BASIC FEATURES OF QUEUING THEORY AND ITS APPLICATIONS IN OUR LIFE

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Abstract : There are many situations in daily life when a queue is formed. Queuing theory is the mathematical study of waiting lines and it is very useful to define Modern information technologies require innovations that are based on modelling, analyzing, designing to deals as well as the procedure of traffic control of daily life of human like telecommunications, reservation counter, super market, big bazaar, Picture Cinema hall ticket window and also to determining the sequence of computer operations, computer performance, health services, airport traffic, airline ticket sales. Queuing theory is the mathematical study of waiting lines and it is very useful for analyzing the procedure of queuing of daily life of human being. Queuing theory applies not only in day to day life but also in sequence of computer programming, networks, medical field, banking sectors etc. In this paper, we analyze the basic features of queuing theory and its applications.

Index Terms **- Queueing theory, Queueing system, Queueing network, Queueing model, Queue length.**

I. INTRODUCTION

Queueing theory is based on mathematical theories and deals with the problems arising due to flow of customers towards the service facility. Queueing theory introduces by A.K. Erlang a Danish mathematician who studied telephone traffic congestion problems in the first decade of the 20th century. Queueing theory very useful in many practical applications in areas such as, e.g., telephone exchange, traffic control, manufacture systems, inventory systems and communication systems, telephone exchange, supermarket, at a petrol station, at computer systems, etc.

A queuing system which consists of the customers and the servers. Waiting line or queues are in the schools, hospitals, bookstores, libraries, banks, post office, petrol pumps, theatres etc., all have Queuing problems. Queues are very familiar in our daily life. Queuing theory is a branch of operations research because the results are used for making decisions about the resources needed to provide service [9]. Many valuable applications of the queuing theory are traffic flow (vehicles, aircraft, people, communications), scheduling (patients in hospitals, jobs on machines, programs on computer), and facility design (banks, post offices, supermarkets). A.K.Erlang (1878-1929) Danish Engineer who is called the father of Queuing theory. He published his articles relating to the study of congestion in telephone traffic. A queuing theory is the Mathematics of waiting lines. A queuing system can be described by the flow of units for service, forming or joining the queue, if service is not available soon, and leaving the system after being served.

Basic elements of Queue

The analysis of queue is based on building a mathematical model representing the process of arrival of Item who joins the queue, the rules by which they are allowed into service, and the time it takes to service. Queueing theory embodies the full scope of such models cover all perceivable systems which incorporate characteristics of a queue. We identify the unit demanding service, whether it is human or otherwise.

Queue: queue is a file or line of persons. "Queue" means to form a line while waiting for something or a waiting line, involves arriving items that wait to be served at the facility that provides the service they seek.

Basic System of Queueing Process:

Arrival at the system Queue Departure from the system Figure 1 Basic Queue Process

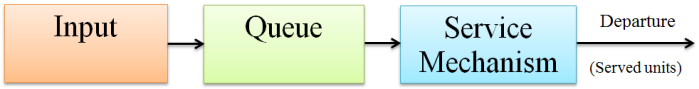

Fig.1 : Basic Queue Process

A Basic queuing system is formed from three general elements (Figure 1)

1. The arrival process of users in the system;
2. The order in which users obtain access to the service facility, once they join the queue.
3. The service process and departure from the system.

- Arrival refers to the average number of customers who require service within a specific period of time.
- Customers can be people, work-in-process inventory, raw materials, incoming digital messages, or any other entities that can be modeled who are to wait for some process to take place it may be infinite or finite also is said size of queue.
- A Server can be a human worker, a machine, or any other entity that can be Processor as executing some process for waiting customers.

II. QUEUE LENGTH

Probability distribution of queue length can be obtained with the help of the given Probability distribution of the arrival and service process. A large queue indicates poor service facility or a need for more space. A small queue indicates excess of service facilities.

Waiting time in Queue : It refers to the time spent by the customer in the queue before the commencement of his service.

