CYLINDRICAL STRONG SHOCK WAVES IN STRONG MAGNETIC FIELD IN SELF-GRAVITATING GAS

Dr. Sarvesh Chandra Yadav Assistant Professor and Head Dept. of

Physics C.L. Jain P.G. College Firozabad. (India) – 283203

Mail ID- drsarveshchandra@gmial.com

ABSTRACT:-

The impact of overtaking disturbance behind the flow on the motion of diverging cylindrical strong shock wave in strong magnetic field in the presence of self gravitating gas having axial and azimuthal components of strong magnetic field as constant with initial density distribution $\rho_0 = \rho' r^{-\omega}$ where ω is a const. ρ' is the density at the plane / axes of symmetry with the help of CCW method, The analytical expression for flow variables in the presence of strong shock have been obtained. Finally The Numerical estimates at **psfl** have been calculated and compared with results obtained by FP. After including of E.O.D and

noted in the change of flow variables with Parameters. r, β^2, ω and ξ.

Key words:

SS→ Strong shock FD→

Free propagation

PSFL→ Permissible shock front location

$\rho_0 = \rho' r^{-\omega}$ → Density distribution

1.1 INTRODUCTION:-

Present work is related to the motion of diverging strong shock waves in a strong magnetic field through self gravitating gas:

Using the CCW method consider the e.o.d behind the flow variable on the propagation of diverging cylindrical shock waves. The expression for flow variables have been obtained for strong shock under the two different ways :

(i) As Purely non magnetic case, when the ratio of densities on the other side or either side of shock.

(ii) when the applied magnetic in large. modified form of analytical expression for flow variable so obtained have been numerically computed only at psfl and compared with FP Through Figure(1-4)

1.2 ELEMENTARY EQUATION :-

The equation governing the cylindrical flow at the gas under the influence of its own gravitating and magnetic field having constant axial and azimuthal components of Magnetic field are written as

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial \rho}{\partial r}+\frac{\mu}{2\rho}\frac{\partial}{\partial r}(H_0^2+H_z^2)+\frac{\mu}{\rho}\frac{H_0^2}{r}+\frac{Gm}{r^2}-0,$$

$$\frac{\partial u}{\partial t}+u\frac{\partial \rho}{\partial r}+\rho\frac{\partial u}{\partial r}\rho\frac{u}{r}=0,$$

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}+\gamma p\frac{\partial u}{\partial r}+\gamma p\frac{u}{r}=0,$$

$$\frac{\partial H_\theta}{\partial t}+u\frac{\partial H_\theta}{\partial r}+H_\theta\frac{\partial u}{\partial r}=0,$$

$$\frac{\partial H_z}{\partial t}+u\frac{\partial H_z}{\partial r}+H_z\frac{\partial u}{\partial r}+H_z\frac{u}{r}=0, \qquad \frac{\partial m}{\partial r}-2\pi\rho r=0 \qquad (1)$$

Where $r\rightarrow$ radial co-ordinate $u,\rho,\rho H_Z,H_\theta,\mu$ and m are. respectively, the particle velocity, thy density. the pressure, azimuthal and axial components at magnetic field permeability at gas, mass inside a cylinder of unit cross-section and unit radius and unit length.

$a_0^2=\frac{\gamma p_o}{\rho_o},\gamma\rightarrow$ Adiabatic index of the gas

1.3 BOUNDARY CONDITION :-

The magneto hydrodynamic conditions can be written in terms of single parameter.

$$\xi=\frac{\rho}{\rho} \quad as \quad \rho=\rho_0\xi, \qquad (2)$$

$$H=H_0\xi, \qquad u=\frac{\xi-1}{\xi}U\ , \qquad (3)$$

$$U^2=\frac{2\xi}{(\gamma+1)-(\gamma-1)\xi}\left[a_0^2+\frac{b_0^2}{2}\{(2-\gamma)\xi+\gamma\}\right] \qquad (4)$$

where 'o' stand for the state immediately ahead of the shock front, U is the shock velocity, a_0 is the sound speed $a_0=\sqrt{\frac{\gamma p_0}{\rho_0}}$ and alfven speed $\left(\frac{\mu H_0^2}{\rho_0}\right)^{\frac{1}{2}}$. (5)

1.4 STRONG SHOCK :-For every weak shock the parameter ξ is written as

Case I: The purely non-magnetic way when $\xi - \dfrac{\gamma+1}{\gamma-1}$ is small

