(2)}$ could lead to defocusing of pump beam. Therefore in order to study dispersion characteristics of the medium, $\chi_r^{(2)}$ can be determined. The nonlinearities which have been taken into account are the nonlinear current and the polarization, which is cause of nonlinear coupling between the density fluctuations and the scattered waves. On following the well-defined procedure, one may deduce an expression for second order nonlinear susceptibility as

$$\chi^{(2)} = \frac{A\varepsilon_L k_a \omega_0 \overline{\omega}_p^2}{2\Gamma m\omega_s\omega_a\gamma_s(\omega_0^2-\omega_c^2)}\left[\Delta_1^2 + i\upsilon\omega_a - \frac{(k_0+k_1)^2\overline{E}^2}{\Delta_2^2 - i\upsilon\omega_s}\right]^{-1} = \chi_r^{(2)} + \chi_i^{(2)} \qquad (7)$$

Here $\Delta_1^2 = \omega_R^2 - \omega_a^2$, $\Delta_2^2 = \omega_R^2 - \omega_s^2$, $K^2 = \frac{\beta^2}{\varepsilon C_{44}}$, $A = k_a^2 K^2 \vartheta_s^2$, $\vartheta_s = \left(\frac{C_{44}}{\rho}\right)^{1/2}$ is the acoustic velocity of the crystal medium (C_{44} is the elastic constant), $\chi_r^{(2)}$ and $\chi_i^{(2)}$ are real and imaginary parts of second order susceptibility. Above equation displays the dependence of nonlinear susceptibility on carrier concentration, external magnetic field and relativistic factor.

2.3 Threshold Pump Electric Field

The sign of imaginary part of $\chi^{(2)}$ determines the gain/absorption characteristics of the medium. Although there are two conditions for nonlinear growth of signal- (1) when $\chi_i^{(2)}$ is negative, (2) when pump field intensity reaches a threshold value which could be determined by setting equation (7) equals to zero, we may obtain

$$E_{th} = \frac{\Gamma m(\omega_c^2 - \omega_0^2)}{ek_a\omega_0^2}\left[(\Delta_1^2 + i\upsilon\omega_a)(\Delta_2^2 - i\upsilon\omega_s)\right]^{0.5} \qquad (8)$$

Threshold pump amplitude could be determined using above equation for a suitable medium. Corresponding threshold pump intensity may be written as $I_{th} = 0.5c\varepsilon_0\varepsilon_L E_{th}^2\eta$, η being the refractive index of the InSb crystal and c is the velocity of light in the vacuum.

III. Results and Discussion

In order to have a numerical appreciation of the theoretical formulation presented in the preceding section, the following set of parameters has been used:

$m = 0.145m_0$ (m_0 being the rest mass of free electrons), $\Gamma = 8$, $\gamma_s = 5\times10^{-10}\,mksunit$, $\rho = 5.8\times10^3\,kgm^{-3}$, $\varepsilon_L = 15.8$, η =3.9, $\beta = 0.054\,Cm^{-2}$, $\upsilon = 3.5\times10^{11}s^{-1}$, $\omega_0 = 1.78\times10^{13}s^{-1}$, $\omega_a = 2\times10^{11}s^{-1}$ and $\vartheta_s = 4\times10^3\,ms^{-1}$ at 77K.

Here n-InSb crystal is assumed to be irradiated by 1.06 μm Nd:YAG laser.

For the improved efficiency and functionality of parametric devices, estimation of threshold condition and optical parametric gain/absorption are the key issues in semiconductor plasma medium ((Fu and Willander 1999, Lutz, 1999). Eq. (8) is utilized for the investigation of favourable threshold characteristics.

Being one of the principal objectives of the present analysis, the nature of the parametric dispersion via real part of the second order optical susceptibility $\chi_r^{(2)}$ has been analyzed utilizing eq. (7).

3.1Threshold Field Analysis

The numerical estimation dealing with the factors influencing the threshold electric field intensity I_{th} required for the onset of the amplification process is plotted in Figure 1.

