

# On the Squeeze Film Characteristics of Finite Journal Bearings Lubricated with Micropolar Fluid.

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**Abstract:** In this paper, the effect on the static and dynamic characteristic of squeeze film lubrication in finite porous journal bearings is studied. The finite modified Reynolds equation is solved numerically by using finite difference technique and the squeeze film characteristics are obtained. According to the results evaluated, the micropolar fluid effect significantly increases the film pressure and load carrying capacity as compared with the Newtonian case. Under cyclic load, the effect of micropolar fluid is to reduce the velocity of the journal centre.

**Key words:** journal bearings, Squeeze films, Micropolar fluids,

## I. INTRODUCTION

The squeeze film lubrication phenomenon is observed in several applications such as gears, bearings, machines tools, rolling elements and automotive engines. The squeeze film action is also seen during approach of faces of disc clutches under lubricated condition. The squeeze film phenomenon arises when the two lubricating surfaces move towards each other in the normal direction and generates a positive pressure and hence supports a load. This is due to the fact that a viscous lubricant present between the two surfaces cannot be instantaneously squeezed out when the two surfaces move towards each other and this action provides a cushioning effect in bearings. The effect of surface roughness on the static and dynamic behaviour of squeeze film lubrication of short journal bearings with micropolar fluids is studied by Naduvinamani et.al.[1].

Porous bearing contains the porous filled with lubricating oil so that the bearing requires no further lubrication during the whole life of the machine self-lubricated bearings or oil retaining bearings exhibit this feature. Self lubricating porous bearings have the advantage of high production rate because, short sintering time is required. Graphite is added to enhance the self lubricating property of the bearings. Porous metal bearings are widely used in home appliance, small motors, instruments and construction equipments because of their low cost and good bearing qualities. The analytical study of porous bearings with hydrodynamic conditions was first made by Morgan and Cameron [2]. There have been numerous studies of various types of porous bearings in literature viz; slider bearings [3-4], journal bearings [5-7], squeeze film bearings [8-13]. An extensive study of porous bearings has been made during the last few decades [14-16].

Recently, the studies of porous bearing are focused on Newtonian lubricants. However, the use of non-Newtonian fluids as lubricants is of growing interest in recent times. The pulsating or reciprocating loads on bearings and bearing surfaces are produced in several machine components. Due to this the oil film breaks down and relatively high friction and wear are to be expected. When the conditions are favorable an oil film is maintained between the contacting surfaces when the relative motion is momentarily zero. When the load is relived or reversed the lubricated film can recover its thickness before the next. Cycle if the bearing has been designed to permit this build up. Such phenomenon is observed in reciprocating machines in which the bearings are subjected to fluctuating dynamic loads. When the bearings are subjected to reciprocating loads the lubricants may become contaminated with dirt and metal particals then the lubricant behaves as a fluid suspension. The classical Newtonian theory will not predict the accurates flow behaviour of fluid suspensions especially when the clearance in the bearing is comparable with average size of the lubricant additives. The Erigen's [17] microcontinuum theory of micropolar fluid accounts for the polar effects.

Based on the Eringen microcontinuum theory, so for no attempt has been made to study the squeeze film characteristics of static and dynamic characteristic with micropolar fluids as a lubricant. Hence, in this paper, an attempt has been made to analyze the effect of micropolar fluid on these bearings.

1 MATHEMATICAL FORMULATION OF THE PROBLEM

The physical configuration of the problem under consideration is shown in the figure 1. The journal of radius R approaches the porous bearing surface at a circumferential section,  $\theta$  with velocity,  $V \left( = \frac{\partial h}{\partial t} \right)$  The film thickness  $h$  is a function of  $\theta$  and is given by

$$h = c + e \cos \theta \tag{1}$$

where ‘c’ is radial clearance and ‘e’ is the eccentricity of the journal centre. The lubricant in the film region and also in the porous region is assumed to be Eringen’s [17] micropolar fluid.