Case II: when $b_0^1 >> a_0^2, i.e., \ \mu H_0^2 >> \gamma p_0 \, i.e.$ i.e. the ambient magnetic pressure is large compared with the ambient field pressure. In terms of ξ, the boundary condition (3.2) become,

$$\rho = \rho_0 \xi, \qquad H_\theta = H_{\theta_0} \xi, \qquad H_z = H_{z_0} \xi, \tag{6}$$

$$U^2 = \frac{2\xi a_0^2}{(\gamma+1)(\gamma-1)\xi}\left[1 + \frac{1}{2}\{(2-\gamma)\xi + \gamma\}\frac{\mu H_0^2}{\gamma p_0}\right] \text{ and} \tag{7}$$

$$P = p_0\left(\chi(\xi)\frac{U^2}{a_0^2} + L\right) \text{ and } u = \frac{\xi+1}{\xi}U \tag{8}$$

where $$\chi(\xi) = \frac{\gamma(\gamma-1)(\xi-1)^3}{2\xi\{(2-\gamma)\xi+\gamma\}} \text{ and } L = \frac{(\gamma-1)\xi - (\gamma-1)}{(\gamma+1)-(\gamma-1)} \tag{9}$$

1.5 CHARACTERISTIC EQUATION:-

For diverging shock. the characteristic form of the system of equation (1) is easily obtained by forming a linear combination of (1) and (3) equation of the system of equation (1) in only one direction in (r, t,) plane Equation (1) and (3) of the system can be written as

$$\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial r} + \frac{\partial_{,}}{\partial r} - \rho\frac{Gm}{r^2} = 0, \tag{10}$$

$$\frac{\partial p_t}{\partial t} + u\frac{\partial p_t}{\partial r} + \rho c^2\frac{\partial u}{\partial r} + \rho\frac{c^2 u}{r} = 0 \tag{11}$$

$$p_1 = p + \frac{u}{2}\left(H_0^2 + H_z^2\right) + \int \mu H_0^2 \frac{dr}{r} \tag{12}$$

is total pressure including magnetic pressure and

$$C^2 = a^2 + b^2 = \frac{\gamma p}{\rho} + \frac{\mu}{\rho}\left(H_0^2 + H_z^2\right) \tag{13}$$

with the help of above equation we get characteristic equation on

$$dp+\mu H_\theta dH_\theta+\mu H_z dH_z+\rho c du+\mu H_\theta^2\frac{dr}{r}+\rho c^2\frac{u}{u+c}\frac{dr}{r}+\rho c\frac{Gm}{4+c}\frac{dr}{r^2}=0$$

(14)

In order to estimate the strength of overtaking disturbances an independents C_+ characteristic is considered the differential relation valid across C_+ disturbances is obtained by replaced c by –c in equation (8) and written as

$$dp+\mu H_\theta dH_\theta+\mu H_z dH_z-\rho c du+\mu H_\theta^2\frac{dr}{r}+\rho c^2\frac{u}{u+c}\frac{dr}{r}-\rho c\frac{Gm}{u+c}\frac{dr}{r^2}=0 \quad (15)$$

equation (9) represents the characteristic form of equation (1) for Conversing shock.

1.6 ANALYTICAL RELATIONS FOR FLOW VARIABLES :-

Considered on initial density distribution of the form viz,

$$\rho_0=\rho^1 r^{-w} \quad (16)$$

For the equilibrium state of the as is assumed

$$\frac{\partial}{\partial t}=0=u$$
$$H_{z_0}=H_{\theta_0}, \quad (17)$$

using (ii) and First equation of the system of equation (1), the hydrostatic equilibrium prevailing in front of shock can be written as

$$\frac{1}{\rho_0}\frac{dp_0}{dr}+\frac{\mu}{2\rho_0}\frac{d}{dr}\left(H_{Z_0}^2+H_{\theta_0}^2\right)+\frac{1}{\rho_0}\frac{\mu H_{\theta_0}^2}{r}+\frac{Gm}{r^2}=0$$
$$\frac{1}{\rho_0}\frac{dp_0}{dr}+\frac{1}{\rho_0}\frac{\mu H_{\theta_0}^2}{r}+\frac{Gm}{r^2}=0 \quad (18)$$

From the sixth equation of (1) can be written as

$$m=2\pi\rho^1\int r^{1-w}$$
$$m=\frac{2\pi\rho^1 r^{2-w}}{2-w} \quad (19)$$

from (10), (12) and (13) we get

$$\frac{p_0}{G\rho^{12}} = K - \beta_2^2 D\log r - K_1 r^{1-2w} \tag{20}$$

why K is constant

$$\frac{a_0}{a^1} = \left(\frac{\gamma r^w}{D}\left(K - \beta_2^2 D\log r - K_1 r^{1-2w}\right)\right)^{\frac{1}{2}} \tag{21}$$

why

(22)

$$K_1 = \frac{2\pi}{(1-2w)(2-w)},$$

$$D = \frac{a^2}{G\rho^2} \quad \beta_2^2 = \frac{\mu H_{\theta_0}^2}{\gamma p} \tag{23}$$

