

The Complete Dissection of Six Sigma with Examples on Statistical Software Minitab

Abstract: This paper gives the summary of the Entire framework of DMAIC of Six Sigma by applying the usage of statistical software tools like Minitab, Sigma XL & Quality Companion Software and discussing the intricacies of Define, Measure, Analyse, and Improve & Control Phase with examples.

1. INTRODUCTION

The paper gives a clear concept of the various phases of Six Sigma and the statistical calculations used in it, it has been derived by referring various books and concisely put together so that it is easy to understand Six Sigma. The Concept of Six Sigma begins with a process model and its implementation is called Business process Management (BPM) and the process of Six Sigma began at Motorola in the 1980's. Six Sigma is the **letter σ** in the Greek alphabet used to denote Standard deviation, a statistical measurement of variation. Standard deviation can be thought of as a comparison between expected results or outcomes in a group of operations versus those that fail. **Bill Smith of Motorola Coined the term Six Sigma and is also called the father of Six Sigma; The Quality leader responsible for the term Total Quality Management is Feigen Baum; robustness in quality management was pioneered by Dr Taguchi; quality leader who did extensive work in Japanese industry is Dr Deming who paved the way for Quality Management Concept; Walter She wart is also one of the contributors in Six Sigma, he is sometimes referred to as father of Statistical Control.** With Six Sigma Standard deviation we arrive at 3.4 defects per million opportunities or 99.9997 percent this would mean that at six sigma, an airline would only loose three pieces of luggage for every one million that it handles. The Three primary elements of Six Sigma are Customers, Processes & Employees.

Six Sigma is basically the application of Statistical formulas and Methods to eliminate defects, variation in a product or a process. For example if you want to find the average height of male population in India, you cannot bring the entire population of more than 2 billion into one room and measure their height for a scenario like this we take samples that is we pick up sample(people) from each state and use statistical formulas to draw the inference about the average height of male population in a population which is more than 2 billion. One more example would be say a company manufactures pistons used in motor cycles the customer demand is that the piston should not a diameter more than 9 cm and less than 5 cm anything manufactured outside this limits is said to be a variation and the six sigma consultant should confirm that the pistons are manufactured within the said limits else if there is variation in the range then the company is not operating at 6 sigma level it is operating at a very low level.

In statistics, the **standard deviation** (SD, also represented by the Greek letter **sigma** σ or the Latin letter s) is a measure that is used to quantify the amount of variation or dispersion of a set of data values.

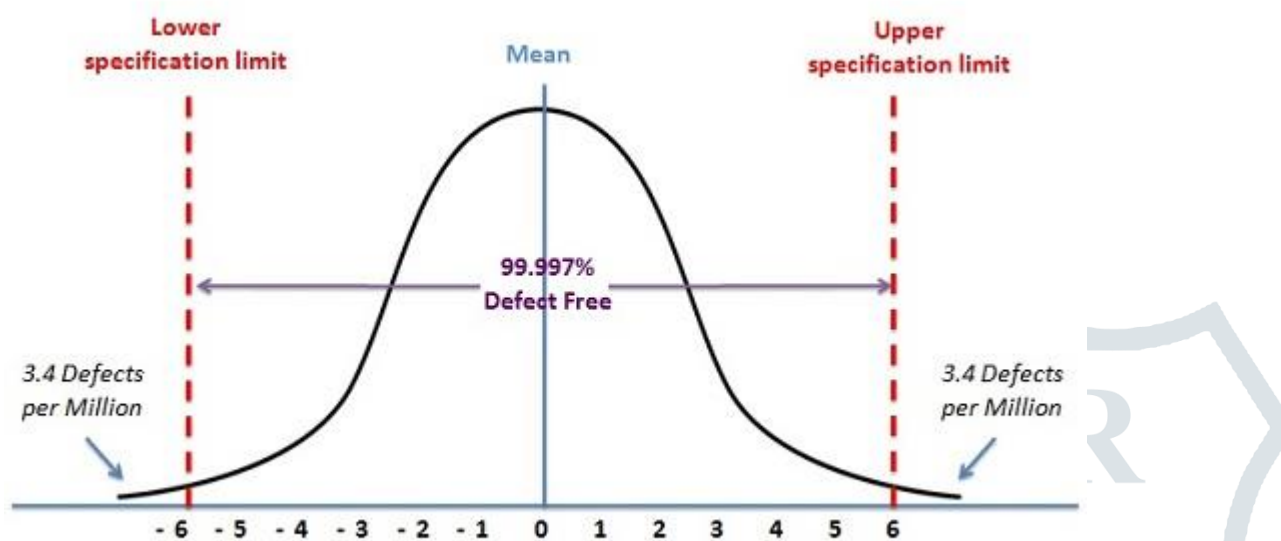
One key innovation of Six Sigma involves the absolute "professionalizing" of quality management functions. Prior to Six Sigma, quality management in practice was largely relegated to the production floor and to **statisticians** in a separate quality department. Formal Six Sigma programs adopt a kind of elite ranking terminology (similar to some martial arts systems, like Kung-Fu and Judo) to define a hierarchy (and special career path) that includes all business functions and levels.