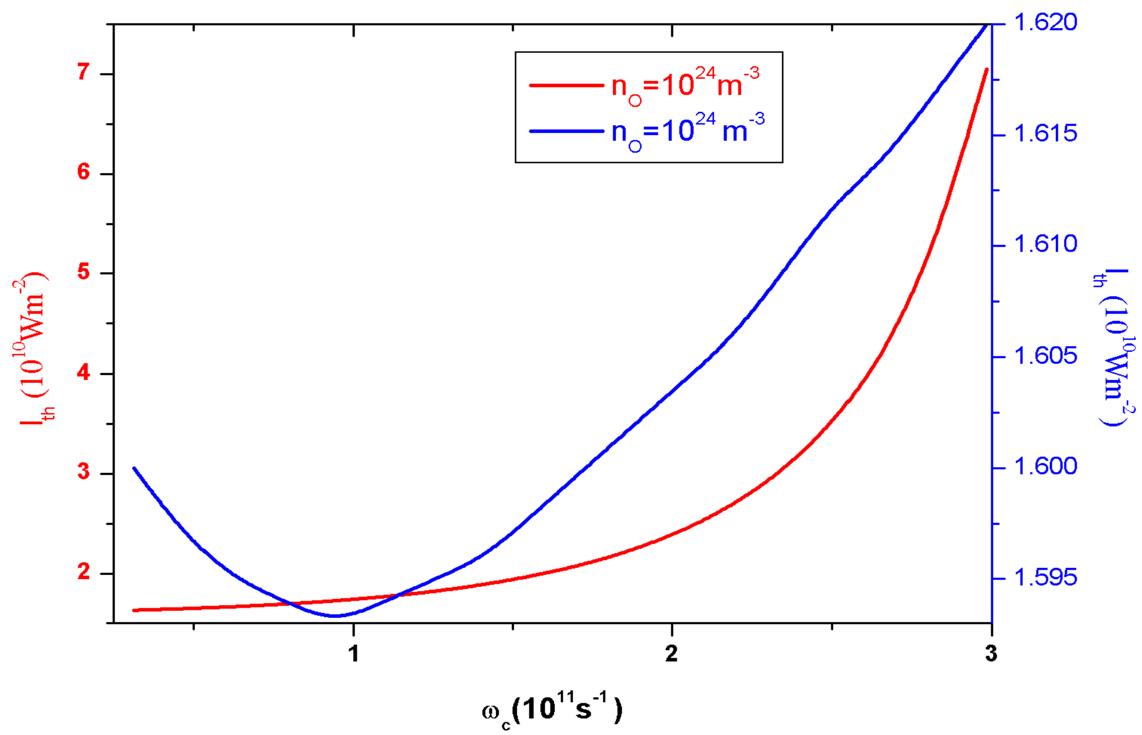

Fig. 1 Threshold Field Intensity Versus Cyclotron Frequency

Fig. 1 displays the variation of threshold electric field intensity with cyclotron frequency. This figure shows the impact of magnetic field on dispersion characteristics at different doping concentrations. Blue and Red curves shows the variation of threshold field intensity with the cyclotron frequency at $n_O=10^{23}m^{-3}$ and $n_O=10^{24}m^{-3}$ respectively. It can be inferred from figure that in the curve-I (Red line), increment in cyclotron frequency causes increment in threshold field intensity whereas in the second curve (Blue line), initially at lower magnitude of cyclotron frequency, threshold field intensity decreases and gets minimum at $\omega_c \approx 1\times10^{11} s^{-1}$ due to resonance between relativistically modified plasma frequency and acoustic wave frequency. Further increments in cyclotron frequency cause an increase in the threshold field intensity.

Hence above curves depicts that comparatively higher pump field intensity is required for higher doping concentration. This result could be utilized for the cost effective manufacturing of parametric devices. Such magnitude of threshold field intensity (i.e., $10^{10}Wm^{-2}$) reported in present paper can be obtained by using Nd:YAG laser which is experimentally feasible and theoretically accepted.