ρ^1 is the density at the plane of symmetry in an unperturbed state and G is a universal Gravitational constant.

equation (14) uprest the variation in the pressure in the unperturbed medium with the distance r-

1.7 STRONG SHOCK WITH STRONG MAGNETIC FIELD:-

$$\frac{dU^2}{dr} + B\frac{U^2}{r} + C_2\beta_2^2 a'^2 r^{w-1} + C_1 G\rho^1 r^{-w} = 0 \tag{24}$$

where ; $B = \frac{1}{A_1}\left(B_1 - \frac{\chi(\xi)}{\gamma}w\right)$, $A_1 = \frac{\chi(\xi)}{\gamma} + \frac{1}{2}(\xi-1)\sqrt{\frac{\chi(\xi)}{\xi}}$ (25)

$$B_1 = \frac{\chi(\xi)(\xi-1)}{(\xi-1)+\sqrt{\chi(\xi).\xi}}, \; C_1 = \frac{C_1^1}{A_1}, \quad C_2 = \frac{\xi^2 - L}{A_1} \tag{26}$$

$$C_1^1 = \frac{\xi\sqrt{\xi\chi(\xi)}}{(\xi-1)+\sqrt{\xi\chi(\xi)}} \cdot \frac{2\pi}{(2-w)} \tag{27}$$

On

integration (3.44), we get.

$$U^2 = r^{-B}\left[K^* - \frac{C_2\beta_2^2 DG\rho^1}{w+B} r^{w+B} - C_1 G\rho^1 \frac{R^{1+B-w}}{1+B-w}\right] \tag{28}$$

Where K* is the constant of integration.

Remember that equation (3.45) describes Free Propagation for the C. Disturbances generated by the shock, the fluid velocity increment using (3.44) into (3.6) may be expressed as

$$du_- = -\frac{\xi-1}{2\sqrt{\xi}}\left[B.\frac{U}{2r} + C_2\beta_2^2 a'^2 r^{w-1} + C_1 G\rho^1 r^{-w}\right]dr \tag{29}$$

On substituting the shock conditions (3.6) into (3.20) and using (3.23), we get

$$\frac{dU^2}{dr} + B^2\frac{U^2}{r} + C_3\beta_2^2 a'^2 r^{w-1} - C_4 G\rho^1 r^{-w} = 0 \tag{30}$$

Where ; $B_2 = \frac{1}{A_1}\left(B_1^1 - \frac{\chi(\xi)}{\gamma}w\right)$, $A_1^1 = \frac{\chi(\xi)}{\gamma} + \frac{1}{2}(\xi-1)\sqrt{\frac{\chi(\xi)}{\xi}}$ (31)

$$B_1^1 = \frac{\chi(\xi)(\xi-1)}{(\xi-1)+\sqrt{\chi(\xi).\xi}},\ C_4 = \frac{L(1-2w)+C_2^1}{A_1^1},\ C_3 = \frac{\xi^2 - L}{A_1^1} \tag{32}$$

$$C_2^1 = \frac{\xi\sqrt{\xi\chi(\xi)}}{(\xi-1)-\sqrt{\xi\chi(\xi)}}.\frac{2\pi}{(2-w)} \tag{33}$$

For the C+ disturbance generated by the shock, the fluid velocity increment using (3.47) into (3.6) may expressed as

$$du_+ = \frac{\xi-1}{2\sqrt{\xi}}\left[-\frac{B_2}{2}\frac{U}{r} - C_3\beta_2^2 a'^2 r^{w-1} + C_4 G\rho^1 r^{-w}\right]dr \tag{34}$$

Now in presence of both C- and C+ disturbances. The fluid velocity increment behind the shock will be related as

$$du_- + du_+ = \frac{\xi-1}{\xi}du \tag{35}$$

Substituting equation (3.46) and (3.48) into (3.49), we get

$$\frac{dU^2}{dr} + \frac{B_3}{r}U^2 + C_5\beta_2^2 a'^2 r^{w-1} + C_6 G\rho'^2 r^{-w} = 0 \tag{36}$$

On integrating, we get

$$U^2 = r^{-B_3}\left[K_2^* - \frac{C_2\beta_2^2 a'^2}{w+B_5} r^{B_5+w} - C_6 G\rho^1 \frac{r^{1-w+B_3}}{1-w+B_3}\right] \tag{37}$$

where is a constant of integration

$$B_3 = B + B_2 \;,\; C_5 = C_2 + C_3 \;,\; C_6 = C_1 - C_4 \tag{38}$$

Equation (3.51) describes the propagation parameter U^2 which includes the e.o.d. behind the flow on the motion of shock.

