

MATHEMATICAL MODELLING OF PULSATILE BLOOD FLOW THROUGH A CATHETERIZED INCLINED STENOSED ARTERY UNDER THE INFLUENCE OF BODY ACCELERATION AND MAGNETIC FIELD

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Abstract: In this paper we have studied the pulsatile blood flow in catheterized artery with axially symmetric stenosis under the influence of body acceleration and uniform transverse magnetic field. Finite difference method is used to solve the Governing equations of motion. Velocity profile w.r.t. radial distance r for different value of body acceleration, Hartmann Number, inclination angle α when a catheter is inserted coaxially in artery. Comparative study of variation of velocity w.r.t. radial distance assuming blood Newtonian, Two layered without catheter and Newtonian with insertion of catheter.

Index Terms: Catheter, Magnetic field, Body acceleration.

I. Introduction: In medicine, a catheter is a thin tube extruded from medical grade materials serving a broad range of functions. Catheters are medical devices that can be inserted in the body to treat diseases or perform a surgical procedure. The method of catheterization is to insert the catheter-tool device in to a peripheral artery and then position the device in to the desired part of the arterial network by passing an appropriate length of the catheter through the artery. Since the catheter tool is used for measurement of arterial blood pressure or pressure gradient and flow rate, X-ray angiography and intravascular ultrasound diagnosis and coronary balloon angioplasty treatment of various arterial diseases, so it is useful to study the blood flow through catheterized artery. Sankar and Hemalatha [2007] investigated the effects of catheterization and non-Newtonian nature of blood in small arteries of diameter less than $100\mu\text{m}$, on velocity, flow resistance and wall shear stress taking blood as a Herschel–Bulkley fluid. Sankar and Lee [2009] analyzed the pulsatile flow of blood through a catheterized artery, assuming blood as a two-fluid model with the suspension of all the erythrocytes in the core region as a Herschel–Bulkley fluid and the peripheral region of plasma as a Newtonian fluid. Perturbation technique is used for obtaining the expressions for shear stress, velocity, flow rate, wall shear stress and longitudinal impedance. They observed that the increase in the longitudinal impedance is significantly lower for the two-fluid model than those of the single fluid model. Chakravarty and Sen [2009] presented theoretical investigation based on the flow characteristics of blood in a catheterized bifurcated artery under stenotic conditions taking blood as Newtonian fluid. Pulsatile flow of blood through a catheterized artery in presence of an axially nonsymmetrical mild stenosis with a velocity slip at stenotic wall taking blood as a Newtonian fluid has been investigated by Biswas and Chakraborty [2010]. The pulsatile flow of blood through catheterized artery assuming blood as a (i) two-fluid Casson model and (ii) two-fluid Herschel–bulkley model is studied by Sankar and Lee [2010]. They concluded that the two-fluid Herschel–bulkley model will have more applicability in analyzing the blood flow through catheterized artery. Verma et al. [2011] proposed a mathematical model for pulsatile flow of blood in a catheterized artery in presence of an axisymmetric stenosis. Siddiqui and Geeta [2013] analyzed the flow of blood through stenosed catheterized artery with the effect of external body acceleration. The pulsatile flow behaviour of blood in an artery subjected to the pulsatile pressure gradient and slip velocity has been studied considering blood as a Newtonian fluid. Dynamic mathematical model of blood flow through a stenosed tapered artery in the presence of a catheter has been studied. Blood is modelled as a homogeneous and incompressible couple stress fluid by Reddy et al. [2014]. Srivastav [2014] studied blood flow through the model of a composite stenosed catheterized artery with permeable wall, has been performed to investigate the blood flow characteristics. Mekheimer and El Kot [2015] analyzed a particle-fluid suspension model for the axis-symmetric flow of blood through curved coaxial tubes where the outer tube with mild overlapping stenosis while the inner tube is uniform rigid representing catheter. Siddiqui and Awasthi [2017] analysed the pulsatile flow of blood through an inclined catheterized stenosed artery is analyzed. Perturbation method is used to solve the implicit system of partial differential equations with suitable boundary conditions. Kakati et al. [2018] developed a mathematical model to study the pulsatile flow of blood through an axially non-symmetric but radially symmetric stenosed inclined catheterized artery with periodic body acceleration and slip at wall with blood is assumed to be Newtonian fluid.

In this paper the pulsatile flow of blood through inclined stenosed artery in which a catheter is introduced coaxially, under the influence of body acceleration and slip velocity and in the presence of magnetic field assuming blood as a Newtonian fluid is studied.