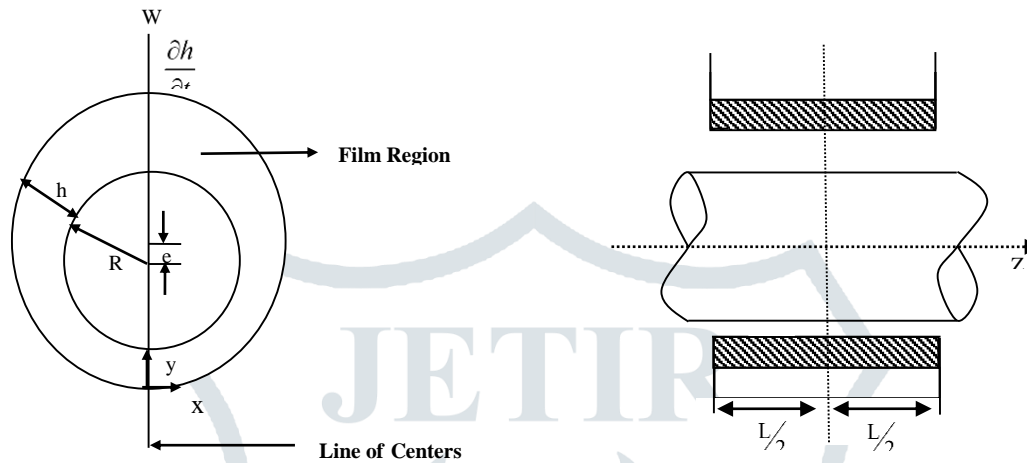


Fig 1. Physical configuration of a finite journal bearing.

The constitutive equations for micropolar fluids proposed by Eringen [17] simplify considerably under the usual assumptions of hydrodynamic lubrication. The resulting equations under steady-state conditions are

Conservation of linear momentum:

$$\left( \mu + \frac{\chi}{2} \right) \frac{\partial^2 u}{\partial y^2} + \chi \frac{\partial v_3}{\partial y} - \frac{\partial p}{\partial x} = 0, \tag{2}$$

$$\left( \mu + \frac{\chi}{2} \right) \frac{\partial^2 w}{\partial y^2} - \chi \frac{\partial v_1}{\partial y} - \frac{\partial p}{\partial y} = 0. \tag{3}$$

Conservation of angular momentum:

$$\gamma \frac{\partial^2 v_1}{\partial y^2} - 2 \chi v_1 + \chi \frac{\partial w}{\partial y} = 0, \tag{4}$$

$$\gamma \frac{\partial^2 v_3}{\partial y^2} - 2 \chi v_3 - \chi \frac{\partial u}{\partial y} = 0. \tag{5}$$

Conservation of mass:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{6}$$

Where  $(u, v, w)$  are the velocity components of the lubricant in the  $x, y$  and  $z$  directions, respectively, and  $(v_1, v_2, v_3)$  are micro rotational velocity components,  $\chi$  is the spin viscosity and  $\gamma$  is the viscosity coefficient for micropolar fluids and  $\mu$  is the Newtonian viscosity coefficient.

The relevant boundary conditions are

(a) at the bearing surface  $(y=0)$

$$u = 0, v = 0, w = 0 \tag{10a}$$

$$v_1 = 0, v_3 = 0 \tag{10b}$$

(b) at the journal surface  $(y=h)$

$$u=0, v=\frac{\partial h}{\partial t}, w=0 \quad (11a)$$

$$v_1=0, v_3=0 \quad (11b)$$

### 3 SOLUTION OF THE PROBLEM

The solution of equations (2) - (5) subject to the corresponding boundary conditions given in the equations (10a), (10b) and (11a), (11b) is obtained in the form.

$$u = \frac{1}{\mu} \left( \frac{y^2}{2} \frac{\partial p}{\partial x} + A_{11} y \right) - \frac{2N^2}{m} \times [A_{21} \sinh(my) + A_{31} \cosh(my)] + A_{41} \quad (12)$$

$$w = \frac{1}{\mu} \left( \frac{y^2}{2} \frac{\partial p}{\partial z} + A_{12} y \right) - \frac{2N^2}{m} \times [A_{22} \sinh(my) + A_{32} \cosh(my)] + A_{42} \quad (13)$$

$$v_1 = \frac{1}{2\mu} \left( y \frac{\partial p}{\partial z} + A_{12} \right) + A_{22} \cosh(my) + A_{32} \sinh(my) \quad (14)$$

$$v_3 = A_{21} \cosh(my) + A_{31} \sinh(my) - \frac{1}{2\mu} \left( y \frac{\partial p}{\partial x} + A_{11} \right) \quad (15)$$