Six Sigma identifies several key roles for its successful implementation.

- **Executive Leadership** includes the CEO and other members of top management. They are responsible for setting up a vision for Six Sigma implementation. They also empower the other role holders with the freedom and resources to explore new ideas for breakthrough improvements by transcending departmental barriers and overcoming inherent resistance to change.
- **Champions** take responsibility for Six Sigma implementation across the organization in an integrated manner. The Executive Leadership draws them from upper management. Champions also act as mentors to Black Belts.
- **Master Black Belts**, identified by Champions, act as in-house coaches on Six Sigma. They devote 100% of their time to Six Sigma. They assist Champions and guide Black Belts and Green Belts. Apart from statistical tasks, they spend their time on ensuring consistent application of Six Sigma across various functions and departments.
- **Black Belts** operate under Master Black Belts to apply Six Sigma methodology to specific projects. They devote 100% of their valued time to Six Sigma. They primarily focus on Six Sigma project execution and special leadership with special tasks, whereas Champions and Master Black Belts focus on identifying projects/functions for Six Sigma.
- **Green Belts** are the employees who take up Six Sigma implementation along with their other job responsibilities, operating under the guidance of Black Belts.
- **Mikel Harry Coined the term Green Belt & Black Belt; he is also one of the Pillars of Six Sigma**
- **Deming on Processes states that eighty five percent of the reasons for failure to meet customer expectations are related to deficiencies in systems and process...Rather than the employee, the role of the management is to change the process rather than badgering individuals to do better.**

Six Sigma Process Table

SIGMA LEVEL	DEFECT RATE	YIELD
2σ	308,770 dpmo	69.10000%
3σ	66,811 dpmo	93.33000%
4σ	6,210 dpmo	99.38000%
5σ	233 dpmo	99.97700%
6σ	3.44 dpmo	99.99966%



An Average american company operates at four Sigma Level.

Noise factors are referred to as random variation or common cause variation.

Common cause variation is a source of variation caused by unknown factors.

Dr Taguchi's Loss Equation :

Taguchi Constructed a loss equation to determine how much society loses every time when the parts produced do not match the specified target the loss function is given below :-

$$L(y) = k(y-T)^2$$

Where $k = \Delta / m^2$

Δ = Cost of a defective product

$$M = |USL - T|$$

Example : Suppose a manufacturer specifies the target for the diameter of a given rivet(A rivet is a permanent mechanical **fastener**. Before being installed, a rivet consists of a smooth **cylindrical** shaft with a head on one end. The end opposite to the head is called the *tail*) to be 6 inches and the lower and upper limits to be 5.98 and 6.02 inches respectively. A bolt measuring 5.99 inches is inserted in its intended hole of a machine. Five months after the machine was sold it breaks down as a result of loose parts . The cost of repair is estimated at \$95. Find the loss to society incurred because of the part not matching its target.