1.8 ANALYTICAL EQUATIONS FOR FLOW VARIABLES FOR SS :- FP:

$$\frac{U}{a_0} = r^{\frac{B}{2}}\left[K^* \frac{C_2\beta_2^2 D}{w+B} r^{w+B} - C_1 \frac{r^{1+B-w}}{1+B-w}\right]^{1/2}\left[\gamma r^w\left(K - \beta_2^2 D\log r - K_1 r^{1-2w}\right)\right]^{\frac{1}{2}}$$

$$\frac{U}{\sqrt{G\rho^1}} = r^{\frac{B}{2}}\left[K^* \frac{C_2\beta_2^2 D}{w+B} r^{w+B} - C_1 \frac{r^{1+B-w}}{1+B-w}\right]^{1/2} \tag{39}$$

$$\frac{p}{G\rho'^2} = \left(K - \beta_2^2 D\log r - K_1 r^{1-2w}\right)\left[x(\xi)\left\{r^{-B}\left(K^* \frac{C_2\beta_2^2 D}{w+B} r^{w+B} - C_1 \frac{r^{1+B-w}}{1+B-w}\right)\right\}\right.$$
$$\left.\left\{\gamma r^w\left(K - \beta_2^2 D\log r - K_1 r^{1-2w}\right)^{-1}\right\} + L\right]$$

$$\frac{u}{\sqrt{G\rho^1}} = \frac{\xi - 1}{\xi}\left[\left(K^* \frac{C_2\beta_2^2 D}{w+B} r^{w+B} - C_1 \frac{r^{1+B-w}}{1+B-w}\right)^{1/2}\right]$$

$$\frac{\rho}{\rho^1} = r^{-w}\xi$$

EOD:

$$\frac{U}{a_0} = r^{\frac{B_3}{2}}\left[K_2^* \frac{C_5\beta_2^2 D}{w+B_5} r^{w+B_5} - C_6 \frac{r^{1+B_3-w}}{1+B_3-w}\right]^{1/2}\left[\gamma r^w\left(K - \beta_2^2 D\log r - K_1 r^{1-2w}\right)\right]^{\frac{1}{2}}$$

$$\frac{U}{\sqrt{G\rho^1}} = r^{\frac{B_3}{2}} \left[K_2^* \frac{C_5 \beta_2^2 D}{w+B_5} r^{w+B_5} - C_6 \frac{r^{1+B_3-w}}{1+B_3-w} \right]^{1/2} \tag{40}$$

$$\frac{U}{\sqrt{G\rho^{/2}}} = \left(K - \beta_2^2 D \log r - K_1 r^{1-2w}\right)\left[x(\xi)\left\{r^{-B_3}\left(K_2^* \frac{C_5 \beta_2^2 D}{w+B_5} r^{w+B_5} - C_6 \frac{r^{1+B_3-w}}{1+B_3-w}\right)\right\}\right.$$

$$\left.\left\{\gamma r^w \left(K - \beta_2^2 D \log r - K_1 r^{1-2w}\right)^{-1}\right\} + L\right]$$

$$\frac{U}{\sqrt{G\rho^1}} = \frac{\xi-1}{\xi}\left[r^{\frac{B_3}{2}}\left\{K_2^* \frac{C_5 \beta_2^2 D}{w+B_5} r^{w+B_5} - C_6 \frac{r^{1+B_3-w}}{1+B_3-w}\right\}^{1/2}\right]$$

1.9 RESULT AND DISCUSSION S.S. :-

The modified analytical expression include the e.o.d. behind the flow on the motion of strong diverging cylindrical shock waves in a self-gravitating gas in presence of strong magnetic field also having constant axial and azimuthal components, these flow variables are dependent, of $r, \beta_2^2, D, w, \beta^2$ and ξ

Taking (i)

$\frac{U}{a_0} = 17.559423, 17,83022,\ 18.09611,\ at\ r = 0.10\ for\ \gamma = 1.4,\ \beta_2^2 = 2.0, 2.0, 2.5, 3.0,$