Where  $A_{11} = 2\mu A_{21}$ ,

$$A_{21} = \frac{A_{31} \sinh(mh) - \frac{h}{2\mu} \frac{\partial p}{\partial x}}{1 - \cosh(mh)},$$

$$A_{12} = -\frac{h}{2\mu} \frac{\partial p}{\partial z} \left[ h \sinh(mh) + \frac{2N^2}{m} (1 - \cosh(mh)) \right] \times \frac{1}{A_5},$$

$$A_{22} = \frac{A_{12}}{2\mu},$$

$$A_{31} = \frac{h}{2\mu} \frac{\partial p}{\partial x} \left[ \frac{h}{2} (\cosh(mh) - 1) + h - \frac{N^2}{m} \sinh(mh) \right] \times \frac{1}{A_5},$$

$$A_{32} = \frac{1}{\mu} \frac{\partial p}{\partial z} \left[ \frac{h}{2} (\cosh(mh) - 1) + h - \frac{N^2}{m} \sinh(mh) \right] \times \frac{1}{A_5},$$

$$A_{41} = \frac{2N^2}{m} A_{31},$$

$$A_{42} = \frac{2N^2}{m} A_{32},$$

$$A_5 = \frac{h}{\mu} \left[ \sinh(mh) - \frac{2N^2}{mh} (\cosh(mh) - 1) \right],$$

in which

$$m = \frac{N}{l}, \quad N = \left( \frac{\chi}{\chi + 2\mu} \right)^{\frac{1}{2}}, \quad l = \left( \frac{\gamma}{4\mu} \right)^{\frac{1}{2}}.$$

Where  $N$  is the non-dimensional parameter called coupling number for it characterizes the coupling of linear and angular momentum equations. When  $N$  is identically zero, the equations of linear and angular momentum are decoupled and the equation of linear momentum reduces to classical Navier – Stokes equations. The parameter  $l$  is called the characteristic length for it characterizes the interaction between micropolar fluid and the film gap. The dimension of parameter  $l$  is length and can be identified as a size of microstructure additives present in the lubricant. In the limiting case of  $l \rightarrow 0$ , the effect of microstructure becomes negligible.

The modified Reynolds equation is obtained by integrating the equation of continuity (6) with respect to  $y$  over the film thickness,  $h$  and replacing  $u$  and  $w$  in equation (6) by their corresponding expressions given in equations (12) and (13) and also using the boundary conditions for  $v$  given in equations (10a) and (11a) in the form.

$$\frac{\partial}{\partial x} \left[ f(N, l, h) \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[ f(N, l, h) \frac{\partial p}{\partial z} \right] = 12\mu \frac{\partial h}{\partial t} \tag{19}$$

Where

$$f(N, l, h) = h^3 + 12l^2 h - 6Nlh^2 \coth\left(\frac{Nh}{2l}\right),$$

$$\frac{\partial h}{\partial t} = c \frac{\partial \varepsilon}{\partial t} \cos \theta .$$

Introducing the non-dimensional scheme into equation (19)

$$\theta = \frac{x}{R}, \quad \bar{z} = \frac{z}{L}, \quad \bar{l} = \frac{l}{c}, \quad \bar{h} = \frac{h}{c} = 1 + \varepsilon \cos \theta ,$$

$$\bar{p} = \frac{p c^2}{\mu R^2 \left(\frac{d\varepsilon}{dt}\right)}, \quad N = \left(\frac{\chi}{\chi + 2\mu}\right)^{\frac{1}{2}}, \quad \lambda = \frac{L}{2R}$$

The modified Reynolds equation (19) can be written in a non-dimensional form as

$$\frac{\partial}{\partial \theta} \left\{ \left[ \bar{f}(N, \bar{l}, \bar{h}) \right] \frac{\partial \bar{p}}{\partial \theta} \right\} + \left(\frac{1}{4\lambda^2}\right) \times \frac{\partial}{\partial \bar{z}} \left\{ \left[ \bar{f}(N, \bar{l}, \bar{h}) \right] \frac{\partial \bar{p}}{\partial \bar{z}} \right\} = 12 \cos \theta \tag{20}$$

where  $\bar{f}(N, \bar{l}, \bar{h}) = \bar{h}^3 + 12\bar{l}^2 \bar{h} - 6N\bar{l}\bar{h}^2 \coth\left(\frac{N\bar{h}}{2\bar{l}}\right)$ .