We must first determine the value of the constant k

Now the equation $L(y) = k (y-T)^2$

Where $k = \Delta / m^2$

T = 6 inches (which is the target)

$Y = 5.99$ (the bolt which is actually inserted)

USL = 6.02 (Upper Specification Limit)

$$m = | USL - T | = 6.02 - 6 = 0.02$$

$\Delta = \$95$ (Δ = Cost of Repair)

$$k = (\$95/0.0004) = \$237,500$$

$$(y-T)^2 = (5.99 - 6)^2 = 0.01^2 = 0.0001$$

Therefore

$$L(y) = k (y-T)^2 = \$237,500 * 0.0001 = \$23.75$$

Not producing a bolt that matches the target would have resulted in a financial loss to the society that would amount to \$23.75.

The 25 patient/day capacity of a hospital is the result of truly remarkable little law, referred convincingly as little law. Little law was proved by John .C.Little of MIT.

That is throughput (TH) is equal to work in process(WIP) divided by cycle time.

$$TH = WIP/CT$$

In our hospital example , WIP is the number of patient the hospital accommodates and cycle time is the average length of the stay.

According to little's law TH can only be increased if

- 1] WIP the amount of work the system can process at one time is increased.**
- 2] CT is decreased example length of stay in hospital.**
- 3] Both**

1.5 by Default Six Sigma Shift

By offsetting normal distribution by a 1.5 standard deviation on either side, the adjustment takes into account what happens to every process over many cycles of manufacturing... Simply put, accommodating shift and drift is our 'fudge factor,' or a way to allow for unexpected errors or movement over time. Using 1.5 sigma as a standard deviation gives us a strong advantage in improving quality not only in industrial process and designs, but in commercial processes as well. It allows us to design products and services that are relatively impervious, or 'robust,' to natural, unavoidable sources of variation in processes, components, and materials."

Statistical Take Away: The reporting convention of Six Sigma requires the process capability to be reported in short-term sigma – without the presence of special cause variation. Long-term sigma is determined by subtracting 1.5 sigma from our short-term sigma calculation to account for the process shift that is known to occur over time.

Comparing Six Sigma and Lean

Essentially, Six Sigma and Lean systems have the same goal. They both seek to eliminate waste and create the most efficient system possible, but they take different approaches toward how achieving this goal. In simplest terms, the main difference between Lean and Six Sigma is that they identify the root cause of waste differently.

Lean practitioners believe that waste comes from unnecessary steps in the production process that do not add value to the finished product, while Six Sigma proponents assert that waste results from variation within the process.

Of course, there is truth in both of these assessments, which is why both Lean and Six Sigma methodologies have been so successful in improving overall business performance in a variety of fields. In fact, these two disciplines have proven to be especially successful when working in tandem - hence the creation of Lean Six Sigma.

Value Stream Mapping

Value stream mapping is a lean manufacturing or lean enterprise technique visualization tool for documenting, analysing and improving the flow of information or materials required to bring a product to market. A value stream map is typically created as a one-page flow chart depicting the current production path. In lean production, value is the ultimate goal for customers. Processes that do not provide any value are called waste. Value stream has been embraced as a great methodology for identifying the inherent waste, and plan to eliminate it, which is an indispensable tool in your six sigma program.

Maturity Models –Process Based

A Wide variety of process-based maturity models have been available for some time. Some are focussed specifically on departments such as Information technology while others include more of the broader aspects of organization. Regardless, most of these models have their roots in the area of software development. Common process based models include:

- ITIL (Information Technology Infrastructure Library)
- ISO/IEC 15504 (Software Process Improvement and Process Capability Determination)
- CMMI (Capability Maturity Model Integration)

Most of the above models are level based. For Example the CMMI model, which has grown significantly in popularity is structured into five levels Initial (Ad Hoc)

Managed (Repeatable)

Defined (Defined and Measured)

Quantitative Managed (Managed)

Optimizing (self-Optimizing)

