IDENTITY FIBONACCI NUMBERS

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ABSTRACT

Fibonacci is famous today the main reason behind it is Edouard Lucas correlate his name with set of +ve integers (infinite sequence) that arose simple problem in Liber Abaci. The sequence {1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89………..} present in nature different number of ways.

We see that there are three petals in Lily while in the five buttercups, thirteen petals in the flowers of marigolds, asters have twenty-one petals while most daisies occur thirty-four, fifty-five and eighty-nine petals. When we see a photo of sunflower and try to count two types of seeds curves, there are curves spirals which twisting clockwise, you found thirty-four spiral and curves spirals which twisting clockwise, you found thirty-four spiral fifty-five spirals. These are examples Fibonacci's sequence numbers in nature.

Keywords – Fibonacci, Area, equation, sequence, cubes.

INTRODUCTION-

FIBONACCI SEQUENCE -A sequence of number it contains Fibonacci numbers

1 1 2 3 5 8 13 21 34 55 89 144 233 377 etc

is consider Fibonacci sequence. We denote 1^{st},2^{nd} ,3^{rd} by a 1=1 , a 2=1 , a 3=2, ………………and a n by n^{th} Fibonacci number. Fibonacci sequence the have following property,

2=1+1 or a 3=a 2+a 1

3=2+1 or a 4=a 3+a 2

5=3+2 or a 5=a 4+a 3

8=5+3 or a 6=a 5+a 4

In general, we can define

$a_1=a_2=1 \quad a_n=a_{n-1}+a_{n-2} \quad \text{for} \quad n\geq 3$

i.e. After the second term ,3^{rd} onwards terms can be calculated by adding previous two terms Also we use recursive sequence for such type of sequence.

The Fibonacci no's increases fast. for this we can consider an example

$a_{5m+2}>10^m$ for $m\geq 1$,

⇨ $a_7>10$, $a_{12}>100$, $a_{17}>1000$, $a_{22}>10000$

Result

We can prove this inequality by principal of mathematical induction method

Step 1.we shall prove this result for m = 1

Since a7 = 13 > 10.

Result is proving obviously

Step 2.we shall assume this result for m = k

i.e. a5(k)+2>10k

Step 2.we shall prove this result for m = k+1

i.e. a5(k+1)+2>10k+1

By repeated application of recursion rule ai=ai-1+ai-2 ,we have

a5k+7=8 a5k+2+5a5k+1 >8 a5k+2+2(a5k+1+a5k)= 10 a5k+2>

10.10k=10k+1

⇨ Result is true for m = k+1

FIBONACCI NUMBERS IDENTITIES

We now discuss some basic Fibonacci numbers which are very important to solve problems.

Result

First we shall prove that sum of first k numbers of this type of sequence is equal to ak+2-1.

In particular, k=8,we have

1+1+2+3+5+8+13+21=54=55-1=a8+2-1

For general solution, consider the relations,

a1=a3- a2 ,

a2=a4- a3 ,

a3=a5- a4 ,

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•

ak-1=ak+1- ak ,

ak=ak+2- ak ,

Now by adding all these, we have

a1+a2+a3+…+ak-1+ak=ak+2-1 (terms cancel in pairs)

Result Prove that Fibonacci numbers satisfy the equality

y2 = yα+1yα-1+ -1α-1

In particular, α =6 and α =7; we have

a62 = 82=13.5-1= a7a5-1

a72 = 132=21.8+1= a8a6+1

For proving the general case, we consider

y2- yα+1yα-1=yα yα-1+ yα-2-yα+1yα-1

=y- yα+1yα-1 +yyα-2

Since yα+1=yα +yα-1 , and so y- yα+1= - yα-1 we have

y2- yα+1yα-1= -1 (yα-12- yyα-2)

We observe that R.H.S. of eq^n is similar to L.H.S. except initial sign of R.H.S. but subscripts decrease one in R.H.S. By Applying & repeating the same steps, we have

yα-12- yyα-2 = -1 (yα-22- yα-1yα-3)

⇨ y2- yα+1yα-1=-1 (yα-22- yα-1yα-3)

After continuing like this (m-2) steps, we have

y2- yα+1yα-1=-1m-2 (y22- y3y1)

=-1m-2 (12- 2.1)

=-1m-1

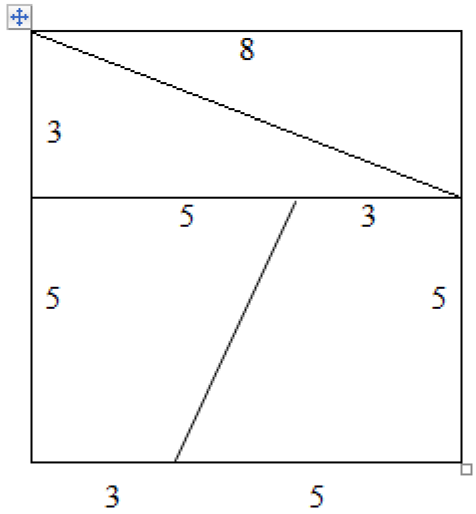
which is required proof.