As the permeability parameter  $\psi \rightarrow 0$ , equation (20) reduces to the corresponding solid case. For the journal bearing, the boundary conditions for the fluid film pressure are

$$\bar{p} = 0 \quad \text{at} \quad \theta = \frac{\pi}{2}, \frac{3\pi}{2} \quad \text{and} \quad \bar{p} = 0 \quad \text{at} \quad \bar{z} = \pm \frac{1}{2} \tag{21}$$

The modified Reynolds equation will be solved numerically by using a finite difference scheme. The film domain under consideration is divided by grid spacing shown in figure 2. In finite increment format, the terms of equation (20) can be expressed as

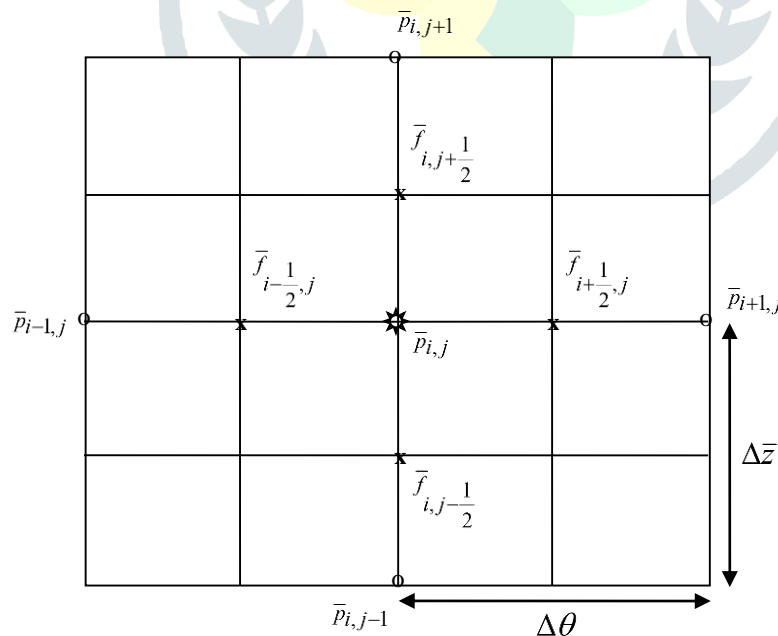


Fig. 2. Grid point notation for film domain

$$\frac{\partial}{\partial \theta} \left[ \left\{ \bar{f} \right\} \frac{\partial \bar{P}}{\partial \theta} \right] = \frac{1}{\Delta \theta} \left[ \left( \bar{f}_{i+1/2,j} \right) \left( \frac{\bar{P}_{i+1,j} - \bar{P}_{i,j}}{\Delta \theta} \right) - \left( \bar{f}_{i-1/2,j} \right) \left( \frac{\bar{P}_{i,j} - \bar{P}_{i-1,j}}{\Delta \theta} \right) \right]$$

$$\frac{1}{4\lambda^2} \times \frac{\partial}{\partial \bar{z}} \left[ \left\{ \bar{f} \right\} \frac{\partial \bar{P}}{\partial \bar{z}} \right] = \frac{1}{4\lambda^2} \times \frac{1}{\Delta \bar{z}} \left[ \left( \bar{f}_{i,j+1/2} \right) \left( \frac{\bar{P}_{i,j+1} - \bar{P}_{i,j}}{\Delta \bar{z}} \right) - \left( \bar{f}_{i,j-1/2} \right) \left( \frac{\bar{P}_{i,j} - \bar{P}_{i,j-1}}{\Delta \bar{z}} \right) \right]$$

Substituting these expressions (22) and (23) into the Reynolds equation (20) we get

$$\bar{P}_{i,j} = C_1 \bar{P}_{i+1,j} + C_2 \bar{P}_{i-1,j} + C_3 \bar{P}_{i,j+1} + C_4 \bar{P}_{i,j-1} + C_5 \quad (24)$$

where

$$C_0 = 4\lambda^2 r^2 \left\{ \left( \bar{f}_{i+1/2,j} \right) + \left( \bar{f}_{i-1/2,j} \right) + \left( \bar{f}_{i,j+1/2} \right) + \left( \bar{f}_{i,j-1/2} \right) \right\}$$

$$C_1 = 4\lambda^2 r^2 \left( \bar{f}_{i+1/2,j} \right) / C_0,$$

$$C_2 = 4\lambda^2 r^2 \left( \bar{f}_{i-1/2,j} \right) / C_0,$$

$$C_3 = \left( \bar{f}_{i,j+1/2} \right) / C_0,$$

$$C_4 = \left( \bar{f}_{i,j-1/2} \right) / C_0,$$

$$C_5 = -48\lambda^2 \cos \theta_i \Delta \bar{z}^2 / C_0, \quad r = \frac{\Delta \bar{z}}{\Delta \theta}.$$

The pressure,  $\bar{p}$  is calculated by using the numerical method with grid spacing of  $\Delta \theta = 9^\circ$  and  $\Delta \bar{z} = 0.05$ .