- Level 0 – Person Dependent Practices

- Level 1- Document Process

- Level 2- Partial Deployment

- level 3- Full Deployment

- Level 4- Measured and Automated

-Level 5- Continuously Improving

Six Sigma Encourages Improvement within the CMMI Framework

Six Sigma success is not equivalent to compliance with standards and models, and vice versa. Industry models and standards frequently demand measurements, monitoring, and control. Frequently used standards include CMMI models, ISO, IEEE standards, and ISO 12207. Six Sigma can be used to achieve compliance with aspects of each of these standards. However, interpreting Six Sigma usage as achievement of model compliance, and likewise assuming Six Sigma when compliance is found, is a mistake. Six Sigma is not limited to use in high maturity organizations. In organizations that primarily use CMMI, many people associate Six Sigma with the high maturity process areas. However, there is a direct connection between Six Sigma and the generic practices, which are used for process areas at all maturity levels. Six Sigma enables a tactical approach to the implementation of the generic practices, and therefore much of the intent of the high-maturity process areas is implemented at lower maturity or within the continuous representation. This drastically accelerates the cycle time required for the final steps to high maturity by putting the building blocks for the high-maturity process areas in place. Six Sigma is not a competitor to CMMI or other process models and standards. There are many domain-specific models and standards. Six Sigma is not domain specific and can be a governance model or a tactical improvement engine. It can provide the problem definition and statement of benefit against which a decision about adopting a technology can be made. It can help solve specific problems and improve specific products or processes within the larger context of overall organizational process improvement. Or, in more general terms, it can serve as an enabler for the successful implementation of domain-specific improvement models.

Integrate CMMI, Six Sigma, and all other improvement initiatives to provide a standard for the execution of every project throughout its life cycle. While the previous three approaches are tactical (i.e., they provide a course of action), this is an approach for setting an organization's strategy. It is longer term and more visionary. It promotes the idea that an organization should take control of its destiny and manage its initiatives rather than be managed by them. Six Sigma methods can be leveraged to design the organization's standard processes, but the focus here is embedding Six Sigma alongside other initiatives in the organizational business and engineering processes. This approach can be executed at any maturity level, with any maturity level as the end goal. When possible, it's best to start while at low maturity

CMMI is used to create an organizational process infrastructure by addressing particular domains, such as software and systems engineering. Six Sigma is a top-down initiative that cuts across the entire enterprise, including areas such as engineering, sales, marketing, and research. Six Sigma is intended to be implemented with a focus on problems and opportunities, often with narrow scopes, that will yield significant business benefits. It focuses on the performance of processes and practices as implemented rather than checking for compliance against a definition or model. While these two improvement initiatives are different by design, they are interdependent in their use. In practice, a back and forth focus is often effective. For instance, Six Sigma could be used to discover that processes need to be more repeatable, CMMI could be used to institute processes based on community best practice, and then Six Sigma could be used to optimize those processes.

1.1.1 Challenges in Implementing Six Sigma

There are several roadblocks if you try to implement Six Sigma in a respective organization there will be resistance stating that Six Sigma is too Statistical, non-cooperation from employees as they don't want to come out from the comfort zone of their work and are afraid whether their performance will be measured during the rigorous process of implementing the

Six Sigma Framework –DMAIC(Define-Measure-Analyse-Improve-Control) because Lean Six Sigma is all about reducing the waste, redundant processes and variation in the process ,so people from all the departments like Production , Finance , Planning have to cooperate in improving the process which they are likely to resist as the people in the organization feel that they might lose their job during the implementation activity. Typically in an Organization the percentages of people having neutral attitude towards the implementation of Six Sigma will be 70% and 15% will be resistant towards the Six Sigma initiative the rest 15% will be supportive for the Six Sigma implementation initiative.

There are some areas where the Six Sigma Consultant finds it difficult to do the formulation of problem statement. Resistance to implement the concepts, in all levels of management, leading to political intelligence coming into play and finding and freezing the right team members for implementing and spearheading the projects selected.

Resistance from the employees or employers will be also like they disagree with benefits and return of investment on a Six Sigma Project, criticizing the statistical tools stating that that they are cumbersome to learn and whether the employees are trainable on these tools, sometimes the employees will delay the implementation to show that Six Sigma is just a fad and there are no benefits by implementing it.