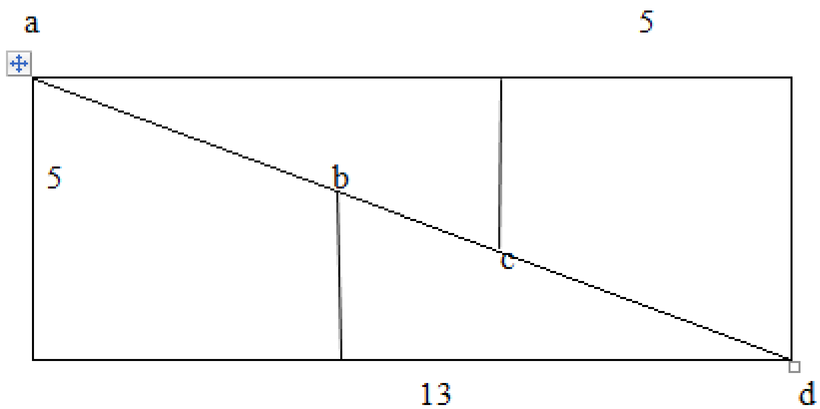
Put m=2k, from above eq^n. we have

$a_{2k}^2 = a_{2k+1} a_{2k-1} - 1$

now we observe that this identity represents a square of size eight units by eight may be cut into parts that cover a rectangle of size five by thirteen units.

For this purpose, divide square into four parts as given figure 1^{st} and rearrange them as 2^{nd} figure below

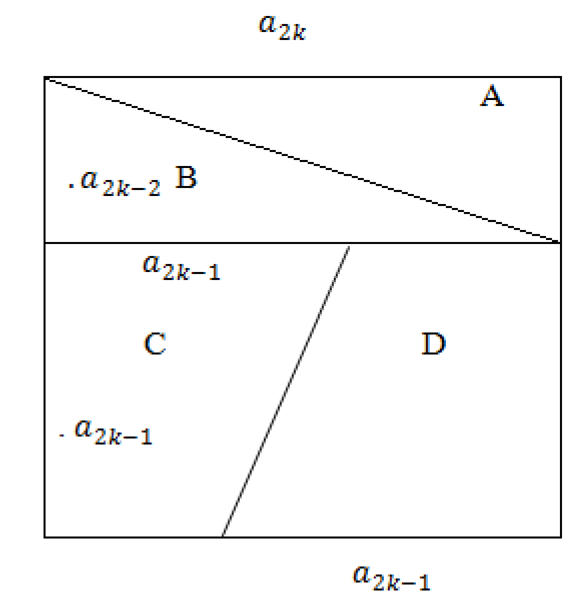


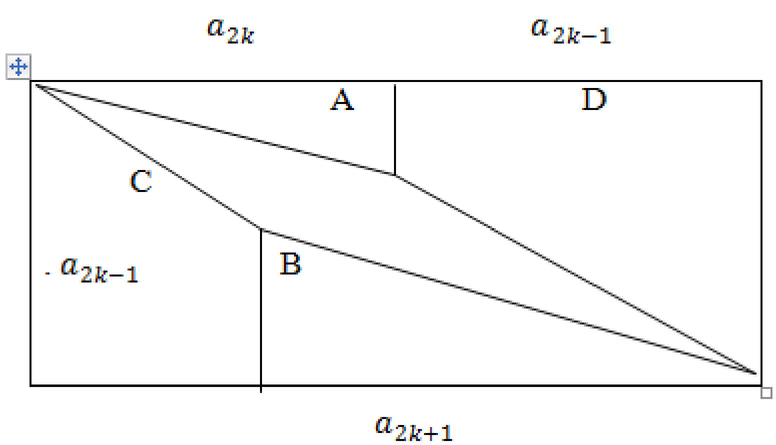


⇨Area of square =64 & Area of rectangle =65

i.e. both have equal constituent parts, area has increase by one square unit.

Also in foregoing observation, we observe that square having sides a2k (Fibonacci number). Divide square into four parts as given figure 1st and rearrange them as 2nd figure below produce a rectangle having a slim parallelogram as a slot as in figure below





⇨ $a_{2k-1}\, a_{2k+1} - 1 = a_{2k}^2$

i.e. Area of rectangle - Area of parallelogram = Area of square

. Also here h = 1a 2k2+a 2k-22 where h denotes parallelogram height

If a2k has logically large In Particular a2k= 144, a2k-2=55),slim slot of parallelogram is narrow impossible to feel from eyes.

CONCLUSION

1.Fibonacci numbers (squares); two only

$a_l = a_2 = 1^2$,

$a_{12} = 12^2$

2. Fibonacci numbers (cubes); three only

$a_1 = a_2 = 1^3$,

$a_6=2^3$

3. Fibonacci numbers (triangular); Five only

$a_1 = a_2 = 1$

$a_4 = 3$, $a_8 = 21$.

$a_{10} = 55$

4. Fibonacci numbers (perfect); Zero

5. Each k^{th} Fibonacci number is in factor of a_k

i.e. In Particular, Each 3^{rd} Fibonacci number factor of $a_3 = 2$

Each 4^{th} Fibonacci number is in factor of $a_4 = 3$

Each 5^{th} Fibonacci number is in factor of $a_5 = 5$

Each 6^{th} Fibonacci number is in factor of $a_6 = 8$

Fibonacci numbers as Factors of Fibonacci numbers

	i	3	4	5	6	7	8	9	10	11	12	...
	Fib(i)	**2**	**3**	**5**	**8**	**13**	**21**	**34**	**55**	**89**	**144**	**...**
Factors	**2=Fib(3)**	✓	✗	✗	✓	✗	✗	✓	✗	✗	✓	Every 3^{rd} Fib number
	3=Fib(4)	✗	✓	✗	✗	✗	✓	✗	✗	✗	✓	Every 4^{th} Fib number
	5=Fib(5)	✗	✗	✓	✗	✗	✗	✗	✓	✗	✗	Every 5^{th} Fib number
	8=Fib(6)	✗	✗	✗	✓	✗	✗	✗	✗	✗	✓	Every 6^{th} Fib number
	F(k)	...										F(all multiples of k)

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