The load carrying capacity of the bearing,  $W$  generated by the film pressure is obtained by

$$W = -LR \int_{\theta=\pi/2}^{\theta=3\pi/2} \int_{z=-1/2}^{z=1/2} p \cos \theta d\theta dz \quad (25)$$

The non-dimensional load carrying capacity,  $\bar{W}$  of the journal bearing is obtained in the form

$$\bar{W} = \frac{Wc^2}{\mu LR^3 \left( \frac{d\varepsilon}{dt} \right)} = - \int_{\theta=\pi/2}^{\theta=3\pi/2} \int_{\bar{z}=-1/2}^{\bar{z}=1/2} \bar{P} \cos \theta_i d\theta d\bar{z} \quad (26)$$

$$\approx \sum_{i=0}^M \sum_{j=0}^N \bar{P}_{i,j} \cos \theta_i \Delta \theta \Delta \bar{z} = g(\varepsilon, \bar{l}, N, \bar{h}) \quad (27)$$

where  $M+1$  and  $N+1$  are the grid point numbers in the  $x$  and  $z$  directions respectively.

Time-height relation is calculated by considering the time taken by the journal to move from  $\varepsilon = 0$  to  $\varepsilon = \varepsilon_1$  can be obtained from equation (27)

$$\frac{d\varepsilon}{d\tau} = \frac{1}{g(\varepsilon, \bar{l}, N, \bar{h})} \quad (28)$$

Where  $\tau = \frac{Wc^2}{\mu LR^3} t$  is the non-dimensional response time.

The first order non-linear differential equation (28) is solved numerically by using the fourth order Runge-Kutta method with the initial conditions  $\varepsilon = 0$  to  $\tau = 0$ .

#### 4 RESULTS AND DISCUSSIONS

To solve squeeze film pressure in the equation (24) the mesh of the film domain has 20 equal intervals along the bearing length and circumference. The co-efficient matrix of the system of algebraic equations is of pentadiagonal form. These equations have been solved by using Scilab tools.

The effect on the static and dynamic behaviors of squeeze film in finite porous journal bearings has been studied on the basis of Eringen microcontinuum theory for micropolar fluids.

The squeeze film lubrication characteristic of a finite porous journal bearings lubricated with micropolar fluids are obtained on the basis of various non-dimensional parameters such as the coupling number,  $N \left( = \frac{\chi}{\chi + 2\mu} \right)^{1/2}$  which characterizes the coupling between the Newtonian and microrotational viscosities, the parameter,  $\bar{l} \left( = \frac{l}{c} \right)$  in which  $\bar{l}$  has the dimension of length and may be considered as chain length of microstructures additives. The parameter  $\bar{l}$ , characterizes the interaction of the bearing geometry with the lubricant properties. In the limiting case as  $\bar{l} \rightarrow 0$  the effect of microstructures becomes negligible. The effect of permeability is observed through the non-dimensional permeability parameter,  $\psi \left( = \frac{kH_0}{c^3} \right)$  and it is to be noted that as  $\psi \rightarrow 0$  the problem reduces to the corresponding solid case and as  $\bar{l}, N \rightarrow 0$  it reduces to the corresponding Newtonian case. The static and dynamic squeeze film characteristics for finite porous journal bearings with no journal rotation have been computed by using equation (24), (27) and (29) and are presented in figures 3 to 11.

#### 4.1 SQUEEZE FILM PRESSURE

The variation of non-dimensional squeeze film pressure  $\bar{p}$  with the circumferential co-ordinate  $\theta$  for different values of  $\bar{l}$  is depicted in the fig .3. It is observed that  $\bar{p}$  increases for increasing values of  $\bar{l}$ . Increases in  $\bar{p}$  is more pronounced for larger value of  $\bar{l}$ . The variation of non-dimensional film pressure  $\bar{p}$  with the circumferential co-ordinate  $\theta$  for different values of  $N$  is depicted in the fig .4. It is observed that  $\bar{p}$  increases for increasing value of  $N$ . Increases in  $\bar{p}$  is more pronounced for larger value of  $N$ . The effect of permeability  $\psi$  on the variation of  $\bar{p}$  is shown in fig.5. It is observed that the increasing values of permeability parameter  $\psi$  decreases  $\bar{p}$ .

