# ECONOMIC ORDER QUANTITY MODEL WITH STOCK DEPENDENT DEMAND UNDER THE EFFECT OF INFLATION

Yatin Kumar Bhatnagar*, Afaq Ahmad, Ajay Kumar Gupta

*Research Scholar & Corresponding Author,

Department of Mathematics, Bhagwant University Ajmer, India.

**ABSTARCT**

In this paper we develop an inventory model under the assumption that a joint pricing, replenishment and preservation technology investment problem for non-instantaneous deteriorating items. Preservation technology affects both the length of non-deterioration period and deterioration rate. Shortages are allowed and partially backlogged. We use price-dependent and stock dependent demand, time-varying deterioration and waiting-time-dependent backlog rates in a general framework to formulate the model with time dependent holding cost.

**Key Words:** Two Warehouse, Decaying Items, time dependent Demand, shortages

## 1. Introduction:

In this chapter we study a joint pricing, replenishment and preservation technology investment problem for non-instantaneous deteriorating items. Preservation technology affects both the length of non-deterioration period and deterioration rate. Shortages are allowed and partially backlogged. We use price-dependent and stock dependent demand, time-varying deterioration and waiting-time-dependent backlog rates in a general framework to formulate the model with time dependent holding cost. We consider two cases: shortages happen after or before the non-deterioration period. We analytically show the existence and uniqueness of the optimal replenishment schedule, price or preservation investment for any given two of them in two cases. We also prove that there exists a global replenishment policy for any given pricing and preservation investment policies. We then provide an iterative algorithm to search for the optimal solution.

Deterioration is a common phenomenon in inventory management, especially for food industry. About 20% of food never reaches consumers' table because of spoilage (Sethi and Shruti 2006). Safeway grocery store stated that 63% of supermarket disposed waste in US comes from food industry and on average each employee throws away 3,000 pounds annually . All food products undergo certain degrees of deterioration due to physical, chemical and microbiological changes. The deterioration/spoilage of food products, such as fruit and vegetables, is no accident but a natural process. The item decomposes from its harvested moment, and maintains its desired quality attributes for a period called "shelf life". At the end of its shelf life, the item deteriorates to a quality point that is below the standard set by the retailer, or the item is not even edible to people. We refer to the time period that no items in a batch need to be disposed as "non-deterioration period".

Our chapter belongs to the area of inventory management for deteriorating items with preservation technology. for this a large number of papers have studied the inventory problem for deteriorating items under various market conditions, such as stock dependent demand rate (e.g., Zhang et al. 2015), time dependent demand rate (e.g., Pervin et al. 2016), price dependent demand rate (e.g., Jaggi et al. 2017) and so on.

This chapter is extension of paper "Pricing, replenishment and preservation technology investment decisions for non-instantaneous deteriorating items"(2019). In this chapter we extend the paper, demand as price and stock dependent, holding cost as time dependent under the effect of inflation.

**2.2 Assumption and Notation:**

This model is developed under the given assumptions and notations.

**2.2.1 Assumptions:**

The assumptions used in this manuscript are as follows:

1. The demand rate of items is stock and price dependent .i.e. (D = a – b*P – c* Q(t))

Where a , b, c are non-zero constants.

2. The time horizon is infinite.

3. The Rate of deterioration is constant .

4. The holding cost varies with time.

5. Shortages are partially backlogged and are fulfilled at the beginning of the next cycle.

6. Replenishment rate is considered to be infinite.

7. Inflation is considered in this Model.

8. Lead time is considered to be negligible.

**2.2.2 Notations:**

The summary of notations:

$Q^o$ Initial Inventory level.

$T_0$ Original non-deterioration period without preservation technology investment.

$T_d$ Non-deterioration period with preservation technology investment.

M Proportion of reduced deterioration rate with preservation technology investment.

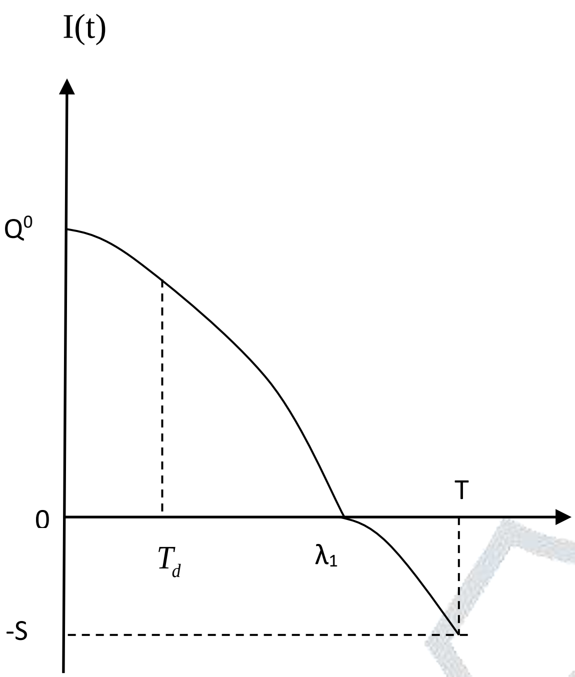
u purchasing cost per unit.

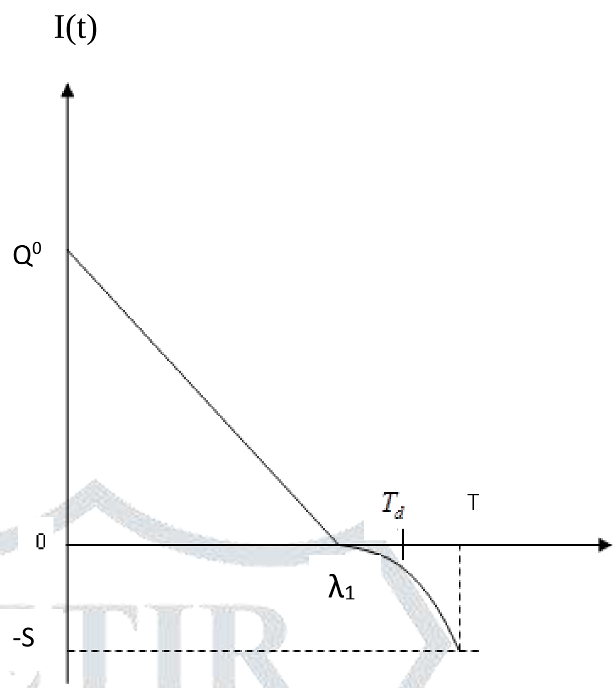
| | |
|---|---|
| p | retail price per unit. |
| D | variable demand rate. |
| P | coefficient of price dependent demand rate. |
| Q(t) | Inventory level at time "t" |
| s | Backlogged demand during shortage period. |
| $\lambda_1$ | Length of inventory holding period with s$\lambda_1 > 0$ |
| $\chi$ | Preservation technology investment per unit time. |
| A | Ordering cost for whole inventory. |
| T | Length of replenishment cycle. |
| H | Holding cost for whole inventory. |
| O | opportunity cost per unit lost sale. |
| r | Rate of inflation $0<r<1$. |
| s` | Shortage cost per unit backordered item per unit. |
| β | Backlogged coefficient $0<\beta<1$. |
| δ | Deterioration cost per unit per unit time. |
| θ | Deterioration rate. |
| $h_1$ ,$h_2$ | Non zero constants used in holding cost. |
| $TP_1$ | Total profit for one replenishment cycle. ($\lambda_1 \geq T_d$) |
| $TP_2$ | Total profit for one replenishment cycle. ($\lambda_1 \leq T_d$) |
| $AP_1$ | Average profit per unit time. ($\lambda_1 \geq T_d$) |
| $AP_2$ | Average profit for per unit time. ($\lambda_1 \leq T_d$) |

**2.3 Mathematical Formulation:**

In case 1, At initial state i.e. t = 0 the inventory level is $Q^0$ units of item. During t = 0 to $T_d$, deterioration is not considered and preservation technology is used. Deterioration starts during t = $T_d$ to $\lambda_1$. After time t = $\lambda_1$ shortage occurs.

In case 2, At initial state t = 0 the inventory is $Q^0$ units of item. Deterioration is not considering in this case. At time t = $T_d$ shortage occurs.