$w = 0.9,\ D = 0.10,\ 0.012, 0.008, \beta^2 = 160, 165, 170,$ and $\xi = 1.5;$

(ii)

$$\frac{U}{a_0} = 25.3476, at\, r = 0.10,\ \gamma = 1.4, \beta_2^2 = 2.0, D = 0.10, w = 0.9, \beta^2 = 160,$$

and $\xi = 3.0;$ (iii)

$$\frac{U}{a_0} = 70.24956\, at\, r = 0.10, \gamma = 1.4, \beta_2^2 = 2.0,\ D = 0.10, w = 0.9, \beta^2 = 60,$$

and $\xi = 4.5$

(iv)

$$\frac{U}{a_0} = 341.8757\, at\, r = 0.10; r = 0.\gamma = 1.4, \beta_2^2 = 2.0,\ D = 0.10, w = 0.9, \beta^2 = 160,$$

and $\xi = 5.9$

it is noted that the shock strength increases whereas the shock velocity, the pressure, the particle velocity and the density decrease with propagation distance r (except $\xi = 4.5$) For increase in β_2^2 from 2.0 to 2.5, all the flow variables decrease and on further increase in β_2^2 their values increase. Increase in β and ξ leads to increase all the flow variables. An increase in D from0.98 to 0.10 leads to increase the shock strength, the shock velocity, the pressure and the density and on further increase of D their velocity, the pressure and the particle velocity decrease with w. however, remained unchanged (vide expression =3.76) Inclusion of the e.o.d. results in overall decrease/increase varying from -2.500% to 1.845%,

-2.500% to 1.845% - 3.141% to velocity, the pressure and the particle velocity

1.10 GRAPH FOR FLOW VARIABLES:-

The graph for flow variables for cylindrical strong shock with strong magnetic field in a S.G.G as below.

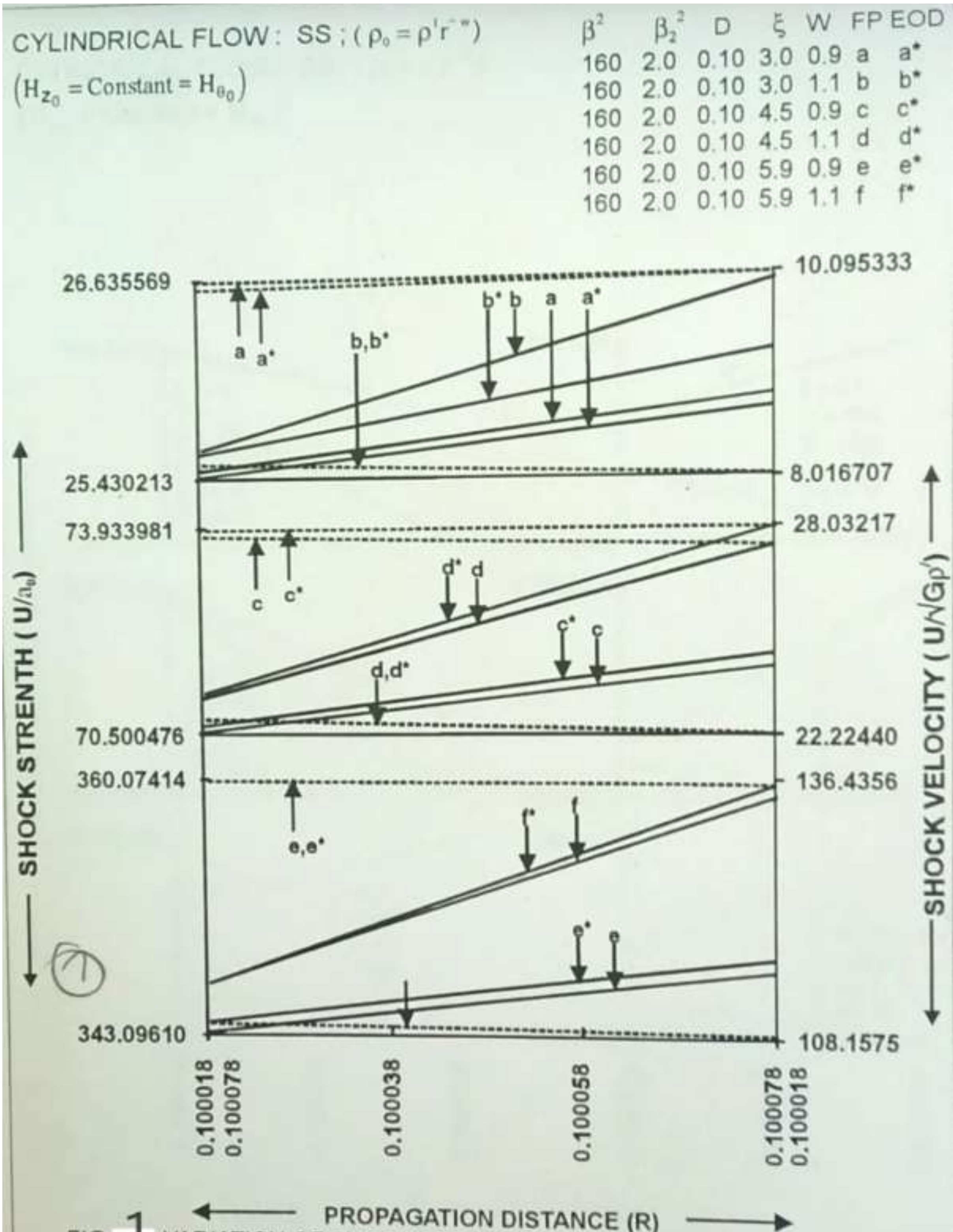

FIG 1 VARIATION OF SHOCK STRENGTH (—) AND SHOCK VELOCITY (---) WITH PROPAGATION DISTANCE (r) FOR CYLINDRICAL SS IN A SGG.