There is also a misunderstanding that Six Sigma is meant for large Manufacturing Companies and not small and Medium Companies so employers of small and medium companies resist to the implementation of Six Sigma.

There can be resistance if the management is not qualified to understand the various business intricacies of their company.

Stakeholders (People who are investing money in the Six Sigma Project) can also resist the change stating that Six Sigma is very Complex and whether the Company has the resources to understand the DMAIC Framework.

Other kind of resistance will be from the workers in the factory because they don't understand the value of implementing Six Sigma and don't want interference in their day to day work.

The degree of resistance to implement Six Sigma can vary from organization to organization depending on the environment, domain of the organization and people working there.

1.1.2 How to overcome the challenges in implementing Six Sigma

These are some strategies as listed below to overcome the challenges while implementing Six Sigma which are as listed below:

1. Convince the Management of the Return of Investment based on Operation Costs and Productivity efficiency also demonstrate the Software Minitab and explain its Simplicity by displaying various graphs in the Software so that they don't avoid statistics.
2. Convince the employees that nobody will be out of job but their participation will incur rewards in the form of financial bonus from the top management
3. Select the best, qualified team in the organization and mentor them and train, certify them to be black belts or green belts.
4. Form a very good Standard Operating Procedure for imparting training to the employees and prepare training manuals which are easily understood by the staff with day to day life examples.
5. As per the respective organizational process assets the Six Sigma Consultant has to bring a cultural and smooth transition in the organization by getting the buy in from the top management and further convincing the staff about the importance of change management for the better and future prospects and not short term benefits.

Effective Management Decision Making

The following questions can be answered although it is more interesting and useful to get a collective response. As you answer make note of your score per question so that you can work out your means which have a self-servicing attribute bias and those that of the observer in answering the questions –choose from the range of 1-4 so that 1= Strongly agree , 2= Agree, 3 = Disagree , 4= Strongly Disagree.

1. Do you feel management issues Contribute to business success?
2. Do you feel individual characteristics contribute to business success?
3. Do you feel financing issues Contribute to business success?
4. Do you feel Marketing activities Contribute to business success?

5. Do you feel HR Issues Contribute to business success ?
6. Do you feel Economic Conditions Contribute to business success?
7. Do you feel Competition Contribute to business success?
8. Do you feel regulations Contribute to business success?
9. Do you feel technology Contribute to business success?
10. Do you feel environmental factors Contribute to business success?
11. Do you feel individual characters impede business success?
12. Do you feel management issues impede business success?
13. Do you feel financing issues impede business success?
14. Do you feel marketing activities impede business success?
15. Do you feel HR Issues impede business success?
16. Do you feel Economic Conditions impede business success?
17. Do you feel competition will impede business success?
18. Do you feel regulations will impede business success?
19. Do you feel Technology will impede business success?
20. Do you feel Environmental factors will impede business success?

- An I (Internal Attribution) to success for questions 1,2,4,5,11,13,14
- An E(External Attribution) to success for questions 3,6,7,8,9,10,12,15,16,17,18,19,20.

A low score of I Suggests you are more likely to attribute to success to factors over which you have control whereas a low E score suggests you are more likely to attribute you success to factors which you have limited control.

Note: Meaning of Impede is Hindrance to the progress.

Six Sigma Deployment & Planning Readiness Assessment

Cultural Assessment	Operations Assessment
How do Senior leaders cascade information How are important Decisions made Who makes the decision How fast are the decision made How are success and failure recognized How does the organization handle failures? Does everyone understand the mission vision and strategy Is everyone aware of critical customer, revenue, and expense issues? How are Corporate goals established? How clear are the goals? Are the goals measurable?	How is success measured? Are the right things measured? How often are things measured? Are there a few metrics that all employees understand and use? Are decision based on assumptions or data? Who owns each critical process? Who owns the data? Are the data on spread sheets, laptops or in a data warehouse? Have the data been validated? Are reports written in simple terms and free from fuzzy language? Are updated process maps available for most of the critical process? Do executives know what a process map is?