II. FORMULATION OF THE PROBLEM:

In this paper, the following assumptions are made:

The blood flow is modeled to be pulsatile, one dimensional, laminar, fully developed and the flowing blood is assumed as homogenous, Newtonian fluid. Stenosis developed in the artery in an axially symmetric manner and depends upon the axial distance z and the height of its growth. A catheter is introduced coaxially in to the artery. Body acceleration and slip velocity on artery wall are taken. Uniform transverse magnetic field is acting along the radius of the tube.

STENOSIS GEOMETRY

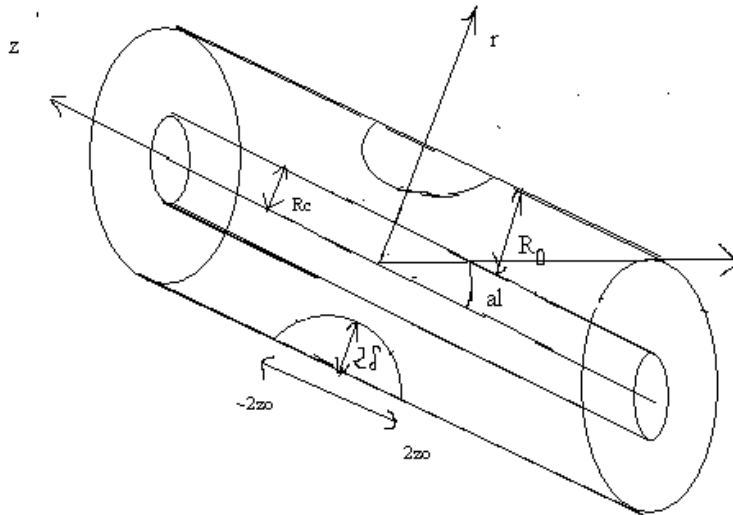


Figure 2.1 Geometry of the stenosis

The stenotic protuberance is assumed to be an axisymmetric surface generated by a cosine curve (Neeraja and Vidya [2012])

$$R(z) = \begin{cases} R_0 - \delta \left(1 + \cos \left(\frac{\pi z}{2z_0} \right) \right) & -2z_0 \leq z \leq 2z_0 \\ R_0 & \text{otherwise} \end{cases} \quad (2.1)$$

where $4z_0$ is the length of the stenotic region, 2δ is the maximum protuberance of the stenotic form of the artery wall and R_0 is the radius of the tube in non-stenotic region.

EQUATION OF PRESSURE GRADIENT AND TIME PERIODIC BODY ACCELERATION

The pressure gradient and the time periodic body acceleration are given by

$$-\frac{\partial p}{\partial z} = A_0 + A_1 \sin(\omega_p t) \quad (2.2)$$

where A_0 is the steady component of the pressure gradient, A_1 is the amplitude of the fluctuating component of the pressure gradient and $\omega_p = 2\pi f_p$, f_p is the pulse frequency in Hz.

$$F(t) = a_0 \cos(\omega_b t + \phi) \quad (2.3)$$

where a_0 is the amplitude and $\omega_b = 2\pi f_b$, f_b is the frequency in Hz and is assumed small so that wave effect can be neglected and ϕ is the phase difference.

III. GOVERNING EQUATION

The governing equation in cylindrical polar coordinates can be written in the form:

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial z} - \frac{1}{r} \frac{\partial}{\partial r} (r\tau) + F(t) + \rho g \sin(al) - B_0^2 \sigma u \quad (3.1)$$

where ρ is density of the fluid, B_0 is the external magnetic field along the radial direction, σ is the conductivity of the fluid, τ is the shear stress and al is small angle of inclination.

The constitutive equation for Newtonian fluid is given as

$$\tau = -\mu \frac{\partial u}{\partial r} \quad (3.2)$$

BOUNDARY CONDITIONS

$$u = u_s \text{ at } r=R \quad \text{and} \quad u = 0 \text{ at } r=R_c \quad (3.3)$$

IV. RESULT AND DISCUSSIONS

In order to get a proper insight in to the flow behavior of blood through a inclined mild stenosed artery in which a catheter is inserted under periodic body acceleration and magnetic field, the variations of u have been computed using Finite Difference Method. Figure 1-3 and table 4.1 demonstrates velocity distribution of Newtonian fluid w.r.t. r for different values of Hartmann

Number H , pressure gradient parameter e , slip velocity u_s , and inclination angle α , when a catheter is inserted in tube. The insertion of catheter into an artery leads the formation of an annular region between the catheter wall and the arterial wall Figure 1 reveals that velocity decreases with increasing values of Hartmann No. H . Figure 2 presents velocity distribution w.r.t. r for different values of pressure gradient parameter e and slip velocity u_s . It shows velocity increases significantly with increasing value of both parameters. Figure 3 shows effect of α on velocity distribution and it shows velocity increases with increasing value of α . Table 1 illustrated that distribution w.r.t. r for different values of body acceleration parameter A . It shows velocity increases with increasing value of A .