Case 1: $\lambda_1 \geq T_d$

Case 2: $\lambda_1 \leq T_d$

$Q^0$

**2.3.1 Case 1:** ($\lambda_1 \geq T_d$) Differential equation for inventory level is given by

$S_1$

$\lambda_1$

$$\frac{dQ_1(t)}{dt} = -D \text{.....................................................} for(0 \le t \le T_d), (1a)$$

$$\frac{dQ_1(t)}{dt} = -(a - bP(s) + cQ_1(t)) \text{.....................................................} (2a)$$

$$Q_1(t) = (a - bP)[(\lambda_1 - T_d) + \frac{c(\lambda_1^2 - T_d^2)}{2} + \frac{\theta(1-m)(\lambda_1^3 + T_d^3)}{6} -$$

$$\frac{T_d(\lambda_1^2 + T_d^2)}{2}]e^{-ct + \frac{\theta(1-m)T_d^2}{2}}$$

$$\frac{dQ_2(t)}{dt} = -D - \theta(1-m)(t - T_d)Q_2(t) \text{....................} (3a)\ for(T_d \le t \le \lambda_1)$$

$$Q_2(t) = (a - bP)((\lambda_1 - t) + \frac{c(\lambda_1^2 - t^2)}{2} + \theta(1-m)((\frac{\lambda_1^3 + t^3}{6}) -$$

$$T_d(\frac{\lambda_1^2 + t^2}{2})))e^{-ct - \theta(1-m)(\frac{t^2}{2} - tT_d)} \text{.....................................................} (4a)$$

$$\frac{dQ_3(t)}{dt} = -\beta D \text{.....................................................} for(\lambda_1 \le t \le T), (5a)$$

$$Q_3(t) = \frac{(a - bP)\beta c(\lambda_1 - t)}{c} \text{.....................................................} (6a)$$

*With initial conditions* $, Q_1(0) = Q^0, Q_1(T_d) = Q_2(T_d),$

$Q_2(\lambda_1) = Q_3(\lambda_1), Q_3(T) = 0$

**2.3.2 Case 2:** ($\lambda_1 \le T_d$) Differential equation for inventory level is given by

$$\frac{dQ_1(t)}{dt} = -D \text{..........} (1b)\ for(0 \le t \le \lambda_1)$$

$$Q_1(t) = \frac{(a - bP)(e^{c(\lambda_1 - t)} - 1)}{c} \text{.....................................................} (1b)$$

$$\frac{dQ_2(t)}{dt} = -\beta D \text{........} (2b)\ for(\lambda_1 \le t \le T)$$

$$Q_2(t) = \frac{\beta(a - bP)(e^{c(\lambda_1 - t)} - 1)}{c} \text{.....................................................} (2b)$$

*With initial conditions* $Q_1(0) = Q^0,\ Q_2(\lambda_1) = 0\ ,$

$Q_2(T) = -S_1$

$$Q^0 = \frac{(a - bP)(e^{c\lambda_1} - 1)}{c}$$

### 2.4 Costs for the Inventory Problem.

**Case: 2.4.1** ($\lambda_1 \geq T_d$) When time $\lambda_1$ is greater than or equal to the time $T_d$. The costs are as follows:

#### 2.4.1.1 Sales revenue cost:

The amount of money which obtained after selling inventory is known as total revenue cost or sales revenue cost. The formula for the cost is given as follows:

$$S.R.C. = \int_0^{\lambda_1} D(t) e^{-rt} dt \,.$$

Total revenue cost for the inventory model for case 1 is given as follows:

$$\begin{aligned}
S.R.C. = {} & (a-bP)\lambda_1 + \frac{c(a-bP)}{c+r}((\lambda_1 - T_d) + c(\lambda_1^2 - T_d)) + \theta(1-m)(\frac{\lambda_1 + T_d^2}{6}) \\
& -T_d(\frac{\lambda_1^2 + T_d^2}{2})(1 - \frac{\theta(1-m)T_d^2}{2})((c+r)T_d) + c(a-bP)((\frac{\lambda_1^2}{2} + \frac{(c-2T_d)}{3})\lambda_1^3 \\
& + \frac{\theta(1-m)\lambda_1^4}{3} - T_d(\lambda_1 + c\lambda_1^2 + \frac{\theta(1-m)\lambda_1^3}{6} - T_d^2(\frac{\lambda_1^2 - 1}{2}) + \frac{T_d^3 c}{6} - \frac{T_d^4}{6}(\theta(1-m) - 1)) \\
& + ((c + r - \theta(1-m)T_d)(\frac{\lambda_1^3}{6} + \lambda_1^4 \frac{(c-3T_d)}{8}) + \frac{7\theta(1-m)\lambda_1^5}{60} - T_d^2(\frac{\lambda_1}{2} + \frac{c\lambda_1^2}{4}) \\
& + T_d^3 \frac{(4 + 3\lambda_1^2)}{12} + \frac{T_d^4 c}{8} + T_d^5 \frac{(15 - 4\theta(1-m))}{120} + (\theta(1-m)(\frac{\lambda_1^4}{24} + \frac{\lambda_1^5(c + 4T_d)}{30} \\
& + \frac{\theta(1-m)\lambda_1^2}{24} - T_d^3 \frac{(2\lambda_1 + c\lambda_1^2)}{12} - \frac{\theta(1-m)\lambda_1^3}{36} - \frac{\lambda_1^2 T_d^4}{12} + \frac{cT_d^5}{20} - \frac{T_d^6(18 + 5\theta(1-m))}{360}
\end{aligned}$$

#### 2.4.1.2. Fixed ordering :

The amount of money which needs to order the inventory material is known as fixed ordering cost. This cost is fixed and let this cost is 'A'.

#### 2.4.1.3. Purchasing cost:

The amount of money which needs for purchase the inventory goods is known as purchasing cost. This amount is equal to 'u'(purchasing cost per unit time) times the sum of initial inventory level and inventory level at third level. The purshasing cost for inventory model is given as follows:

$$P.C. = u(Q^0 + Q_3)$$

$$\begin{aligned}
\text{P.C.} = {} & u((a-bP)(\lambda_1 - T_d) + \frac{c(\lambda_1^2 - T_d^2)}{2} + \frac{\theta(1-m)(T_d^3 + \lambda_1^3)}{6} \\
& - \frac{T_d(T_d^2 + \lambda_1^2)}{2} + (a-bP)\beta(\lambda_1 - t))
\end{aligned}$$

### 2.4.1.4. Preservation technology investment:

The amount of money which needs to stabilized the facility for preservation of inventory is known as preservation technology investment.

The cost for preservation technology investment in this model is given by '$\chi\, \lambda_1$'.

### 2.4.1.5. Holding cost:

The amount of money which needs to hold the inventory is known as holding cost. In this model we considered holding cost as time dependent as the expression for finding holding cost is as follows:

$$H.C. = \int_0^{\lambda_1} (h_1 + th_2) Q(t) e^{-rt} dt$$

$$H.C. = h_1(\frac{(a-bP)}{(c+r)}((\lambda_1 - T_d) + c(\lambda_1^2 - T_d)) + \theta(1-m)(\frac{\lambda_1 + T_d^2}{6}) - T_d(\frac{\lambda_1^2 + T_d^2}{2})$$

$$(1-\frac{\theta(1-m)T_d^2}{2})((c+r)T_d) + (a-bP)((\frac{\lambda_1^2}{2} + \frac{(c-2T_d)}{3})\lambda_1^3 +$$

$$\frac{\theta(1-m)\lambda_1^4}{3} - T_d(\lambda_1 + c\lambda_1^2 + \frac{\theta(1-m)\lambda_1^3}{6} - T_d^2(\frac{\lambda_1^2 - 1}{2}) + \frac{T_d^3 c}{6} - \frac{T_d^4}{6}(\theta(1-m)-1))$$

$$+((c+r-\theta(1-m)T_d)(\frac{\lambda_1^3}{6} + \lambda_1^4\frac{(c-3T_d)}{8}) + \frac{7\theta(1-m)\lambda_1^5}{60} - T_d^2(\frac{\lambda_1}{2} + \frac{c\lambda_1^2}{4})$$

$$+T_d^3\frac{(4+3\lambda_1^2)}{12} + \frac{T_d^4 c}{8} + T_d^5\frac{(15-4\theta(1-m))}{120} + (\theta(1-m)(\frac{\lambda_1^4}{24} + \frac{\lambda_1^5(c+4T_d)}{30} + \frac{\theta(1-m)\lambda_1^2}{24}$$

$$-T_d^3\frac{(2\lambda_1 + c\lambda_1^2)}{12} + \frac{\theta(1-m)\lambda_1^3}{36} - \frac{\lambda_1^2 T_d^4}{12} + \frac{cT_d^5}{20} - \frac{T_d^6(18+5\theta(1-m))}{360})$$

$$+h_2((a-bP)((\lambda_1 - T_d) + (c(\lambda_1^2 - T_d^2)) + \frac{\theta(1-m)(\lambda_1^3 + T_d^3)}{6} - \frac{T_d(\lambda_1^2 + T_d^2)}{2}$$