Waiting time in system : A basic concept in the analysis of a queuing theory is that of a state of the system. It involves study of a system's behavior overtime.

Motive of the study : This article gives the basic ideas of some important concepts and applications of queuing theory in the field. This paper aims to study of the applications of library management, bank ATMs, hospitals, traffic system, banking, toll plaza, railway station and computer system.

Library management : A library is an organized collection of books, and some special materials like audio or visual materials, CDs, cassettes, video tape, DVDs, e-books, audio books and many other types of electronic resources. Scope of Queuing application in Libraries are circulation of books, counter service and allied services like reprography. The basic tasks in library are stacks maintenance, membership management, selection of library materials, and planning the acquisition of materials.

Bank ATMs : In ATM, bank customers arrive randomly and the service time i.e. the time customer takes to do transaction in ATM, is also random. We use queuing model to derive the arrival rate, service rate, utilization rate, waiting time in the queue and the average number of customers in the queue. Queuing can help bank ATM to increase its quality of service, by anticipating, if there are many customers in the queue.

Hospitals : Queuing models using for estimating waiting time of a patient, utilization of service, models system design, and models for evaluating appointment systems. A queuing system helps minimizing the waiting time of patients and maximizing the utilization of the servers i.e. doctors, nurses, hospital beds etc. Queuing is not new but recently hospitals has begun to use it effectively.

Traffic system : The vehicular traffic flow and explore could be minimized using queuing theory in order to reduce the delay on the roads. The role of transportation in human life cannot be overemphasized. A basic model of vehicular traffic based on queuing theory. It will determine the best times of the red, amber, and green lights to be either on or off in order to reduce traffic congestion on the roads. Queuing also helps to reduce fuel consumption thereby saving money for the Government to tackle problem of other sectors of the economy.

Banking : Today banks are one of the most important units of the public. Most banks used standard queuing models. It is very useful to avoid standing in a queue for a long time or in a wrong line and to give tickets to all customers. Bank is an example of unlimited queue length. Queuing is used to generate a sequence of customers' arrival time and to choose randomly between three different services: open an account, transaction, and balance, with different period of time for each service.

Toll plaza : Computer simulation is one of the popular approaches to the design of toll plazas. Toll plaza configurations such as toll collection methods, number of toll booths, and types of vehicles have been used here. Toll plaza performance measures such as average queue length, average waiting time, maximum queue length, and maximum waiting time at the tolls were compared between two different types of representations of projected traffic volumes. Toll plaza designing factors such as lane selection options, electronic toll collection(ETC)rates, and number of manual tolls were combined with traffic flow measure the specified toll performances. Finding appropriate values of input parameters for a traffic simulation model is always a challenge to simulation model builders as well as to traffic engineers. For generating traffic flow in a simulation model, deterministic traffic counts for a time period can be used as an input parameter into the model rather than considering a probabilistic distribution.

Railway station : In the country like India where Railway is one of the most popular and cheapest means of transportation, it is always difficult to book confirmed tickets for the journey. The population that the country has it doesn't match up with number of trains running various routes especially those connecting the metro cities. Indian Railway is trying to meet the ever increasing demand of over billion people. The queuing system is used to avoid the inconvenience of passengers and it is feasible and the results are effective and practical.

Computer system : Many jobs arrive sequentially at a computer system in accordance with a Poisson arrival process, and the execution time of a job is a random variable. Jobs are executed in the order of arrival, if the computer is busy when a job arrives, the job is placed in a queue. In the terminology of queuing theory the computer is the "server" and the jobs are "customers". The logical structure of the single server queuing model can be restored with a simple device. Let the server represent the combined resources of computer and operator.

III. STATE OF THE QUEUING SYSTEM

A queuing system can be completely described by

- The input (or arrival Patten)
- The service mechanism (or service pattern)

- The queuing discipline
- Customer's behavior.

A queuing system is said to be in transient state when its operating characteristics, arrivals, waiting time and service time of the customers are dependent on time.