#### 5 CONCLUSIONS

The effect of micropolar on the static and dynamic characteristics of squeeze film lubrication of finite porous journal bearings is theoretically studied by using the Eringen's constitutive equations for micropolar fluid theory. The film pressure distribution is solved numerically by using finite difference technique with a grid spacing of  $\Delta\theta = 9^0$  and  $\Delta\bar{z} = 0.05$ . According to the results evaluated of the following conclusions are drawn below.

- 1) The micropolar effect significantly increases the film pressure and load carrying capacity as compared with the corresponding Newtonian case.
- 2) Under a cyclic load, the effect of micropolar is to reduce the velocity of the journal centre of the squeeze film.
- 3) The effect of porous parameter causes reduction in pressure, load and enhancement in journal centre velocity.

#### NOMENCLATURE

$c$	radial clearance
$e$	eccentricity
$h$	film thickness ( $h = c + e \cos \theta$ )
$\bar{h}$	non-dimensional film thickness ( $= h/c$ )
$\bar{h}_0$	minimum film height
$H_0$	porous layer thickness
$k$	permeability of the porous matrix
$l$	characteristic length of the polar suspension $\left( = \left( \frac{\gamma}{4\mu} \right)^{1/2} \right)$
$\bar{l}$	non-dimensional form of $l$ ( $= l/c$ )
$L$	bearing length
$N$	coupling number $\left( = \left( \frac{\chi}{\chi + 2\mu} \right)^{1/2} \right)$

$p$  lubricant pressure

$\bar{p}$  non-dimensional pressure  $\left( = \frac{p c^2}{\mu R^2 \left( \frac{\partial \varepsilon}{\partial t} \right)} \right)$

$R$  radius of the journal

$t$  time

$u, v, w$  components of fluid velocity in  $x, y$  and  $z$  directions, respectively

$v_1, v_2, v_3$  microrotational velocity components in the  $x, y$  and  $z$  directions

$r, \theta, z$  cylindrical co-ordinates

$V$  squeeze velocity,  $\frac{\partial h}{\partial t} \left( = c \frac{\partial \varepsilon}{\partial t} \cos \theta \right)$

$W(s)$  steady load

$W(t)$  applied load

$\bar{W}$  non-dimensional load carrying capacity  $\left( = \frac{W(s) c^2}{\mu L R^3 \left( \frac{\partial \varepsilon}{\partial t} \right)} \right)$

$W_0$  amplitude of the applied cyclic load

$x, y, z$  Cartesian co-ordinates

$\varepsilon$  eccentricity ratio ( $= e/c$ )

$\chi$  spin viscosity

$\gamma$  viscosity co-efficient for micropolar fluids

$\mu$  viscosity co-efficient

$\tau$  dimensionless response time  $\left( = \left( \frac{2\pi N}{60} t \right) \right)$

$\theta$  circumferential co-ordinate ( $= x/R$ )

$\lambda$  length to diameter ratio ( $= L/2R$ )