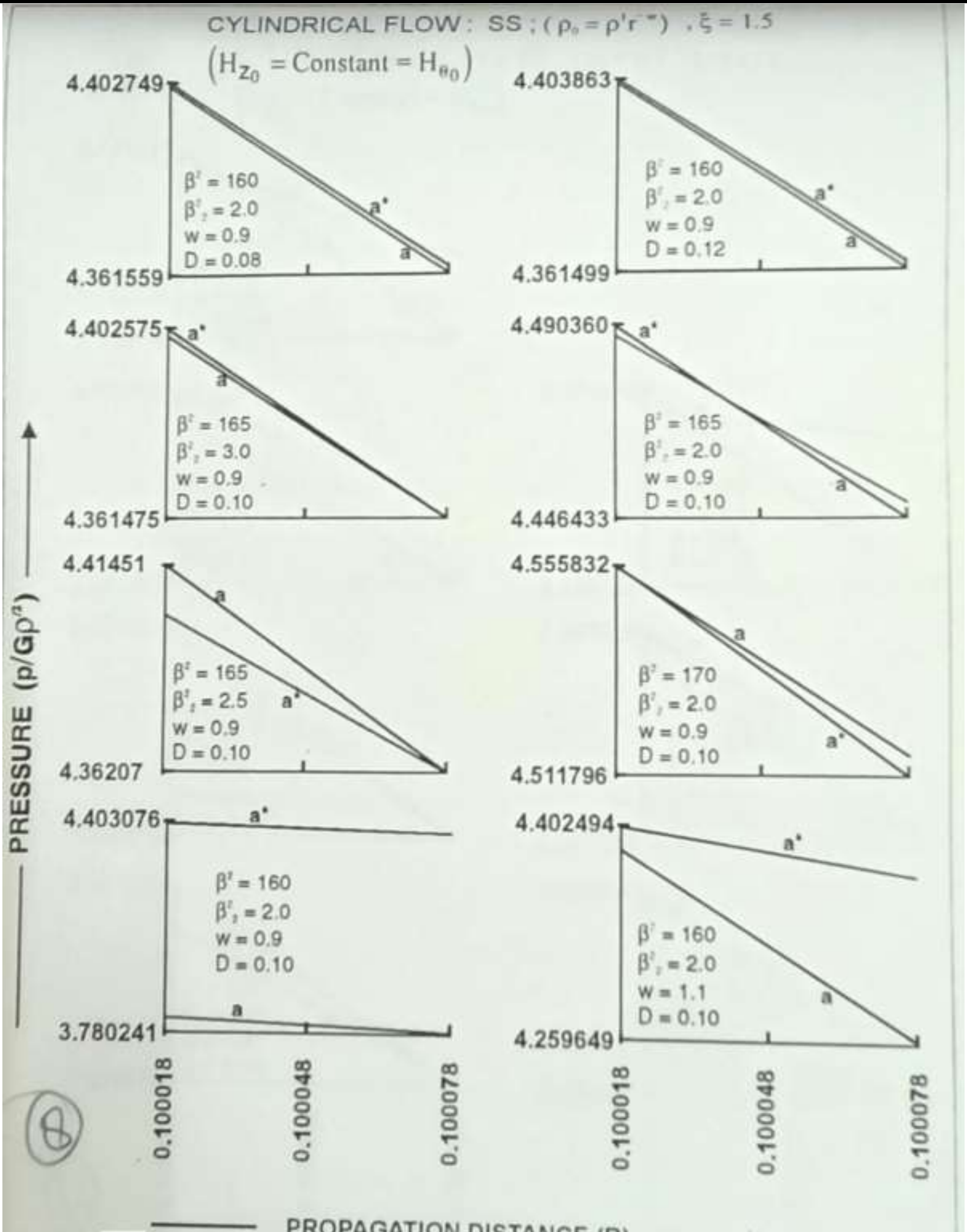

FIG : 2 VARIATION OF PRESSURE WITH PROPAGATION DISTANCE (r) FOR CYLINDRICAL SS IN A SGG