A queuing system is said to be in a steady state when its operating characteristics, arrivals, waiting time and service time of the customers are independent on time.

If the arrival rate of the system is more than its servicing rate, the length of the queue will go on increasing with the time and will tend to infinity.

IV. Classification of Queuing Models

- Model I (M/M/I) : (∞/FCFS)
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- Model II (M/M/I): (∞/SIRO)
- Model III (Birth-Death process) (M/M/I) : (∞/FCFS)
- Model IV (M/M/I): (N/FCFS)
- Model V (M/M/C): (∞/FCFS)
- Model VI (M/E/I): (∞/FCFS)
- Model VII (M/M/R): (K/GD); K < R
- Model VIII – Power supply Model
- Model IX – D/D/I
- Model X – M/D/I
- Model XI (M/G/I) : (∞/FCFS)

V. Kendall Notation

A/S/m/B/K/SD

A : arrival process

S : service time distribution

m : number of servers

B : number of buffers (system capacity)

K : population size

SD: service discipline

Arrival process : The number of customers arriving per unit of time is called arrival rate. Random arrivals are described by the Poisson distribution.

• Jobs/customer arrival pattern

• τ form a sequence of Independent and Identically Distributed(IID) random variables

– Arrival times : t1, t2, …, tj

– Interarrival times : T_j=tj-tj-1

• Arrival models

– Exponential + IID (Poisson)

– Erlang

– Hyper-exponential

– General : results valid for all distributions

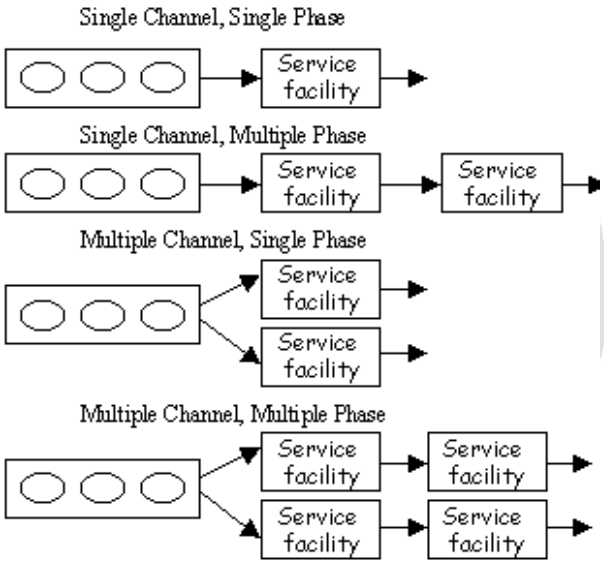
Service time distribution : The number of customers served per unit of time is called service time. The service times are described by the exponential distribution.

• Time each user spends at the terminal
• IID
• Distribution model
– Exponential
– Erlang
– Hyper-exponential
– General
• cf.
– Jobs = customers
– Device = service center = queue
– Buffer = waiting position

Service Discipline : This is the manner by which customers are selected for service when a queue has formed. The most common discipline is FCFS – First Come, First Served. The other disciplines are LCFS – Last Come First Served and SIRO – Service In Random Order and GD – general service discipline and including priority.

Number of Servers

A queuing system is called one server model, when the system has server only and a multi- server model when the system has a number of parallel channels each with one server.



Notations used in queuing system

1. n = number of customers in system
2. μ = mean service rate
3. λ = mean arrival rate
4. $P_n(t)$ = probability of n customers in system at time *t*
5. Probability of one arrival in the system during Δt = Δt+ O(h)
6. Probability of more than one arrival in the system during Δt = O(h)
7. Probability of no arrival in the system during Δt=1- Δt+ O(h)
8. Probability of one customer being service in time Δt= μ Δt+ O(h)
9. Probability of more than one customer being service in time Δt= O(h)
10. Probability of not a single customer being service in time Δt=1- μ Δt+ O(h)

Little's formula

In his connection, it is relevant to mention one of the important and useful relationship in queuing theory which holds under fairly quite general conditions. It is known as Little's formula, a rigorous proof of

which was given by Little (1961). It is given by $L_q = \lambda W$ where λ is arrival rate, $L_q = E(L)$ is the expected queue length under steady state and W is the steady state expected waiting time in the system.

