ANALYTIC AND NON- ANALYTIC FUNCTION AND THEIR CONDITIONS, PROPERTIES AND CHARACTERIZATION

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ABSTRACT

In Mathematics, an analytic function is a function that is locally given by a convergent power series. There exist both real analytic function & complex analytic functions. We can also say that A function is a Analytic iff it's Taylor series about x_o in its domain. Sometimes there exist smooth real functions that are not Analytic known as Non- Analytic smooth function.In fact there are many such functions.

Function is Analytic & Non- Analytic at some conditions which are mentioned. Properties and Characterization of Analytic and Non- Analytic functions are very vast and these are very helpful to solve our problems in many ways. Some of these properties are same as in Analytic function with several variables.

Keywords:- Analytic, Non- Analytic, function, meromorphic, entire, holomorphic, polygenic, non-monogenic, real, complex, continuous, differential, Quasi-Analytic function, etc.

Introduction:- The outlines of the theory of non-analytic functions of a complex variable, called also polygenic functions, have been stated in recent years in a number of articles.! Indeed, from the very first of the modern study of the theory of functions, going back at least as far as the famous inaugural dissertation of Riemann, the beginnings of the subject have been mentioned essentially, if for no other purpose than to state the conditions under which a function of a complex variable is analytic, and to delimit the field of functions to be studied.

In the present address, such preliminary details will be mentioned only briefly, with references ; but enough of them must be stated to develop a notation, and to give the proper setting. More detailed attention will be given to those developments which have taken place during the last decade, and to some hitherto unpublished facts. A brief review of some of the historical background will serve both its obvious purpose, and also that of introducing the necessary preliminary details and notations.

Historical Background. As was stated above, every careful presentation of the classical theory of functions of a complex variable did include in a measure the elementary ideas for the general case of *any* function of a complex variable. A function

$w = f(z) = \Phi\ (x, y) + \Psi\ (x, y) \ldots\ldots\ldots\ldots\ldots\ldots.(1)$

where $w = u+iv$ and $z=x+iy$, is said to be defined for a given region (or set of values) of z if w is determined whenever z is assigned a value in that region (or set). The equation (1) is then equivalent, of course, to the two real simultaneous equations

$u = \Phi\ (x, y),\ v = \Psi\ (x, y) \ldots\ldots\ldots\ldots\ldots\ldots.(2)$

which themselves express a transformation of the xy plane onto the uv plane. In general, we shall assume that Φ and Ψ are Continuous, and that they possess continuous first partial derivatives, unless the contrary is stated.

Analytic Function: - A complex function is said to be Analytic on a region of it is complex differentiable at every point in. The terms holomorphic function, differentiable function, & complex differentiable function are sometimes used interchangeably with "Analytic function". If a complex function is analytic on a region, it is infinitely differentiable in. A complex function may fail to be analytic at one or more points through the presence of singularities, or along lines or line segments through the presence of branch cuts. A complex function that is Analytic at all finite points of the complex plane is said to be entire or we can also say that if it is Analytic in whole complex plane known as entire function. Sinz, Cosz, e^z are entire function. If f(z) is analytic in region D except at finite number of poles known as Meromorphic function. All trigonometry functions are Meromorphic function.

Analyticity & Differentiability:- If f is analytic then f is continuous & differentiable but converse is not true i.e. continuous & differentiable function need not to be Analytic function.

Any Analytic function (real or complex) is Infinitely differentiable.

Conditions at which function is said to be Analytic and Non- Analytic function:- If f(z) = u + iv is analytic in D then f(z) is constant in D if any one of the following conditions holds

(a) f'(z) vanishes Identically in D.

(b) Re(f(z)) = u = constant

(c) Im (f(z)) = v = constant

(d) |f(z)| = constant

(e) arg f(z) = constant

(f) f(z) is Analytic at z= ∞

For Non- constant Analytic function: - non- constant analytic function cannot have uncountable number of zeros in C. It is unbounded also. If path is not continuous & differentiable then it is not smooth path then it is known as Non-constant analytic function. Non- constant Analytic function is also known as polygenic function.

Properties of Analytic function: - properties of Analytic functions are very vast but some of these are as:

- If f and g are analytic function then f+ g, f-g, f.g are also Analytic.
- 1/f is Analytic function if it is nowhere zero.
- Any analytic function is smooth is Infinitely differentiable but converse is not true.
- Analytic is Independent of path & Indefinite Integral exist.
- Analytic function is continuous, differentiable, Infinitely differentiable.
- In an analytic function omits two values, then it is constant.
- f(z) is analytic in domain D the $f^n(z)$ exist in D.
- A polynomial cannot be zero at too many points unless it is the zero polynomial.
- Let f & g be analytic in domain D, let each have zeros of order m & n at $z= z_o$. Then order of zeros of fg at $z= z_o$ is m + n. Similarly order of zeros of f+g is ≥ min{ m, n }.
- An Analytic function with constant modulus is constant.
- An analytic function with constant argument is also constant.
  i.e. if arg f(z) = constant then f(z) = constant.

Characteristics of Analytic function :- Analytic function implies many naming theorems which are used in many fields and we can get many methods to solve our problem. Some of are listed as :-

- ★ If f(z) is analytic Inside & on simple closed curve C & f(z) is not constant then maximum value of |f(z)| occurs on C.

- ★ If f(z) is analytic Inside & on a simple closed curve C, f(z) is not zero inside C then |f(z)| assume its minimum value on C.
  Above these two characteristics is also known as maximum modulus theorem and minimum modulus theorem respectively.
- ★ f is analytic in a domain D, if the set of zeros of f has a limit point in D, then f(z) ≡ 0 in D.
  - f & g are analytic in D, if the set of zeros of f-g has limit point in D , then f(z) ≡ g(z) ʊ z ε D.

  These states the theorem of Identity theorem.

- ★ One of the important characteristics of analytic function is used to find the number of roots known as Rouche's theorem which stated as f(z) & g(z) are Analytic inside & on a simple closed curve C and if |g(z)|<|f(z)| on C then f(z)+ g(z) & f(z) have same number of zeros inside C.

Characteristics of Non- constant Analytic function :- these are as

- ★ f is non- constant analytic on D then for any open set U in D, f(U) is open. i.e. It sends open subset of U to open subset of C. This is also known as Open- mapping Theorem.
- ★ Every Non-constant Analytic function omits at most one complex number as its value. This is known as Picard's theorem.
- ★ f(z) = u + iv & u not equals to v then f(U) is open for U¢G , Also f(1/n) = f(-1/n) = $1/n^2$ ʊ n ε N.

Conclusion: - As we discussed above Analytic and non- Analytic function in brief. Analytic function related with entire function, meromorphic function, holomorphic function and also with Cauchy Reiman conditions. Analytic and non- Analytic function differ with each other. Their properties, Characterization, and conditions under which they are Analytic or non- Analytic are also different. But some of properties of Analytic function with one variable is same as of Analytic functions with several variables. These are also used in rectifiable curve, piecewise continuous, branch cuts, logarithmic function, etc. This also shows that how it is used in Quasi-Analytic function with one or several variables.

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