$\omega$  frequency of applied cyclic load

$\Delta$  gradient operator

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**FIGURE CAPTIONS**

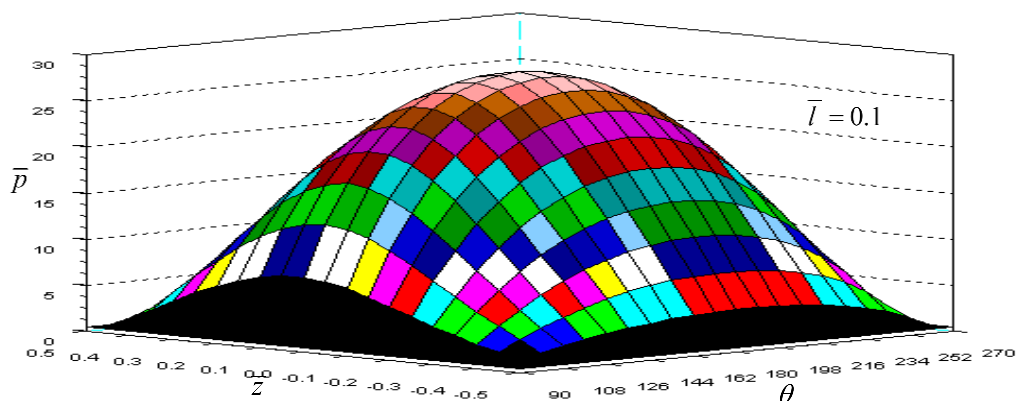
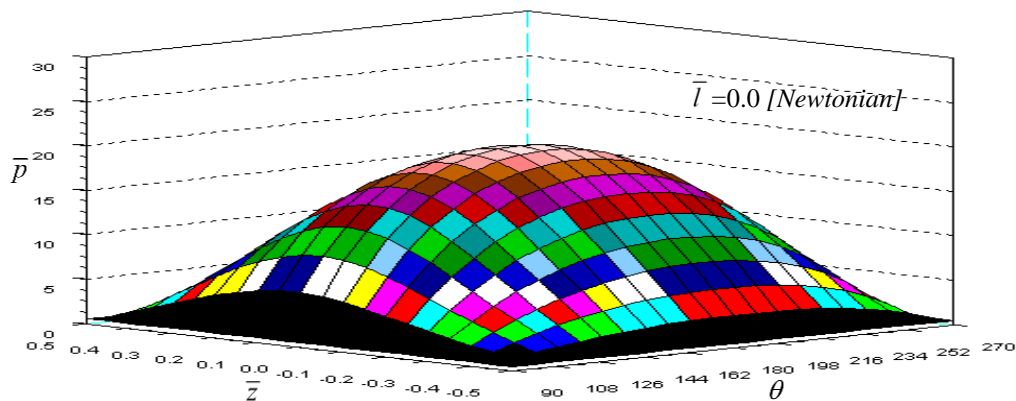
Fig 1: Physical configuration of a finite porous journal bearing.

Fig 2: Grid point notation for film domain.

Fig 3: Non-dimensional film pressure  $\bar{p}$  for different values of  $\bar{l}$  with  $N = 0.6, \lambda = 0.75, \varepsilon = 0.2$  and  $\psi = 0.01$

Fig 4: Non-dimensional film pressure  $\bar{p}$  for different values of  $N$  with  $\bar{l} = 0.2, \lambda = 0.75, \varepsilon = 0.2$  and  $\psi = 0.01$

Fig 5: Non-dimensional film pressure  $\bar{p}$  for different values of  $\psi$  with  $\bar{l} = 0.2, \lambda = 0.75, \varepsilon = 0.2$  and  $N = 0.6$





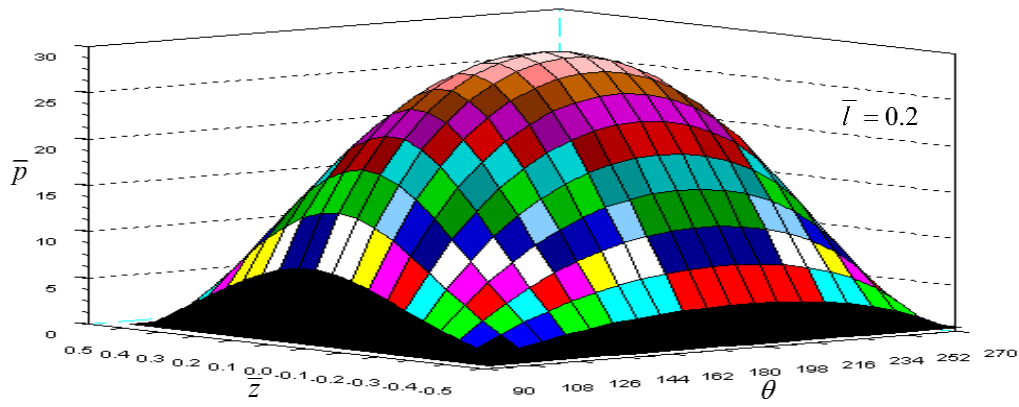


Figure 3. Non-dimensional film pressure  $\bar{p}$  for different values of  $\bar{l}$  with  $N = 0.6, \lambda = 1.2$  and  $\varepsilon = 0.4$ .