Table 4.1**Variation of velocity profile w.r.t. radial distance r for different value of body acceleration**

sr.no.	radial distance	velocity at A=0.0	velocity at A=1.0
1	0.0857	0	0
2	0.1143	0.0891	0.0892
3	0.1429	0.1551	0.1552
4	0.1715	0.2055	0.2059
5	0.2000	0.2452	0.2455
6	0.2286	0.2760	0.2764
7	0.2572	0.2999	0.3003
8	0.2857	0.3178	0.3183
9	0.3143	0.3307	0.3312
10	0.3429	0.3391	0.3396
11	0.3714	0.3434	0.3439
12	0.4000	0.3440	0.3445
13	0.4286	0.3411	0.3416
14	0.4572	0.3350	0.3355
15	0.4857	0.3259	0.3264
16	0.5143	0.3138	0.3143
17	0.5429	0.2990	0.2994
18	0.5714	0.2814	0.2818
19	0.6000	0.2612	0.2615
20	0.6286	0.2384	0.2387
21	0.6571	0.2131	0.2133
22	0.6857	0.1854	0.1855
23	0.7143	0.1551	0.1553
24	0.7429	0.1225	0.1226
25	0.7714	0.0874	0.0875
26	0.8000	0.0500	0.0500

5.8 CONCLUSION

In present paper blood flow through artery when a catheter is inserted coaxially assuming blood as Newtonian fluid is investigated. Figure 4 shows the comparison between the velocity profiles of one layered Newtonian fluid flow, one layered catheterized artery flow, two-layered blood flow taking Bingham-plastic fluid in core region and two-layered blood flow taking Casson fluid in core region. For this purpose we studied the problems at same platform assuming $H=0$ in catheterized artery and in one layered Newtonian flow, radius of the catheter is equal to plug radius in two layered flow and keeping other parameters same.

It shows that the of one layered Newtonian fluid occupies highest velocity and two-layered blood flow taking Casson fluid in core region occupies lowest velocity at the same platform. It is also noticeable that insertion of catheter reduces velocity to more than half of velocity observed in case of without insertion of catheter. It is because, the insertion of a catheter in to an artery alters the flow field, modifies the pressure distribution and hence increases the resistance to flow, Dash et.al.[1999] and Sankar and Hemalatha[2007].

Main object of the present study is to find out the velocity profile in different cases. This study is useful to determine low velocity region and shows the behavior of blood flow in different conditions.