$$(\frac{T_d^2}{2} - \frac{(c+r)T_d^3}{3} - \frac{\theta(1-m)T_d^4}{4}) + (a-bP)((\frac{\lambda_1^3}{6} + \frac{c\lambda_1^4}{8} + \frac{7\theta(1-m)\lambda_1^5}{60} - \frac{3T_d\lambda_1^4}{8}$$

$$-\frac{(3\lambda_1 T_d^2 - 2T_d^3)}{6} - \frac{c(2\lambda_1^2 T_d^2 - T_d^4)}{8} - \theta(1-m)(\frac{\lambda_1^3 T_d^2}{12} + \frac{T_d^5}{30}) + \frac{T_d(2\lambda_1^2 T_d^2 + T_d^4)}{8}) -$$

$$(r+c+\theta(1-m)T_d)(\frac{\lambda_1^4}{12} + \frac{c\lambda_1^5}{15} + \frac{\theta(1-m)\lambda_1^6}{12}) - \frac{4T_d\lambda_1^5}{15} - \frac{(4\lambda_1 T_d^3 - 3T_d^4)}{12} - \frac{c(5\lambda_1^2 T_d^3 - 3T_d^5)}{30}$$

$$-\frac{\theta(1-m)(2\lambda_1^3 T_d^3 + T_d^6)}{36} + \frac{T_d(5\lambda_1^2 T_d^3 + 3T_d^5)}{30}) + \theta(1-m)(\frac{\lambda_1^5}{40} + \frac{c\lambda_1^6}{48} + \frac{2\theta(1-m)\lambda_1^7}{21} -$$

$$\frac{5T_d\lambda_1^6}{48} - \frac{(5\lambda_1 T_d^4 - 4T_d^5)}{40} - \frac{c(3\lambda_1^2 T_d^4 - 2T_d^6)}{24} - \frac{\theta(1-m)(7\lambda_1^3 T_d^4 + T_d^7)}{84} + T_d\frac{(3T_d^4\lambda_1^2 + 2T_d^6)}{48}))$$

### 2.4.1.6. Shortage cost:

The amount of money which is lost due to shortage of inventory is known as shortage cost. The shortage cost for this model is given as follows

$$S.C. = -s\int_{\lambda_1}^{T} Q_3(t)e^{-rt}dt$$

$$S.C. = -s\beta(a-bP)(\lambda_1(T+\frac{rT^2}{2})+\lambda_1{}^2\frac{(\lambda_1 r-1)}{2}-\frac{r\lambda_1{}^3}{3}-\frac{T^2}{2}+\frac{rT^3}{3}))$$

### 2.4.1.7. Deterioration cost:

The amount of money which is lost due to deterioration or damage of inventory is known as deterioration cost. The expression for deteriorating cost is given as follows:

$$D.C. = d\int_{T_d}^{\lambda_1} \theta(t-T_d)Q_2(t)e^{-rt}dt$$

$$\begin{aligned}
D.C. = {} & d\theta((a-bP)((\frac{\lambda_1^3}{6}+\frac{c\lambda_1^4}{8}+\frac{7\theta(1-m)\lambda_1^5}{60}-\frac{3T_d\lambda_1^4}{8}-\frac{(3\lambda_1T_d^2-2T_d^3)}{6}-\frac{c(2\lambda_1^2T_d^2-T_d^4)}{8} \\
& -\theta(1-m)(\frac{\lambda_1^3T_d^2}{12}+\frac{T_d^5}{30})+\frac{T_d(2\lambda_1^2T_d^2+T_d^4)}{8})-(r+c+\theta(1-m)T_d)(\frac{\lambda_1^4}{12}+\frac{c\lambda_1^5}{15}+\frac{\theta(1-m)\lambda_1^6}{12}) \\
& -\frac{4T_d\lambda_1^5}{15}-\frac{(4\lambda_1T_d^3-3T_d^4)}{12}-\frac{c(5\lambda_1^2T_d^3-3T_d^5)}{30}-\frac{\theta(1-m)(2\lambda_1^3T_d^3+T_d^6)}{36}+\frac{T_d(5\lambda_1^2T_d^3+3T_d^5)}{30}) \\
& +\theta(1-m)(\frac{\lambda_1^5}{40}+\frac{c\lambda_1^6}{48}+\frac{2\theta(1-m)\lambda_1^7}{21}-\frac{5T_d\lambda_1^6}{48}-\frac{(5\lambda_1T_d^4-4T_d^5)}{40}-\frac{c(3\lambda_1^2T_d^4-2T_d^6)}{24}- \\
& \frac{\theta(1-m)(7\lambda_1^3T_d^4+T_d^7)}{84}+T_d\frac{(3T_d^4\lambda_1^2+2T_d^6)}{48})-T_d+(a-bP)((\frac{\lambda_1^2}{2}+\frac{(c-2T_d)}{3})\lambda_1^3+ \\
& ((c+r-\theta(1-m)T_d)(\frac{\lambda_1^3}{6}+\lambda_1^4\frac{(c-3T_d)}{8})+\frac{7\theta(1-m)\lambda_1^5}{60}-T_d^2(\frac{\lambda_1}{2}+\frac{c\lambda_1^2}{4})\frac{\theta(1-m)\lambda_1^4}{3} \\
& -T_d(\lambda_1+c\lambda_1^2+\frac{\theta(1-m)\lambda_1^3}{6}-T_d^2(\frac{\lambda_1^2-1}{2})+\frac{T_d^3c}{6}-\frac{T_d^4}{6}(\theta(1-m)-1))+T_d^3\frac{(4+3\lambda_1^2)}{12}+\frac{T_d^4c}{8} \\
& +T_d^5\frac{(15-4\theta(1-m))}{120}+(\theta(1-m)(\frac{\lambda_1^4}{24}+\frac{\lambda_1^5(c+4T_d)}{30}+\frac{\theta(1-m)\lambda_1^2}{24}-T_d^3\frac{(2\lambda_1+c\lambda_1^2)}{12} \\
& \frac{\theta(1-m)\lambda_1^3}{36})-\frac{\lambda_1^2T_d^4}{12}+\frac{cT_d^5}{20}-\frac{T_d^6(18+5\theta(1-m)}{360}))
\end{aligned}$$

### 2.4.1.8. Opportunity Cost:

The amount of money which needs to pay by the inventor for penalty due to not completing the demand on time is known as opportunity cost or penalty cost. The expression for this cost is given as follows:

$$O.P.=o\int_{\lambda_1}^{T}(1-\beta)D(t)dt$$

$$=o(1-\beta)(a-bP)(\frac{r(T-\lambda_1)}{r}+c\beta(\lambda_1(T+\frac{rT^2}{2})+\lambda_1{}^2\frac{(\lambda_1 r-1)}{2}$$

$$-\frac{r\lambda_1{}^3}{3}-\frac{T^2}{2}+\frac{rT^3}{3})$$

**2.4.2 Case 2: ($\lambda_1 \le T_d$)** When time $\lambda_1$ is less than or equal to the time $T_d$. The costs are as follows:

### 2.4.2.1 Sales revenue cost:

The amount of money which obtained after selling inventory is known as total revenue cost or sales revenue cost. The formula for the cost is given as follows:

$$S.R.C.=\int_{0}^{\lambda_1}D(t)e^{-rt}dt$$

$$S.R.C.=\frac{(a-bP)(e^{c\lambda_1}-e^{-r\lambda_1})}{(c+r)}$$

### 2.4.2. 2. Fixed ordering:

The amount of money which needs to order the inventory material is known as fixed ordering cost. This cost is fixed and let this cost is 'A'.

### 2.4.2.3. Holding cost :

The amount of money which needs to hold the inventory is known as holding cost. In this model we considered holding cost as time dependent so the expression for finding holding cost is as follows:

$$H.C.=\int_0^{\lambda_1}(h_1+th_2)Q(t)e^{-rt}dt$$

$$H.C.=h_1\frac{(a-bP)(e^{c\lambda_1}-e^{-r\lambda_1})}{(c+r)}+h_2(a-bP)(\frac{(e^{-r\lambda_1}-1)}{(c+r)^2}-\lambda_1\frac{e^{-r\lambda_1}}{(c+r)})$$

### 2.4.2.4. Purchasing cost:

The amount of money which needs for purchase the inventory goods is known as purchasing cost. This amount is equal to 'u' (purchasing cost per unit time) times the invent tory level at first state. The purchasing cost for this model is given as follows:

$$P.C.=uQ_1$$

$$P.C.=u\frac{(a-bP)}{c}(e^{c(\lambda_1-t)}-1)$$

**2.4.2.5. Preservation technology investment:**

The amount of money which needs to stabilized the facility for preservation of inventory is known as preservation technology investment.

The cost for preservation technology investment in this model is given by '$\chi \lambda_1$'.