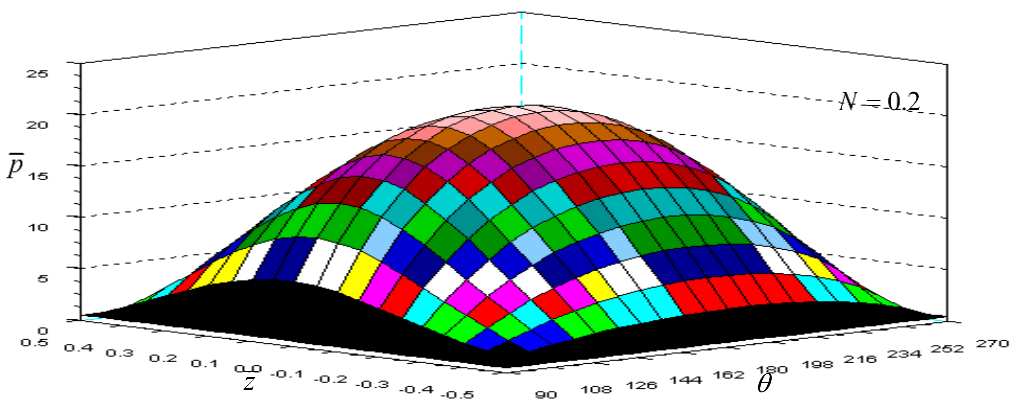
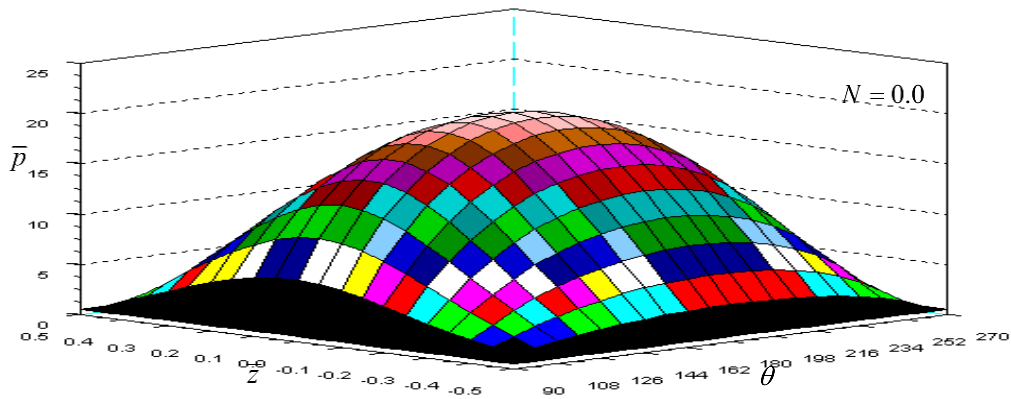


Figure 4: Non-dimensional film pressure  $\bar{p}$  for different values of  $N$  with  $\bar{l} = 0.2, \lambda = 0.75, \varepsilon = 0.2$  and  $\psi = 0.01$

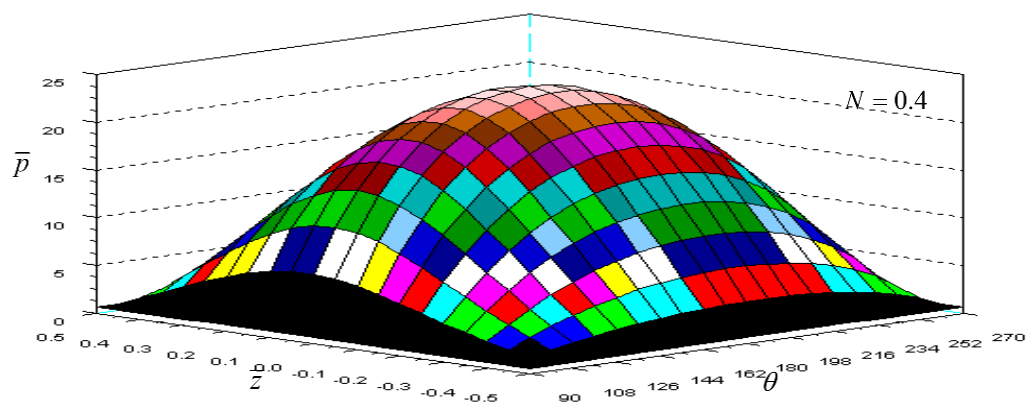


Figure 5: Non-dimensional film pressure  $\bar{p}$  for different values of  $\psi$  with  $\bar{l} = 0.2, \lambda = 0.75, \varepsilon = 0.2$  and  $N = 0.6$