Budgets

Budgeting for a lean Six Sigma project can be difficult since the path from start to finish is unknown. In lean Six Sigma projects we follow data. Consequently, the path we follow could be circuitous. Fortunately there are several items we may be able to budget for directly or alternately establish a reserve budget. For example we have an assembled team, with an estimated percentage contribution to the project per week and estimated project duration. Therefore we can budget labour dollars for the project and in instances where doubt exists it may be possible to establish a project reserve budget. Such budgets may be set as a percentage (for example, 10%, 15%, and 20%) of the actual dollar budget. These budgets are typically set aside as contingency dollars to help mitigate risk

Supply/Demand Management

Two important aspects of supply and demand management are the ability to keep the pipeline full and the development of trained resources' to sustain the lean six sigma deployment initiative.

1.1.3 Basic Questions to be asked before Implementing Six Sigma

Six Sigma has been successfully applied in banking, healthcare, Military Fast Food Chains, airlines, Hotels, retail stores if there's a repetitive process with a problem you can apply Six Sigma.

1.1.4 Below is some basic Questionnaire which can be used by a Six Sigma Consultant

1. List four Problems your business is facing right now?
2. Now think about the day to day operations of your organization list four of the major repetitive processes
3. Next think about the defects affecting your products or services or outputs what four defects you see on a regular basis list them out
4. Finally what variations you see in your business processes list the four variations it might include the differences in the way you do things from day to day or in your outputs.

Dr Taguchi's Loss Equation: Dr Taguchi Defined quality as "loss imparted by the product to society from the time the product is shipped". This loss includes not only to the company but also costs to the customer through poor product performance and reliability, leading to further loss to the manufacturer.


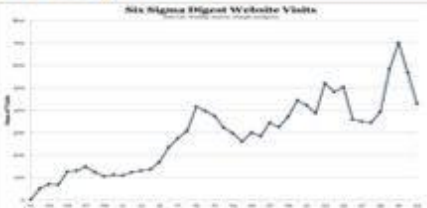
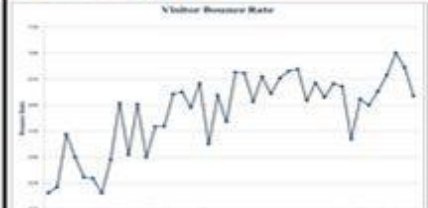
According to Dr Taguchi the mathematical equation for quality loss function is given by $L=k(Y-T)^2$ where L is Loss, K is a Constant and Y-T is the distance of the characteristic from the target thus increase in loss is proportional to the square of the distance from the target. Taguchi's Contribution later became the foundation for design for Six Sigma (DFSS).

2.DEFINE PHASE SOME TOPICS

2.1 The objectives within the Define Phase which is first phase in DMAIC framework of Six Sigma are:-

Define the Project Charter

- Define scope, objectives, and schedule
- Define the Process (top-level) and its stake holders
- Select team members
- Obtain Authorization from Sponsor
- Assemble and train the team.