**2.4.2.6. Shortage cost:**

The amount of money which is lost due to shortage of inventory is known as shortage cost. The shortage cost for this model is given as follows

$$S.C. = -s\int_{\lambda_1}^{T} Q_2(t)e^{-rt}dt$$

$$S.C. == -s\beta\frac{(a-bp)}{c}\left(\frac{(e^{-r\lambda_1}-e^{c\lambda_1-(c+r)T})}{(c+r)}+\frac{(e^{-r\lambda_1}-e^{-r\lambda_1})}{r}\right)$$

**2.4.2.7. Opportunity Cost:**

The amount of money which needs to pay by the inventor for penalty due to not completing the demand on time is known as opportunity cost or penalty cost. The expression for this cost is given as follows:

$$O.C. = o\int_{\lambda_1}^{T}(1-\beta)D(t)e^{-rt}dt$$

$$= o(1-\beta)(a-bP)\left(\frac{(e^{-r\lambda_1}-e^{-rT})}{r}+\frac{(e^{-r\lambda_1}-e^{c\lambda_1-(c+r)T})}{(c+r)}+(e^{-rT}-e^{-r\lambda_1})\right)$$

**2.5 Total Inventory Costs for the Inventory Problem:** Total cost for inventory problem in both case are given as follows:

**2.5.1 Total cost ($TP_1$) for case 1:**

Seles revenue cost - Fixed ordering cost - Purchasing cost- Holding cost - Preservation technology investment cost - Deterioration cost - Shortage cost - Opportunity cost.

**2.5.2 Total cost ($TP_2$) for case 2:**

Seles revenue cost - Fixed ordering cost - Purchasing cost- Holding cost - Preservation technology investment cost - Shortage cost - Opportunity cost.

$$
\begin{aligned}
&\mathrm{T.P}_1(\lambda_1,T,P,\chi)=[\{(a-bP)\lambda_1+\frac{c(a-bP)}{c+r}((\lambda_1-T_d)+c(\lambda_1^2-T_d))+\theta(1-m)\\
&(\frac{\lambda_1+T_d^2}{6})\frac{\theta(1-m)\lambda_1^4}{3}-T_d(\lambda_1+c\lambda_1^2+\frac{\theta(1-m)\lambda_1^3}{6}-T_d^2(\frac{\lambda_1^2-1}{2})+\frac{T_d^3c}{6}-\frac{T_d^4}{6}\\
&(\theta(1-m)-1))-T_d(\frac{\lambda_1^2+T_d^2}{2})(1-\frac{\theta(1-m)T_d^2}{2})((c+r)T_d)+c(a-bP)((\frac{\lambda_1^2}{2}+\\
&\frac{(c-2T_d)}{3})\lambda_1^3+((c+r-\theta(1-m)T_d)(\frac{\lambda_1^3}{6}+\lambda_1^4\frac{(c-3T_d)}{8})+\frac{7\theta(1-m)\lambda_1^5}{60}-T_d^2\\
&(\frac{\lambda_1}{2}+\frac{c\lambda_1^2}{4})+T_d^3\frac{(4+3\lambda_1^2)}{12}+\frac{T_d^4c}{8}+T_d^5\frac{(15-4\theta(1-m))}{120}+(\theta(1-m)(\frac{\lambda_1^4}{24}+\\
&\frac{\lambda_1^5(c+4T_d)}{30}+\frac{\theta(1-m)\lambda_1^2}{24}-T_d^3\frac{(2\lambda_1+c\lambda_1^2)}{12}-\frac{\theta(1-m)\lambda_1^3}{36})-\frac{\lambda_1^2T_d^4}{12}+\frac{cT_d^5}{20}-\\
&\frac{T_d^6(18+5\theta(1-m))}{360}-A-\{u((a-bP)(\lambda_1-T_d)+\frac{c(\lambda_1^2-T_d^2)}{2}+\frac{\theta(1-m)(T_d^3+\lambda_1^3)}{6}\\
&-\frac{T_d(T_d^2+\lambda_1^2)}{2}+(a-bP)\beta(\lambda_1-t))\}\frac{\lambda_1^3}{6}+\lambda_1^4\frac{(c-3T_d)}{8})+\frac{7\theta(1-m)\lambda_1^5}{60}-T_d^2\\
&(\frac{\lambda_1}{2}+\frac{c\lambda_1^2}{4})+h_1(\frac{(a-bP)}{(c+r)}((\lambda_1-T_d)+c(\lambda_1^2-T_d))+\theta(1-m)(\frac{\lambda_1+T_d^2}{6})-T_d\\
&(\frac{\lambda_1^2+T_d^2}{2})(1-\frac{\theta(1-m)T_d^2}{2})+((c+r)T_d)+(a-bP)((\frac{\lambda_1^2}{2}+\frac{(c-2T_d)}{3})\lambda_1^3-\\
&\frac{\theta(1-m)\lambda_1^4}{3}-T_d(\lambda_1+c\lambda_1^2+\frac{\theta(1-m)\lambda_1^3}{6}-T_d^2(\frac{\lambda_1^2-1}{2})+\frac{T_d^3c}{6}-\frac{T_d^4}{6}\\
&(\theta(1-m)-1))+((c+r-\theta(1-m)T_d)(\frac{\lambda_1^3}{6}+\lambda_1^4\frac{(c-3T_d)}{8})+\frac{7\theta(1-m)\lambda_1^5}{60}\\
&-T_d^2(\frac{\lambda_1}{2}+\frac{c\lambda_1^2}{4})+T_d^3\frac{(4+3\lambda_1^2)}{12}+\frac{T_d^4c}{8}+T_d^5\frac{(15-4\theta(1-m))}{120}
\end{aligned}
$$

$$+(\theta(1-m)(\frac{\lambda_1^4}{24}+\frac{\lambda_1^5(c+4T_d)}{30}+\frac{\theta(1-m)\lambda_1^2}{24}-T_d^3\frac{(2\lambda_1+c\lambda_1^2)}{12}-\frac{\theta(1-m)\lambda_1^3}{36}-\frac{\lambda_1^2T_d^4}{12}+$$

$$\frac{cT_d^5}{20}-\frac{T_d^6(18+5\theta(1-m))}{360}+h_2((a-bP)((\lambda_1-T_d)+(c(\lambda_1^2-T_d^2))+\frac{\theta(1-m)(\lambda_1^3+T_d^3)}{6}-$$

$$\frac{T_d(\lambda_1^2+T_d^2)}{2}(\frac{T_d^2}{2}-\frac{(c+r)T_d^3}{3}-\frac{\theta(1-m)T_d^4}{4})+(a-bP)((\frac{\lambda_1^3}{6}+\frac{c\lambda_1^4}{8}+\frac{7\theta(1-m)\lambda_1^5}{60}-\frac{3T_d\lambda_1^4}{8}-$$

$$\frac{(3\lambda_1T_d^2-2T_d^3)}{6}-\frac{c(2\lambda_1^2T_d^2-T_d^4)}{8}-\theta(1-m)(\frac{\lambda_1^3T_d^2}{12}+\frac{T_d^5}{30})+\frac{T_d(2\lambda_1^2T_d^2+T_d^4)}{8})-(r+c+\theta(1-m)T_d)$$

$$(\frac{\lambda_1^4}{12}+\frac{c\lambda_1^5}{15}+\frac{\theta(1-m)\lambda_1^6}{12}-\frac{4T_d\lambda_1^5}{15}-\frac{(4\lambda_1T_d^3-3T_d^4)}{12}-\frac{c(5\lambda_1^2T_d^3-3T_d^5)}{30}-\frac{\theta(1-m)(2\lambda_1^3T_d^3+T_d^6)}{36}+$$

$$\frac{T_d(5\lambda_1^2T_d^3+3T_d^5)}{30})+\theta(1-m)(\frac{\lambda_1^5}{40}+\frac{c\lambda_1^6}{48}+\frac{2\theta(1-m)\lambda_1^7}{21}-\frac{5T_d\lambda_1^6}{48}-\frac{(5\lambda_1T_d^4-4T_d^5)}{40}$$

$$-\frac{c(3\lambda_1^2T_d^4-2T_d^6)}{24}-\frac{\theta(1-m)(7\lambda_1^3T_d^4+T_d^7)}{84}+T_d\frac{(3T_d^4\lambda_1^2+2T_d^6)}{48}))\}-\lambda_1\chi$$

$$d\theta((a-bP)((\frac{\lambda_1^3}{6}+\frac{c\lambda_1^4}{8}+\frac{7\theta(1-m)\lambda_1^5}{60}-\frac{3T_d\lambda_1^4}{8}-\frac{(3\lambda_1T_d^2-2T_d^3)}{6}-\frac{c(2\lambda_1^2T_d^2-T_d^4)}{8})$$