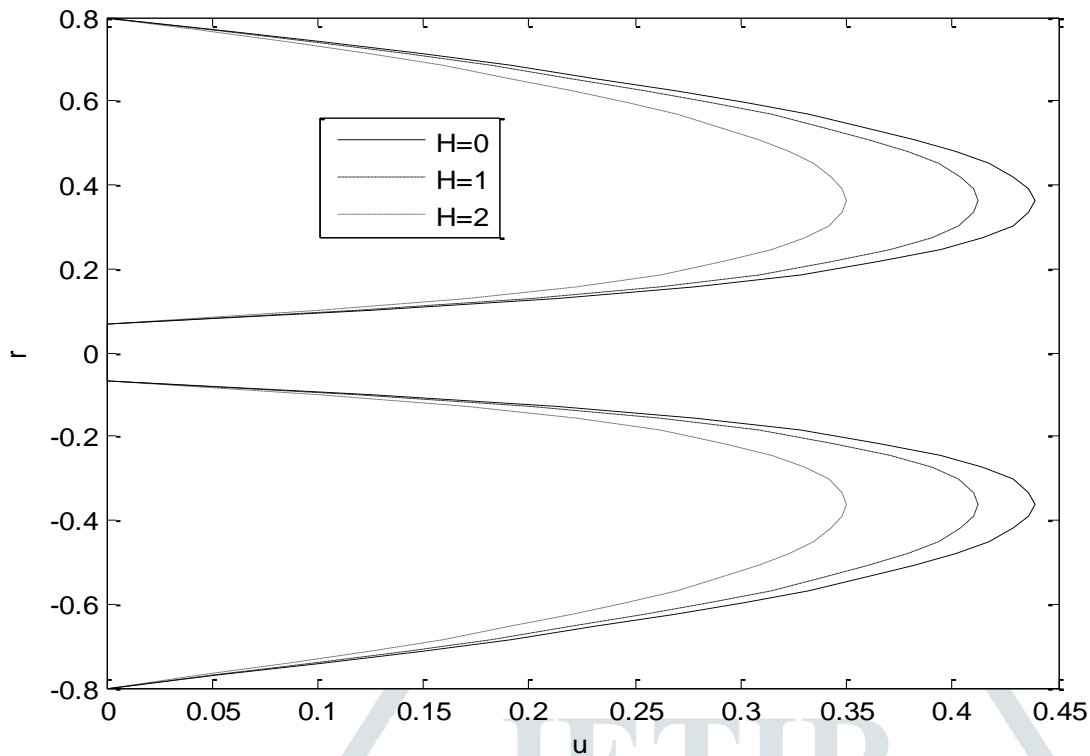


Figure 1: Variation of velocity w.r.t. radial distance r for different values of Hartmann Number H when a catheter is inserted coaxially in artery

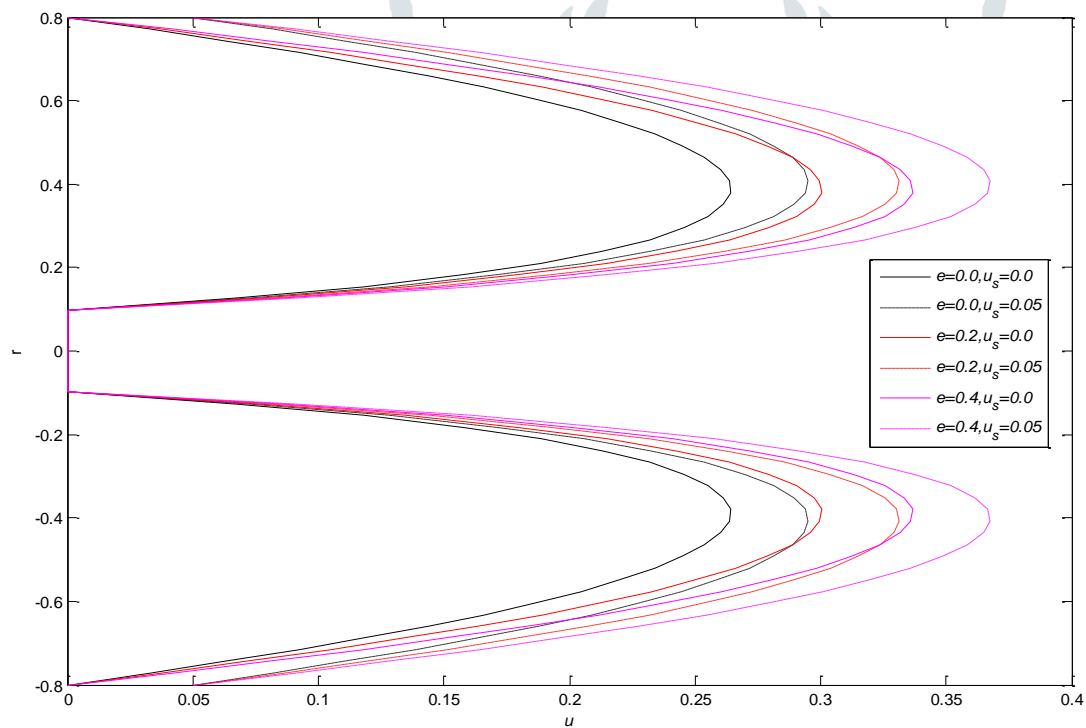


Figure 2 : Variation of velocity w.r.t. radial distance r for different values of pressure gradient parameter e and slip velocity u_s when a catheter is inserted coaxially in artery