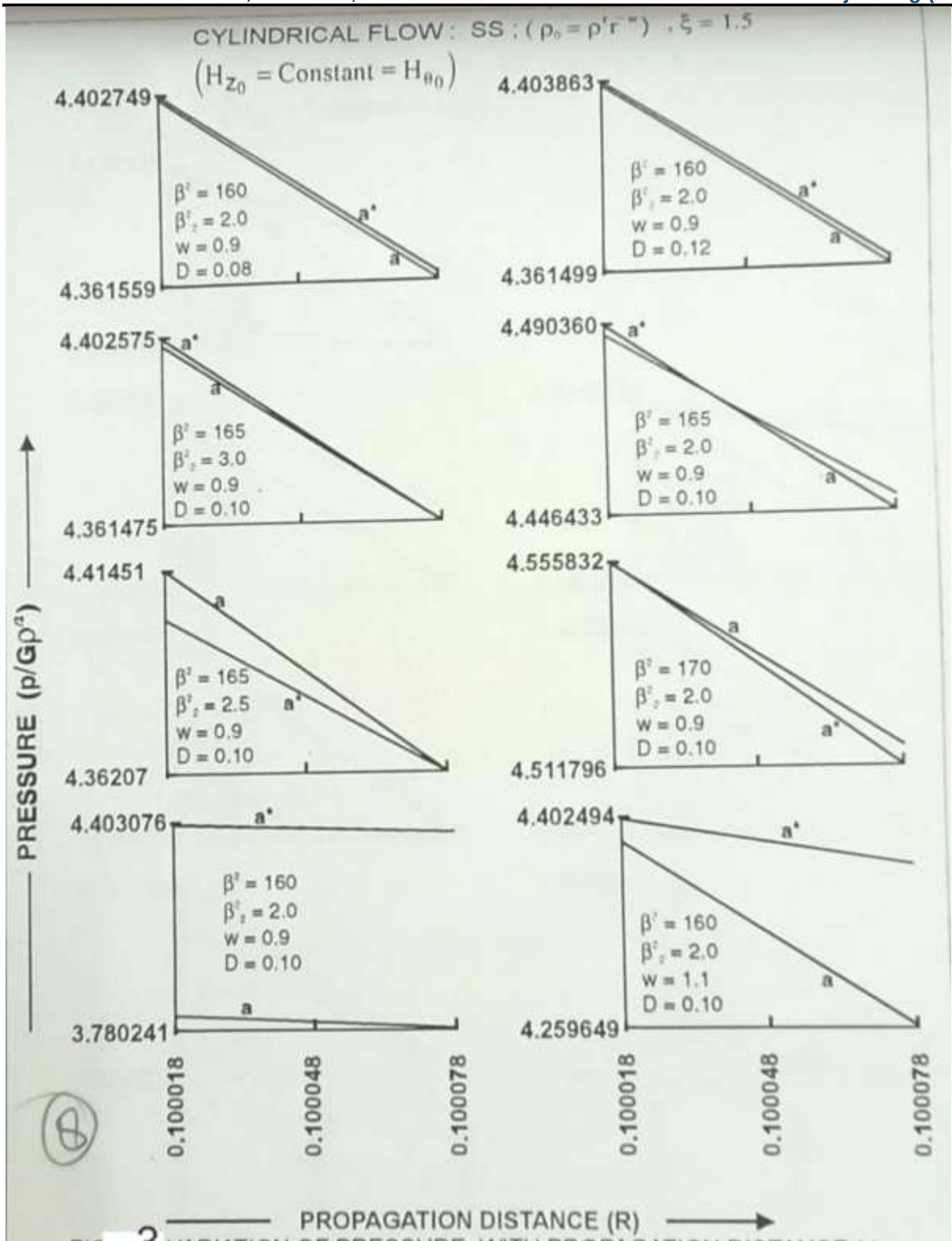

FIG 3 VARIATION OF PRESSURE WITH PROPAGATION DISTANCE (r) FOR CYLINDRICAL SS IN A SGG.