$$-\theta(1-m)(\frac{\lambda_1^3T_d^2}{12}+\frac{T_d^5}{30})+\frac{T_d(2\lambda_1^2T_d^2+T_d^4)}{8})-(r+c+\theta(1-m)T_d)(\frac{\lambda_1^4}{12}+\frac{c\lambda_1^5}{15}+\frac{\theta(1-m)\lambda_1^6}{12}$$

$$-\frac{4T_d\lambda_1^5}{15}-\frac{(4\lambda_1T_d^3-3T_d^4)}{12}-\frac{c(5\lambda_1^2T_d^3-3T_d^5)}{30}-\frac{\theta(1-m)(2\lambda_1^3T_d^3+T_d^6)}{36}+\frac{T_d(5\lambda_1^2T_d^3+3T_d^5)}{30})$$

$$+\theta(1-m)(\frac{\lambda_1^5}{40}+\frac{c\lambda_1^6}{48}+\frac{2\theta(1-m)\lambda_1^7}{21}-\frac{5T_d\lambda_1^6}{48}-\frac{(5\lambda_1T_d^4-4T_d^5)}{40}-\frac{c(3\lambda_1^2T_d^4-2T_d^6)}{24}-$$

$$\frac{\theta(1-m)(7\lambda_1^3T_d^4+T_d^7)}{84}+T_d\frac{(3T_d^4\lambda_1^2+2T_d^6)}{48})-T_d+(a-bP)((\frac{\lambda_1^2}{2}+\frac{(c-2T_d)}{3})\lambda_1^3$$

$$+((c+r-\theta(1-m)T_d)(\frac{\lambda_1^3}{6}+\lambda_1^4\frac{(c-3T_d)}{8})+\frac{7\theta(1-m)\lambda_1^5}{60}-T_d^2(\frac{\lambda_1}{2}+\frac{c\lambda_1^2}{4})$$

$$+T_d^3\frac{(4+3\lambda_1^2)}{12}+\frac{T_d^4c}{8}+T_d^5\frac{(15-4\theta(1-m))}{120}+(\theta(1-m)(\frac{\lambda_1^4}{24}+\frac{\lambda_1^5(c+4T_d)}{30}+\frac{\theta(1-m)\lambda_1^2}{24}$$

$$-T_d^3\frac{(2\lambda_1+c\lambda_1^2)}{12}\frac{\theta(1-m)\lambda_1^3}{36})-\frac{\lambda_1^2T_d^4}{12}+\frac{cT_d^5}{20}-\frac{T_d^6(18+5\theta(1-m)}{360}))\}-\{s\beta(a-bP)(\lambda_1(T+\frac{rT^2}{2})$$

$$+\lambda_1^2\frac{(\lambda_1r-1)}{2}-\frac{r\lambda_1^3}{3}-\frac{T^2}{2}+\frac{rT^3}{3}))\}\frac{\theta(1-m)\lambda_1^3}{36})-\frac{\lambda_1^2T_d^4}{12}+\frac{cT_d^5}{20}-\frac{T_d^6(18+5\theta(1-m)}{360}))\}$$

$$-\{o(1-\beta)(a-bP)(\frac{r(T-\lambda_1)}{r}+c\beta(\lambda_1(T+\frac{rT^2}{2})+\lambda_1^2\frac{(\lambda_1r-1)}{2}-\frac{r\lambda_1^3}{3}-\frac{T^2}{2}+\frac{rT^3}{3})\}]$$

$$\mathrm{TP}_2(\lambda_1,T,P,\chi)=[\{\frac{(a-bP)(e^{c\lambda_1}-e^{-r\lambda_1})}{(c+r)}\}-A-\{h_1\frac{(a-bP)(e^{c\lambda_1}-e^{-r\lambda_1})}{(c+r)}+h_2(a-bP)$$

$$(\frac{(e^{-r\lambda_1}-1)}{(c+r)^2}-\lambda_1\frac{e^{-r\lambda_1}}{(c+r)})\}-\{u\frac{(a-bP)}{c}(e^{c(\lambda_1-t)}-1)\}-\lambda_1\chi-\{o(1-\beta)(a-bP)$$

$$(\frac{e^{-r\lambda_1}-e^{c\lambda_1-(c+r)T}}{c+r})\}-\{-s\beta\frac{(a-bp)}{c}(\frac{(e^{-r\lambda_1}-e^{c\lambda_1-(c+r)T})}{(c+r)}+\frac{(e^{-r\lambda_1}-e^{-r\lambda_1})}{r})\}]$$

### 2.5 Total average cost for the inventory problem:

The total average cost for inventory problem is given by dividing the total inventory cost to the total time horizon ‘T’.

$$
\begin{aligned}
&\mathrm{A.P}_1.(\lambda_1,T,P,\chi)=\frac{1}{T}[\{(a-bP)\lambda_1+\frac{c(a-bP)}{c+r}((\lambda_1-T_d)+c(\lambda_1^2-T_d))+\theta(1-m)\\
&(\frac{\lambda_1+T_d^2}{6})\frac{\theta(1-m)\lambda_1^4}{3}-T_d(\lambda_1+c\lambda_1^2+\frac{\theta(1-m)\lambda_1^3}{6}-T_d^2(\frac{\lambda_1^2-1}{2})+\frac{T_d^3c}{6}-\frac{T_d^4}{6}\\
&(\theta(1-m)-1))-T_d(\frac{\lambda_1^2+T_d^2}{2})(1-\frac{\theta(1-m)T_d^2}{2})((c+r)T_d)+c(a-bP)((\frac{\lambda_1^2}{2}+\\
&\frac{(c-2T_d)}{3})\lambda_1^3+((c+r-\theta(1-m)T_d)(\frac{\lambda_1^3}{6}+\lambda_1^4\frac{(c-3T_d)}{8})+\frac{7\theta(1-m)\lambda_1^5}{60}-T_d^2\\
&(\frac{\lambda_1}{2}+\frac{c\lambda_1^2}{4})+T_d^3\frac{(4+3\lambda_1^2)}{12}+\frac{T_d^4c}{8}+T_d^5\frac{(15-4\theta(1-m))}{120}+(\theta(1-m)(\frac{\lambda_1^4}{24}+\\
&\frac{\lambda_1^5(c+4T_d)}{30}+\frac{\theta(1-m)\lambda_1^2}{24}-T_d^3\frac{(2\lambda_1+c\lambda_1^2)}{12}-\frac{\theta(1-m)\lambda_1^3}{36})-\frac{\lambda_1^2T_d^4}{12}+\frac{cT_d^5}{20}-\\
&\frac{T_d^6(18+5\theta(1-m))}{360}-A-\{u((a-bP)(\lambda_1-T_d)+\frac{c(\lambda_1^2-T_d^2)}{2}+\frac{\theta(1-m)(T_d^3+\lambda_1^3)}{6}\\
&-\frac{T_d(T_d^2+\lambda_1^2)}{2}+(a-bP)\beta(\lambda_1-t))\}\frac{\lambda_1^3}{6}+\lambda_1^4\frac{(c-3T_d)}{8})+\frac{7\theta(1-m)\lambda_1^5}{60}-T_d^2\\
&(\frac{\lambda_1}{2}+\frac{c\lambda_1^2}{4})+h_1(\frac{(a-bP)}{(c+r)}((\lambda_1-T_d)+c(\lambda_1^2-T_d))+\theta(1-m)(\frac{\lambda_1+T_d^2}{6})-T_d\\
&(\frac{\lambda_1^2+T_d^2}{2})(1-\frac{\theta(1-m)T_d^2}{2})+((c+r)T_d)+(a-bP)((\frac{\lambda_1^2}{2}+\frac{(c-2T_d)}{3})\lambda_1^3-\\
&\frac{\theta(1-m)\lambda_1^4}{3}-T_d(\lambda_1+c\lambda_1^2+\frac{\theta(1-m)\lambda_1^3}{6}-T_d^2(\frac{\lambda_1^2-1}{2})+\frac{T_d^3c}{6}-\frac{T_d^4}{6}\\
&(\theta(1-m)-1))+((c+r-\theta(1-m)T_d)(\frac{\lambda_1^3}{6}+\lambda_1^4\frac{(c-3T_d)}{8})+\frac{7\theta(1-m)\lambda_1^5}{60}\\
&-T_d^2(\frac{\lambda_1}{2}+\frac{c\lambda_1^2}{4})+T_d^3\frac{(4+3\lambda_1^2)}{12}+\frac{T_d^4c}{8}+T_d^5\frac{(15-4\theta(1-m))}{120}
\end{aligned}
$$