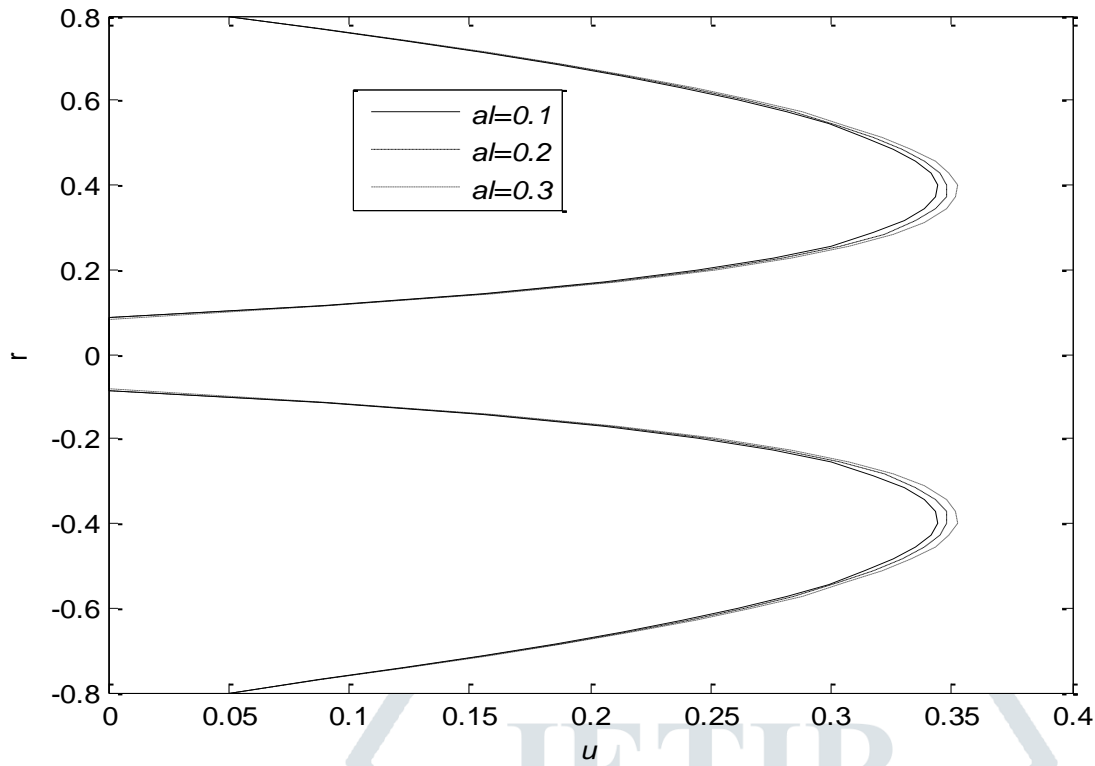


Figure 3 : Variation of velocity w.r.t. radial distance r for different values of inclination angle αl when a catheter is inserted coaxially in artery

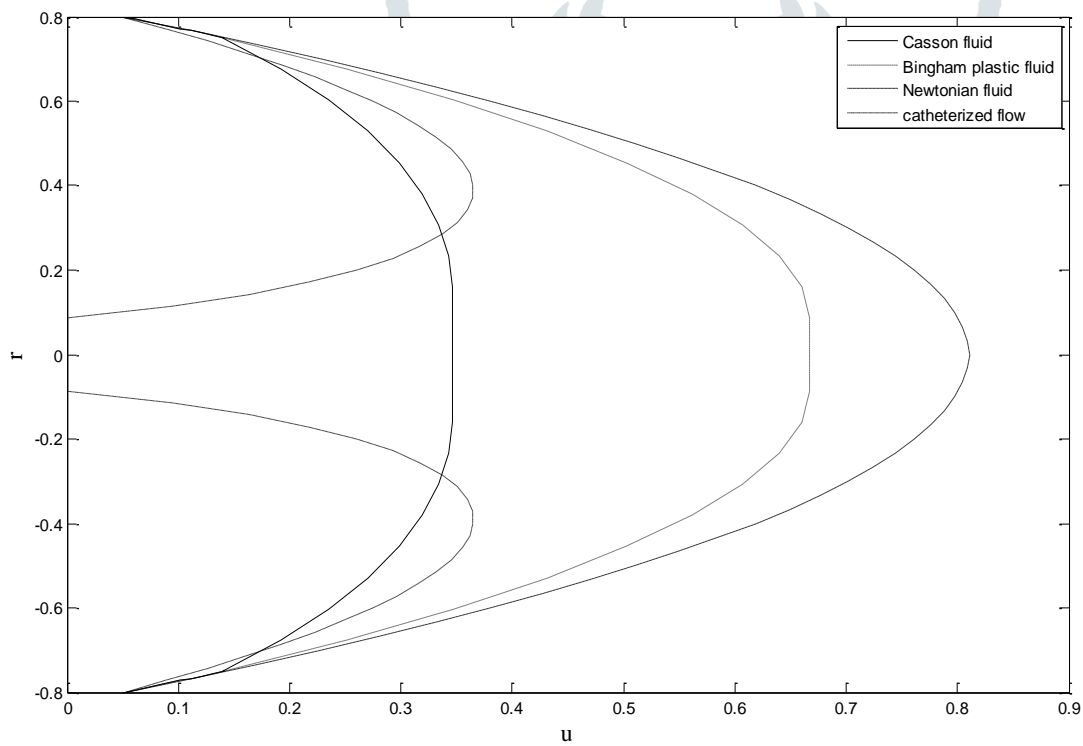


Figure 4 : Comparative study of variation of velocity w.r.t. radial distance r

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