VI. QUEUE NETWORKS

The queueing systems considered in the preceding chapters had customers demanding service from a single facility. But there are many real-world systems in which customers are served in more than one station arranged in a network structure, which is a collection of nodes connected by a set of paths. Computer, communication, and manufacturing systems, where queueing theory has found major application areas, abound with such networks. Network of Queues can be described as a group of nodes where each of the nodes represents a service facility of some type. Customers can arrive from outside the system to any node and may leave the system from any node. Therefore, customers may enter the system at some node, can transverse from node to node in the system and finally can leave the system at some other node, not all customer necessarily enter and leave at the some node or taking the same path once after entering the system. Also customers can return to nodes already visited, skip some nodes entirely or even choose to remain in the system forever.

Classification of Networks

(i) Open Networks
(ii) Closed Networks
(iii) Mixed Networks

Open Networks : In an open queuing network, customers enter the system from outside and after service at one or more queues, eventually leave the system.
Closed Networks : A closed queuing network does not have any external arrivals or departures. It represents a situation where a fined number of jobs circulate in the system, moving from one queue to the next, getting served at individual queues. No jobs enter the system nor does any job leave the system.
Mixed Networks : Network has multiple job classes and is open with respect to some classes but closed with repeat to the others.

Serious Queues

A series queue is a series of service stations through which the arriving customer may get the services in a sequence before leaving the system. In a series queue a customer might not revisit the station where he has got the service already. In Queuing model in which there are a series of service stations through which each calling unit must progress prior to leaving system. There are several types of network of Queues for which customers are not allowed to visit previously visited nodes. The first series model to be considered is the sequence of queues with no restriction on the waiting's room capacity between the stations. For example, a physical examination for a patient undergoes through a series of stages lab tests, electrocardiogram, chest X -ray, etc.

Fuzzy queues

The fuzzy queues are first analyzed by R.J.Lie and E.S.Lee in 1989. In poisson arrival queuing system is a fairly reasonable assumption, the arrival rate and service rate are really more possibilistic than probabilistic. The fuzzy set theory has been applied to investigate the queuing model of two priority classes by using triangular fuzzy numbers. Fuzzy set theory has been applied to some queuing systems to provide wider applications in instrumentation technology, information technology and communication technology. Based on Zadeh's extension principle, system performances of interest for the expected waiting time in the

queue of priority classes and the expected number of customers in the queue when the arrival rates of two priority classes with the same service rates are triangular fuzzy numbers. Furthermore, in many practical situations the parameters λ (arrival rate) and μ (service rate) are frequently fuzzy and cannot be expressed in exact terms. Thus linguistic expressions for these parameters such as "the mean arrival rate is approximately 5" and "the mean service rate is approximately 10" are much more realistic under these circumstances. Fuzzy queues are much more realistic than the commonly used crisp queues.

VII. CONCLUSION

With help of this theory complete details can be obtained regarding setting up workstations, requirement of manpower and number of people who would visit that domain. However, all these details are based on the concept of probability. With help of this theory, a scientific acknowledgement of the problem can be developed which would further help in making optimal usage of facilities that are available thereby reducing the total waiting time. Hence, with these set of benefits available, problems associated with these areas can be easily sorted and waiting time limit can be reduced considerably. The formation of queue is a common phenomenon which occurs whenever the current demand for a service exceeds the current capacity to provide that service. Queuing systems are useful throughout society. The capability of these systems can have an important result on the quality of human life and productivity of the process. Queuing systems are successfully used for the performance analysis of different systems such as computer, communications, transportation networks and manufacturing. In addition, examples of queuing theory applications are given. This analysis provides some fundamental concepts of queuing theory and their applications.

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