$$+(\theta(1-m)(\frac{\lambda_1^4}{24}+\frac{\lambda_1^5(c+4T_d)}{30}+\frac{\theta(1-m)\lambda_1^2}{24}-T_d^3\frac{(2\lambda_1+c\lambda_1^2)}{12}-\frac{\theta(1-m)\lambda_1^3}{36}-\frac{\lambda_1^2T_d^4}{12}+$$

$$\frac{cT_d^5}{20}-\frac{T_d^6(18+5\theta(1-m))}{360}+h_2((a-bP)((\lambda_1-T_d)+(c(\lambda_1^2-T_d^2))+\frac{\theta(1-m)(\lambda_1^3+T_d^3)}{6}-$$

$$\frac{T_d(\lambda_1^2+T_d^2)}{2}(\frac{T_d^2}{2}-\frac{(c+r)T_d^3}{3}-\frac{\theta(1-m)T_d^4}{4})+(a-bP)((\frac{\lambda_1^3}{6}+\frac{c\lambda_1^4}{8}+\frac{7\theta(1-m)\lambda_1^5}{60}-\frac{3T_d\lambda_1^4}{8}-$$

$$\frac{(3\lambda_1T_d^2-2T_d^3)}{6}-\frac{c(2\lambda_1^2T_d^2-T_d^4)}{8}-\theta(1-m)(\frac{\lambda_1^3T_d^2}{12}+\frac{T_d^5}{30})+\frac{T_d(2\lambda_1^2T_d^2+T_d^4)}{8})-(r+c+\theta(1-m)T_d)$$

$$(\frac{\lambda_1^4}{12}+\frac{c\lambda_1^5}{15}+\frac{\theta(1-m)\lambda_1^6}{12}-\frac{4T_d\lambda_1^5}{15}-\frac{(4\lambda_1T_d^3-3T_d^4)}{12}-\frac{c(5\lambda_1^2T_d^3-3T_d^5)}{30}-\frac{\theta(1-m)(2\lambda_1^3T_d^3+T_d^6)}{36}+$$

$$\frac{T_d(5\lambda_1^2T_d^3+3T_d^5)}{30})+\theta(1-m)(\frac{\lambda_1^5}{40}+\frac{c\lambda_1^6}{48}+\frac{2\theta(1-m)\lambda_1^7}{21}-\frac{5T_d\lambda_1^6}{48}-\frac{(5\lambda_1T_d^4-4T_d^5)}{40}$$

$$-\frac{c(3\lambda_1^2T_d^4-2T_d^6)}{24}-\frac{\theta(1-m)(7\lambda_1^3T_d^4+T_d^7)}{84}+T_d\frac{(3T_d^4\lambda_1^2+2T_d^6)}{48}))\}-\lambda_1\chi$$

$$d\theta((a-bP)((\frac{\lambda_1^3}{6}+\frac{c\lambda_1^4}{8}+\frac{7\theta(1-m)\lambda_1^5}{60}-\frac{3T_d\lambda_1^4}{8}-\frac{(3\lambda_1T_d^2-2T_d^3)}{6}-\frac{c(2\lambda_1^2T_d^2-T_d^4)}{8})$$

$$-\theta(1-m)(\frac{\lambda_1^3T_d^2}{12}+\frac{T_d^5}{30})+\frac{T_d(2\lambda_1^2T_d^2+T_d^4)}{8})-(r+c+\theta(1-m)T_d)(\frac{\lambda_1^4}{12}+\frac{c\lambda_1^5}{15}+\frac{\theta(1-m)\lambda_1^6}{12}$$

$$-\frac{4T_d\lambda_1^5}{15}-\frac{(4\lambda_1T_d^3-3T_d^4)}{12}-\frac{c(5\lambda_1^2T_d^3-3T_d^5)}{30}-\frac{\theta(1-m)(2\lambda_1^3T_d^3+T_d^6)}{36}+\frac{T_d(5\lambda_1^2T_d^3+3T_d^5)}{30})$$

$$+\theta(1-m)(\frac{\lambda_1^5}{40}+\frac{c\lambda_1^6}{48}+\frac{2\theta(1-m)\lambda_1^7}{21}-\frac{5T_d\lambda_1^6}{48}-\frac{(5\lambda_1T_d^4-4T_d^5)}{40}-\frac{c(3\lambda_1^2T_d^4-2T_d^6)}{24}-$$

$$\frac{\theta(1-m)(7\lambda_1^3T_d^4+T_d^7)}{84}+T_d\frac{(3T_d^4\lambda_1^2+2T_d^6)}{48})-T_d+(a-bP)((\frac{\lambda_1^2}{2}+\frac{(c-2T_d)}{3})\lambda_1^3$$

$$+((c+r-\theta(1-m)T_d)(\frac{\lambda_1^3}{6}+\lambda_1^4\frac{(c-3T_d)}{8})+\frac{7\theta(1-m)\lambda_1^5}{60}-T_d^2(\frac{\lambda_1}{2}+\frac{c\lambda_1^2}{4})$$

$$+T_d^3\frac{(4+3\lambda_1^2)}{12}+\frac{T_d^4c}{8}+T_d^5\frac{(15-4\theta(1-m))}{120}+(\theta(1-m)(\frac{\lambda_1^4}{24}+\frac{\lambda_1^5(c+4T_d)}{30}+\frac{\theta(1-m)\lambda_1^2}{24}$$

$$-T_d^3\frac{(2\lambda_1+c\lambda_1^2)}{12}\frac{\theta(1-m)\lambda_1^3}{36})-\frac{\lambda_1^2T_d^4}{12}+\frac{cT_d^5}{20}-\frac{T_d^6(18+5\theta(1-m)}{360}))\}-\{s\beta(a-bP)(\lambda_1(T+\frac{rT^2}{2})$$

$$+\lambda_1^2\frac{(\lambda_1r-1)}{2}-\frac{r\lambda_1^3}{3}-\frac{T^2}{2}+\frac{rT^3}{3}))\}\frac{\theta(1-m)\lambda_1^3}{36})-\frac{\lambda_1^2T_d^4}{12}+\frac{cT_d^5}{20}-\frac{T_d^6(18+5\theta(1-m)}{360}))\}$$

$$-\{o(1-\beta)(a-bP)(\frac{r(T-\lambda_1)}{r}+c\beta(\lambda_1(T+\frac{rT^2}{2})+\lambda_1^2\frac{(\lambda_1r-1)}{2}-\frac{r\lambda_1^3}{3}-\frac{T^2}{2}+\frac{rT^3}{3})\}]$$

$$\mathrm{A.P_2.}(\lambda_1,T,P,\chi)=\frac{1}{T}[\{\frac{(a-bP)(e^{c\lambda_1}-e^{-r\lambda_1})}{(c+r)}\}-A-\{h_1\frac{(a-bP)(e^{c\lambda_1}-e^{-r\lambda_1})}{(c+r)}+h_2(a-bP)$$

$$(\frac{(e^{-r\lambda_1}-1)}{(c+r)^2}-\lambda_1\frac{e^{-r\lambda_1}}{(c+r)})\}-\{u\frac{(a-bP)}{c}(e^{c(\lambda_1-t)}-1)\}-\lambda_1\chi-\{o(1-\beta)(a-bP)$$

$$(\frac{e^{-r\lambda_1}-e^{c\lambda_1-(c+r)T}}{c+r})\}-\{-s\beta\frac{(a-bp)}{c}(\frac{(e^{-r\lambda_1}-e^{c\lambda_1-(c+r)T})}{(c+r)}+\frac{(e^{-r\lambda_1}-e^{-r\lambda_1})}{r})\}]$$