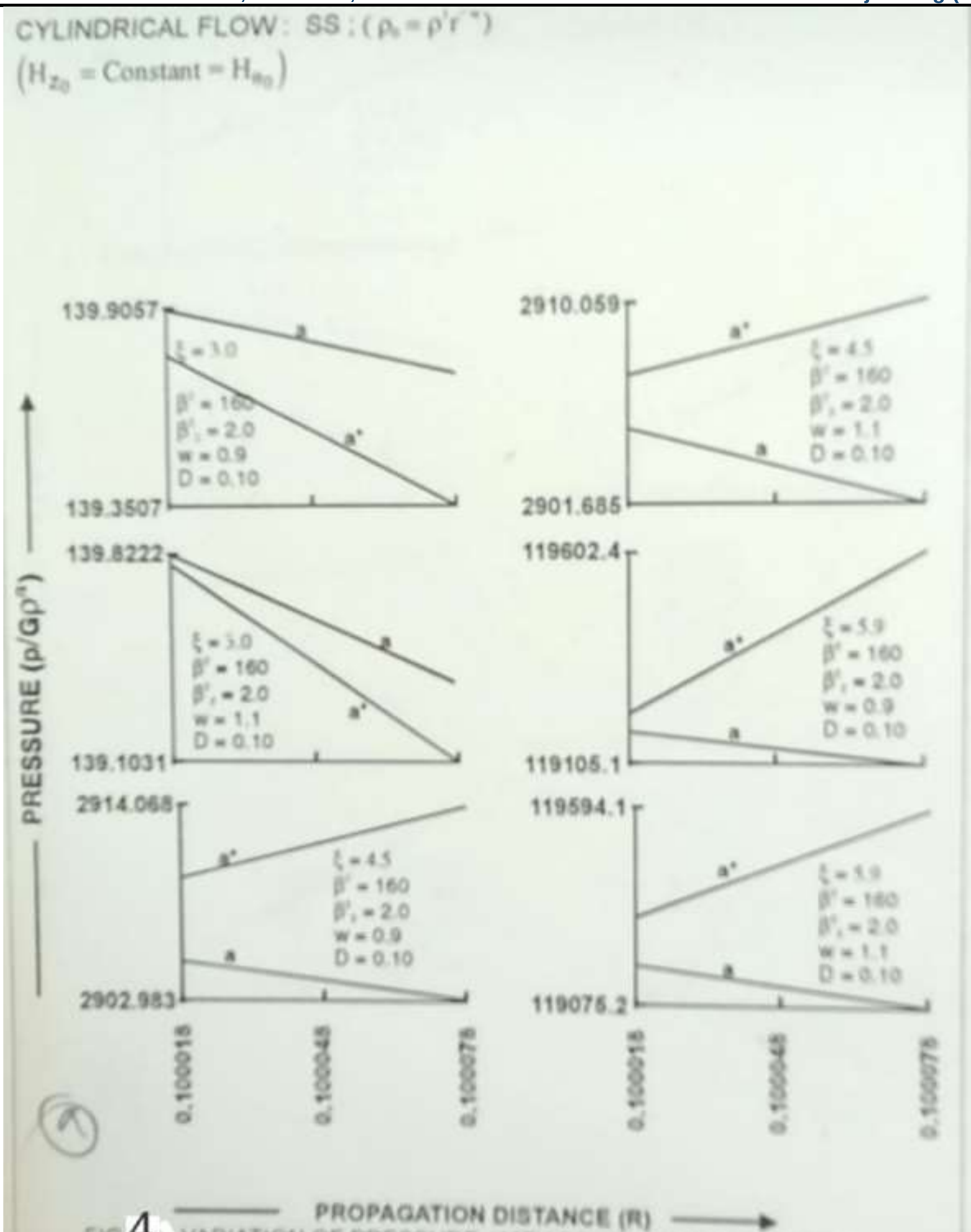

FIG 4 VARIATION OF PRESSURE WITH PROPAGATION DISTANCE (r) FOR CYLINDRICAL SS IN A SGG

References

[1] Chester, W., Phil Mag., 45(7): 1293 (1954).

[2] Chisnell, R.F., Proc. Roy. Soc. (London). A 3, 350 (1955).

[3] Whitham, G.B., J. Fluid Mech., 4, 337(1958).

[4] Yousaf, M.J., Fluid Mech., 66-77 (1974).

[5] Yousaf, M.J., Phy. Fluid Mech., 28(6), 1659 (1985).

[6] Yadav, R.P., Gupta, M. and Kumar, S., Astrophysics Space Science, 235, 27-39(1996).

[7] J.B., Bhowmick, Astrophysics and Space Science, 60, 183-190 (1979)

[8] B.C., Verma, J.P., Visrmakarnia and V. Sliaran, Astrophysics and Space Science, 88, 125-134 (1982)

[9] Bhatnagar, P.L. and Sachdev, P.L., II. Nuovo ciinento. 44(I), pp 15-30 (1966).

[10] Kumar, S. and Kulshrestha, A.K., Zamp, Angew, Math. Phys. 33, 326(1982)

[11] Levin, V.A. and Zhuravskaya, T.A.. Shock Waves, 6, 177-182(1996).

[12] Dinshaw, S. Balasara, Astro physical JI. Supplement series 116; I33E 153(1998).

[13] S. Kumar, M.K. Rana and A.K. Gautam, Jl. I1SPS, 21(3), 651-666(2009)

[14] Singh, L.K., Saxena, A.K. and Agarwal. R.K. RRST, 3(7), 116-119 (2011).

[15] G. Nath. Ain shams engg. Journal. 3. 393-401 (2012)