3.2 Nonlinear Dispersion Characteristics

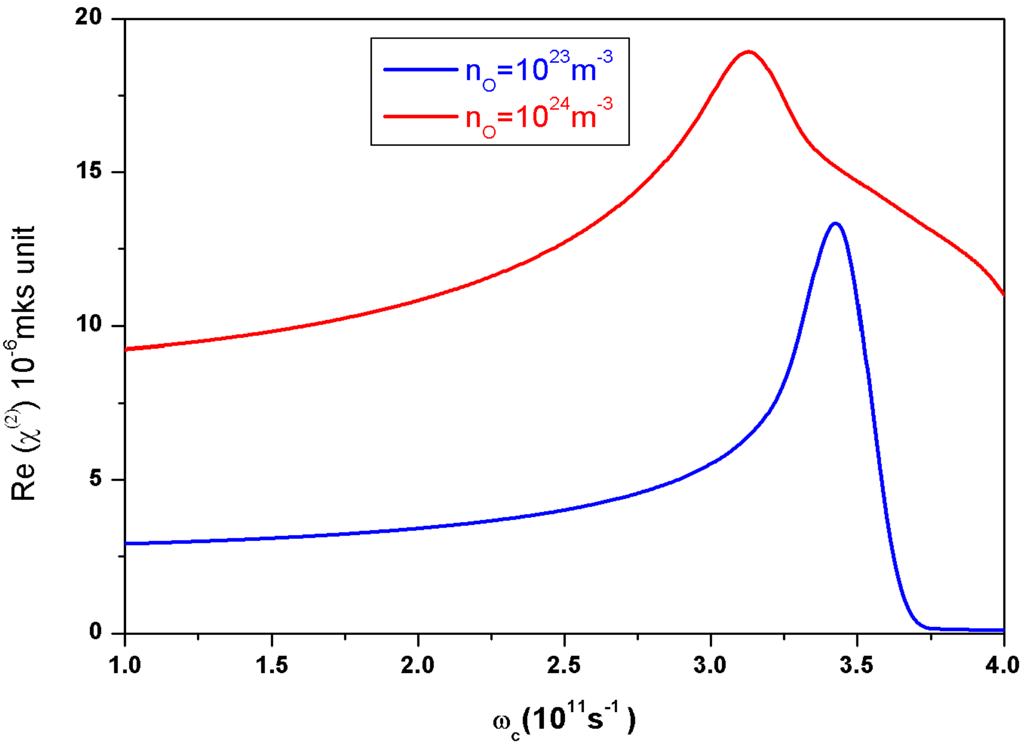

Fig. 2: Dispersion Coefficient Versus Cyclotron Frequency

Figure 2 displays influence of magnetic field on dispersion characteristics of the medium at different doping concentrations. Both the curves are nearly identical in behaviour with respect to variation in magnetic field, the only difference lies in the fact that position of maxima of $\chi_r^{(2)}$ gets shifted towards lower value of magnetic field for the higher doping concentration.

One more significant observation is evident from the curves that 10times higher $\chi_r^{(2)}$ is obtained at lower doping concentration. Hence n-type doping in the crystal with lower concentration is found to be beneficial. However, to avoid the total reflection of the electromagnetic pump wave in the intervening medium, the condition $\omega_p^2 > \omega_0^2$ is considered.

Estimated magnitudes of effective second order nonlinear optical susceptibility has been illustrated in table (1)-

Table 1: Influence of magnetic field on Second-order nonlinear susceptibility at $n_O \approx 2\times10^{23} m^{-3}, k_a \approx 2.11\times10^{11} m^{-1}$

	With B (B=0.14T)	Without B (B=0)
I_{th}	$\approx 10^{10} Wm^{-2}$	$\approx 10^{11} Wm^{-2}$
$\chi_r^{(2)}$	$\approx 2.8\times10^{-6}$ *mksunit*	$\approx 8.82\times10^{-7}$ *mksunit*
$\chi^{(2)}$	$\approx 2.86\times10^{-6}$ *mksunit*	$\approx 8.65\times10^{-7}$ *mksunit*

Table 1 illustrates that presence of magnetic field significantly affects the magnitude of second order nonlinear optical susceptibility and threshold field intensity. It is notable that comparatively 10 times higher effective susceptibility is observed with magnetic field. However, magnitude of $\chi^{(2)}$ is found to agree with other studies (Chouhan et al., 2019, Ghosh et al., 2010, Ghosh et al., 2018, Neogi and Ghosh, 1989). It may be concluded that heavily doped semiconductors are the appropriate hosts to study the relativistic effects on second order optical nonlinearities of semiconductor plasma medium. Presence of magnetic field increases optical nonlinearity in the relativistic laser plasma medium. Inclusion of magnetic field is found to have favourable threshold and dispersion characteristics. Hence it can be concluded that interdependence of doping concentration and magnetic field leads to some interesting results. Nd:YAG laser system utilized in this study is capable to achieve the reported amplitude of threshold field intensity. This fact could drastically reduce the operating cost of parametric amplifiers and other related nonlinear devices based on this interaction.

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