**2.7 Solution Procedure:** Our aim is to find out the best possible value of P, $\chi$, T, $\lambda_1$ such that **$AP_1$ and $AP_2$** is maximal. For this we use bordered hessian matrix method for solving non-linear programming problem. There are four variables on which objective function depends.so, there will be 4*4 order hessian matric. Since, this problem is of maximization, so for optimality bordered hessian matrix should be negative definite. The given bordered hessian matrix is as follows:

$$[H]_B = \begin{bmatrix} \frac{\partial^4(A.P_2)}{\partial P^4} & \frac{\partial^4(A.P_2)}{\partial P^3\partial\chi} & \frac{\partial^4(A.P_2)}{\partial P^3\partial T} & \frac{\partial^4(A.P_2)}{\partial P^3\partial\lambda_1} \\ \frac{\partial^4(A.P_2)}{\partial\chi^3\partial P} & \frac{\partial^4(A.P_2)}{\partial\chi^4} & \frac{\partial^4(A.P_2)}{\partial\chi^3\partial T} & \frac{\partial^4(A.P_2)}{\partial\chi^3\partial\lambda_1} \\ \frac{\partial^4(A.P_2)}{\partial T^3\partial P} & \frac{\partial^4(A.P_2)}{\partial T^3\partial\chi} & \frac{\partial^4(A.P_2)}{\partial T^4} & \frac{\partial^4(A.P_2)}{\partial T^3\partial\lambda_1} \\ \frac{\partial^4(A.P_2)}{\partial\lambda_1^3\partial P} & \frac{\partial^4(A.P_2)}{\partial\lambda_1^3\partial\chi} & \frac{\partial^4(A.P_2)}{\partial\lambda_1^3\partial T} & \frac{\partial^4(A.P_2)}{\partial\lambda_1^4} \end{bmatrix}$$

$$here, \left|\frac{\partial^4(A.P_2)}{\partial P^4}\right| \le 0$$

$$\begin{vmatrix} \frac{\partial^4(A.P_2)}{\partial P^4} & \frac{\partial^4(A.P_2)}{\partial P^3\partial\chi} \\ \frac{\partial^4(A.P_2)}{\partial\chi^3\partial P} & \frac{\partial^4(A.P_2)}{\partial\chi^4} \end{vmatrix} \ge 0$$

$$\begin{vmatrix} \frac{\partial^4(A.P_2)}{\partial P^4} & \frac{\partial^4(A.P_2)}{\partial P^3\partial\chi} & \frac{\partial^4(A.P_2)}{\partial P^3\partial T} \\ \frac{\partial^4(A.P_2)}{\partial\chi^3\partial P} & \frac{\partial^4(A.P_2)}{\partial\chi^4} & \frac{\partial^4(A.P_2)}{\partial\chi^3\partial T} \\ \frac{\partial^4(A.P_2)}{\partial T^3\partial P} & \frac{\partial^4(A.P_2)}{\partial T^3\partial\chi} & \frac{\partial^4(A.P_2)}{\partial T^4} \end{vmatrix} \le 0$$

$$[H]_B \ge 0$$

The given matrix is negative definite. So, optimality will obtained on its critical points which will obtained by putting partial derivative of $A.P_2$ with respect to parameters T, P, $\chi$, $\lambda_1$.

That is,

$$\frac{\partial A.P_2}{\partial T} = 0$$

$$\frac{\partial A.P_2}{\partial P} = 0$$

$$\frac{\partial A.P_2}{\partial\chi} = 0$$

$$\frac{\partial A.P_2}{\partial\lambda_1} = 0$$

This is analytical method to solve this problem, but i did all these calculations by the help of mathematica and develop a numerical on it, which are given as follows.

**2.8 Numerical Example:**

**2.8.1** The above given result are illustrated through the numerical examples. To illustrate the model we consider the following input data.

Let a=70, b=1.8, c=0.25, s=30, o=0.55, u=0.42, A=350, r=0.45, $\beta$=0.095, $\gamma$=0.75, $\delta$=0.45, $\theta$=0.4,

$T_o$=1.8, $h_1$=1.1, $h_2$=1.92.

Answer: Applying the solution process of the given last section for case 1, we find the following results: T=2, P=45, $TP_1$=1362.99, $\lambda_1$=1.

**2.8.2** Same example for case 2 is given as:

Let a=70, b=1.8, c=0.25, s=30, o=0.55, u=0.42, A=350, r=0.45, $\beta$=0.095, $\gamma$=0.75, $\delta$=0.45, $\theta$=0.4,

$T_o$=1.8, $h_1$=1.1, $h_2$=1.92.

Answer: Applying the solution process of the given last section for case 2, we get the following results, T=3, P=95, $TP_2$=66.4215, $\lambda_1$=0.5.

**2.10 Sensitivity analysis:**

To see, how optimal solution is affected by the values of parameters, we originate the sensitivity analysis for some of the parameters. The particular values of some parameter decreased to -5%, -10%, -15%, -20% and then increased to 5%, 10%, 15%, 20%.

**2.10.1 Sensitivity analysis for parameter '$h_1$':**

| $h_1$ | T | P | $\lambda_1$ | $\chi$ | $AP_1$ |
|---|---|---|---|---|---|
| 1.320 | 2.0001 | 45.0013 | 0.9998 | 4.9998 | 1475.39 |
| 1.265 | 2.0002 | 45.0011 | 0.9999 | 4.9999 | 1447.29 |
| 1.210 | 2.0001 | 45.0010 | 0.9997 | 4.9999 | 1419.19 |
| 1.155 | 2.0003 | 45.0009 | 0.9999 | 4.9998 | 1391.09 |
| 1.100 | 2.0015 | 45.0011 | 0.9999 | 4.9999 | 1362.99 |
| 1.045 | 2.0011 | 45.0012 | 0.9995 | 4.9998 | 1334.88 |
| 0.990 | 2.0001 | 45.0010 | 0.9999 | 4.9999 | 1306.78 |
| 0.935 | 2.0000 | 45.0000 | 0.9999 | 4.9999 | 1278.67 |
| 0.880 | 2.0000 | 45.0000 | 0.9996 | 4.9998 | 1250.57 |

### 2.10.2 Sensitivity analysis for parameter '$h_2$'

| $h_2$ | T | P | $\lambda_1$ | $\chi$ | $AP_1$ |
|---|---|---|---|---|---|
| 2.304 | 2.0001 | 45.0008 | 0.9996 | 4.9998 | 1460.00 |
| 2.208 | 2.0002 | 45.0007 | 0.9999 | 4.9997 | 1435.74 |
| 2.112 | 2.0001 | 45.0006 | 0.9999 | 4.9999 | 1411.49 |
| 2.016 | 2.0003 | 45.0009 | 0.9997 | 4.9997 | 1387.24 |
| 1.920 | 2.0002 | 45.0008 | 0.9999 | 4.9996 | 1362.99 |
| 1.824 | 2.0001 | 45.0007 | 0.9998 | 4.9999 | 1338.73 |
| 1.728 | 2.0004 | 45.0009 | 0.9999 | 4.9998 | 1314.48 |
| 1.632 | 2.0003 | 45.0008 | 0.9999 | 4.9999 | 1290.23 |
| 1.536 | 2.0002 | 45.0009 | 0.9998 | 4.9997 | 1265.95 |

### 2.10.3 Sensitivity analysis for parameter '$\gamma$'

| $\gamma$ | T | P | $\lambda_1$ | $\chi$ | $AP_1$ |
|---|---|---|---|---|---|
| 0.9000 | 2.0001 | 45.0011 | 0.9998 | 4.9998 | 1392.83 |
| 0.8625 | 2.0003 | 45.0010 | 0.9999 | 4.9999 | 1387.31 |
| 0.8250 | 2.0002 | 45.0015 | 0.9997 | 4.9999 | 1380.66 |
| 0.7875 | 2.0000 | 45.0018 | 0.9999 | 4.9997 | 1372.65 |
| 0.7500 | 2.0001 | 45.0009 | 0.9999 | 4.9996 | 1362.99 |
| 0.7125 | 2.0002 | 45.0017 | 0.9998 | 4.9998 | 1351.34 |
| 0.6750 | 2.0000 | 45.0011 | 0.9999 | 4.9999 | 1337.31 |
| 0.6375 | 2.0000 | 45.0010 | 0.9996 | 4.9997 | 1320.41 |
| 0.6000 | 2.0003 | 45.0009 | 0.9999 | 4.9999 | 1300.05 |

### 2.10.4 Sensitivity analysis for parameter '$\beta$'

| $\beta$ | T | P | $\lambda_1$ | $\chi$ | $AP_1$ |
|---|---|---|---|---|---|
| 0.11400 | 2.0010 | 45.0009 | 0.9998 | 4.9998 | 1464.44 |
| 0.10925 | 2.0020 | 45.0008 | 0.9999 | 4.9999 | 1439.01 |
| 0.10450 | 2.0009 | 45.0007 | 0.9997 | 4.9999 | 1413.62 |
| 0.09975 | 2.0010 | 45.0009 | 0.9999 | 4.9997 | 1388.28 |
| 0.09500 | 2.0010 | 45.0008 | 0.9996 | 4.9999 | 1362.98 |
| 0.09025 | 2.0009 | 45.0007 | 0.9999 | 4.9996 | 1337.74 |
| 0.08550 | 2.0018 | 45.0009 | 0.9997 | 4.9999 | 1312.54 |
| 0.08075 | 2.0011 | 45.0006 | 0.9999 | 4.9996 | 1287.39 |
| 0.07600 | 2.0009 | 45.0009 | 0.9998 | 4.9998 | 1262.29 |

### 2.10.5 Sensitivity analysis for parameter 'r'

| **r** | T | P | $\lambda_1$ | $\chi$ | $AP_1$ |
|---|---|---|---|---|---|
| 0.5400 | 2.0001 | 45.0010 | 0.9998 | 4.9999 | 1381.20 |
| 0.5175 | 2.0002 | 45.0020 | 0.9999 | 4.9998 | 1376.65 |
| 0.4950 | 2.0001 | 45.0010 | 0.9997 | 4.9999 | 1372.09 |
| 0.4725 | 2.0001 | 45.0030 | 0.9998 | 4.9999 | 1367.54 |
| 0.4500 | 2.0000 | 45.0020 | 0.9997 | 4.9999 | 1362.98 |
| 0.4275 | 2.0002 | 45.0000 | 0.9997 | 4.9997 | 1358.43 |
| 0.4050 | 2.0001 | 45.0020 | 0.9999 | 4.9999 | 1353.88 |
| 0.3825 | 2.0000 | 45.0010 | 0.9996 | 4.9999 | 1349.32 |
| 0.3600 | 2.0000 | 45.0030 | 0.9999 | 4.9996 | 1344.77 |

### 2.10.6 Sensitivity analysis for parameter 'a'

| **a** | T | P | $\lambda_1$ | $\chi$ | $AP_1$ |
|---|---|---|---|---|---|
| 264 | 2.0001 | 45.0010 | 0.9998 | 4.9997 | 1850.10 |
| 253 | 2.0002 | 45.0020 | 0.9999 | 4.9999 | 1728.33 |
| 242 | 2.0000 | 45.0010 | 0.9997 | 4.9999 | 1606.54 |
| 231 | 2.0001 | 45.0009 | 0.9999 | 4.9998 | 1484.76 |
| 220 | 2.0003 | 45.0030 | 0.9999 | 4.9998 | 1362.98 |
| 209 | 2.0000 | 45.0040 | 0.9996 | 4.9999 | 1241.20 |
| 198 | 2.0000 | 45.0010 | 0.9998 | 4.9998 | 1119.42 |
| 187 | 2.0002 | 45.0030 | 0.9999 | 4.9997 | 997.64 |
| 176 | 2.0001 | 45.0020 | 0.9997 | 4.9998 | 875.86 |

### 2.10.7 Sensitivity analysis for parameter 'b'

| **b** | T | P | $\lambda_1$ | $\chi$ | $AP_1$ |
|---|---|---|---|---|---|
| 2.16 | 2.0001 | 45.0002 | 0.9998 | 4.9996 | 1183.64 |
| 2.07 | 2.0002 | 45.0000 | 0.9999 | 4.9999 | 1228.47 |
| 1.98 | 2.0001 | 45.0001 | 0.9996 | 4.9997 | 1273.31 |
| 1.89 | 2.0000 | 45.0000 | 0.9999 | 4.9998 | 1318.14 |
| 1.80 | 2.0003 | 45.0003 | 0.9996 | 4.9999 | 1362.98 |
| 1.71 | 2.0002 | 45.0000 | 0.9999 | 4.9997 | 1407.82 |
| 1.62 | 2.0000 | 45.0002 | 0.9995 | 4.9999 | 1452.60 |
| 1.53 | 2.0002 | 45.0000 | 0.9999 | 4.9998 | 1497.49 |
| 1.44 | 2.0003 | 45.0001 | 0.9998 | 4.9999 | 1542.34 |

### 2.10.8 Sensitivity analysis for parameter ‘c’

| c | T | P | $\lambda_1$ | $\chi$ | $AP_1$ |
|---|---|---|---|---|---|
| 0.2890 | 2.0001 | 45.0020 | 0.9998 | 4.9995 | 1323.93 |
| 0.2875 | 2.0000 | 45.0030 | 0.9999 | 4.9999 | 1325.43 |
| 0.2750 | 2.0002 | 45.0010 | 0.9999 | 4.9994 | 1337.99 |
| 0.2625 | 2.0000 | 45.0040 | 0.9997 | 4.9999 | 1350.50 |
| 0.2500 | 2.0000 | 45.0020 | 0.9996 | 4.9999 | 1362.99 |
| 0.2375 | 2.0004 | 45.0030 | 0.9999 | 4.9999 | 1375.43 |
| 0.2250 | 2.0000 | 45.0040 | 0.9997 | 4.9995 | 1387.85 |
| 0.2125 | 2.0001 | 45.0020 | 0.9999 | 4.9999 | 1400.22 |
| 0.2000 | 2.0002 | 45.0030 | 0.9998 | 4.9998 | 1412.56 |

### 2.10.9 Sensitivity analysis for parameter ‘θ’

| θ | T | P | $\lambda_1$ | $\chi$ | $AP_1$ |
|---|---|---|---|---|---|
| 0.48 | 2.0001 | 45.0011 | 0.9998 | 4.9999 | 1371.24 |
| 0.46 | 2.0000 | 45.0013 | 0.9997 | 4.9995 | 1369.25 |
| 0.44 | 2.0002 | 45.0010 | 0.9998 | 4.9999 | 1367.19 |
| 0.42 | 2.0003 | 45.0011 | 0.9997 | 4.9996 | 1365.10 |
| 0.40 | 2.0000 | 45.0010 | 0.9996 | 4.9999 | 1362.99 |
| 0.38 | 2.0001 | 45.0012 | 0.9999 | 4.9997 | 1360.84 |
| 0.36 | 2.0000 | 45.0014 | 0.9997 | 4.9999 | 1358.66 |
| 0.34 | 2.0003 | 45.0010 | 0.9999 | 4.9999 | 1356.45 |
| 0.32 | 2.0002 | 45.0013 | 0.9998 | 4.9998 | 1354.21 |

### 2.10.10 Sensitivity analysis for parameter ‘A’

| A | T | P | $\lambda_1$ | $\chi$ | $AP_1$ |
|---|---|---|---|---|---|
| 418.80 | 2.0001 | 45.0010 | 0.9997 | 4.9996 | 1328.59 |
| 401.60 | 2.0003 | 45.0020 | 0.9999 | 4.9999 | 1337.19 |
| 384.40 | 2.0002 | 45.0010 | 0.9998 | 4.9997 | 1345.79 |
| 367.20 | 2.0001 | 45.0015 | 0.9999 | 4.9999 | 1354.39 |
| 350.00 | 2.0000 | 45.0018 | 0.9997 | 4.9998 | 1362.98 |
| 332.80 | 2.0003 | 45.0023 | 0.9999 | 4.9999 | 1371.59 |
| 316.60 | 2.0002 | 45.0021 | 0.9999 | 4.9996 | 1379.69 |
| 309.40 | 2.0000 | 45.0019 | 0.9997 | 4.9999 | 1383.29 |
| 292.20 | 2.0001 | 45.0018 | 0.9998 | 4.9997 | 1391.89 |

### 2.10.11 Sensitivity analysis for parameter 'δ'

| δ | T | P | $\lambda_1$ | $\chi$ | $AP_1$ |
|---|---|---|---|---|---|
| 0.5400 | 2.0001 | 45.0030 | 0.9996 | 4.9995 | 1381.20 |
| 0.5175 | 2.0000 | 45.0025 | 0.9999 | 4.9999 | 1376.65 |
| 0.4950 | 2.0002 | 45.0020 | 0.9995 | 4.9994 | 1372.09 |
| 0.4725 | 2.0000 | 45.0040 | 0.9999 | 4.9999 | 1367.54 |
| 0.4500 | 2.0003 | 45.0030 | 0.9998 | 4.9996 | 1362.98 |
| 0.4275 | 2.0000 | 45.0018 | 0.9999 | 4.9999 | 1358.43 |
| 0.4050 | 2.0000 | 45.0020 | 0.9998 | 4.9995 | 1353.88 |
| 0.3825 | 2.0001 | 45.0030 | 0.9999 | 4.9999 | 1349.32 |
| 0.3600 | 2.0002 | 45.0025 | 0.9997 | 4.9996 | 1344.77 |

## 2.11 Conclusion:

In this chapter we study a joint pricing, ordering and preservation technology investment problem for a retailer who operates a non-instantaneous deteriorating inventory system. The retailer invests in preservation technology to reduce losses due to deterioration. We stated that that preservation technology can both reduce the deterioration rate and lengthen the non-deterioration period. We formulate a mathematical model with price-dependent and stock demand rate, time-varying deterioration rate with time varying holding cost and partially backlogged shortages. We characterize the properties of the optimal solution. Our numerical examples show that it is optimal for the retailer not to invest in preservation technology when the deterioration rate, ordering cost or purchasing cost are small, or when holding cost is high. In this chapter, we mainly focus on the impact of preservation technology investment on the operational policy with varying parameters. Although we take the pricing decision into account in the model, the attention on the pricing policy is limited since it is assumed to be a constant for items whose quality is maintaining at one level during the whole replenishment cycle.

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