		Project Title: Project Digest			
		Project Lead	Project Champion	Executive Sponsor	MBB/Mentor
		Michael Parker	Michael Parker	Michael Parker	Michael Parker
Primary Metric		Secondary Metric			
 <p>Primary Metric: Website Visits Data Type: Continuous Frequency: Weekly Baseline: 471 vsits/week (last 10 weeks) Goal: 3,500 visits/week Timeline: 2012</p>		 <p>Secondary Metric: Bounce Rate Data Type: Attribute~Proportion Frequency: Weekly Baseline: 63% (last 10 weeks) Goal: <= 60% Timeline: 2012</p>			
Problem Statement & Objective		Business Case			
Six Sigma Digest currently averages 471 weekly visits to it's website. The goal is 3,500 weekly visitors. This leaves a gap of 3,029 weekly visitors to be closed by the end of the year 2012.		Six Sigma Digest is a new online start-up that has only been open for business since 8/21/2011. In the year leading up to our opening we made significant investments of time and finances to establish our business plan, curriculum, website, learning management system etc. Our marketing budget is low and we need to fully leverage as many effective online and virtual sales channels as possible. The trick is figuring out which channels and avenues provide the best ROI and most impact to our business growth.			
High Level Project Timeline		Constraints & Dependencies		Project Risks	
Phase	Start	Finish	Constraints: Marketing Budget Dependencies: N/A		Other Diagnostics
Define	11/1/2011	12/1/2011	1. Disclosure Risk: Six Sigma Digest runs the risk of sharing too much information regarding its business. 2. Y=f(x) correlation assumption: To mitigate some disclosure risk we have chosen visits as our primary metric instead of our true primary metric which is sales.		1. Unique visitors 2. Avg time on site 3. Pageviews 4. Pages/visit 5. Clicks & CTR 6. Cost per click
Measure	12/1/2011	1/1/2012			
Analyze	1/1/2012	2/1/2012			
Improve	2/1/2012	3/1/2012			
Control	3/1/2012	4/1/2012			
Approval/Steering Committee		Stakeholders & Advisors		Project Team & SME's	
Name	Organization	Name	Organization	Name	Organization
Michael Parker	Six Sigma Digest	Instructors	Six Sigma Digest	Blog Commentors	Six Sigma Digest
		President	Six Sigma Digest	Survey Respondants	Six Sigma Digest
		Customers	Six Sigma Digest	Customers	Six Sigma Digest
				Newsletter Subscribers	Six Sigma Digest
				Instructors	Six Sigma Digest

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Project charters the charter documents the why, how, who and when of a project include the following elements

- Problem Statement
- The business case is one of the elements of project charter which asks the following questions why is the project worth doing? Why is it important to do now? What are the consequences of not doing the project? What activities have higher or equal priority? How does it fit with business initiatives and targets?
- Project objective or purpose, including the business need addressed
- Scope
- Deliverables
- Sponsor and stakeholder groups
- Team members
- Project schedule (using GANTT or PERT as an attachment)
- Other resources required

2.2 Work break down Structure

It is a process for defining the final and intermediate products of a project and their relationship. Defining Project task is typically complex and accomplished by a series of decomposition followed by a series of aggregations it is also called top down approach and can be used in the Define phase of Six Sigma framework.

2.3 The Second is Pareto Analysis which is a vertical bar graph showing problems in a prioritized order so it, so it can be determined which problems or opportunities should be tackled first. In the define phase the Pareto diagram is used in conjunction with the WBS to quantify the opportunities inherent in each of the tiny elements resulting from the WBS.

2.3.1 Deliverables

Six Sigma project focus on one or more of the following three deliverables: Cost, Quality and schedule. Factors Critical to Cost (CTC), include parameters that impact work in progress, finished goods inventory, overhead, delivery, material and labour, even when the costs can be passed on to the customer. Critical to Quality (CTQ) factors are perhaps most familiar to operational personnel since they directly impact the functional requirement specified by the internal and external customers. Critical to Schedule (CTS) factors impact the delivery time of the product or service.

Net Present Value

The net present value (NPV) of an amount to be received in the future is given by the formula $P = A(1+i)^{-n}$

Where P = net present value; A = Amount to be received n years from now; i= annual interest rate expressed as decimal ; this formula assumes that the money is compounded annually.

Example:-

In four years \$2500 will be available; what is the NPV of that money assuming an annual interest rate of 8% ?

$$P = 2500(1+0.08)^{-4}$$

$$P = 1837.57$$

Therefore \$1837.57 invested at 8% compounded annually will be worth \$2500 after 4 years

Cost of Quality

The standard categories of quality costs are as follows

- **Appraisal Cost** – Expenses involved in the inspection process.
- **Prevention Cost**- Cost of all activities whose purpose is to prevent failures.
- **Internal failure Cost**- Costs incurred when a failure occurs in-house.
- **External Failure Cost**- Costs incurred when a failure occurs when the customer owns the product.

Cost of Poor Quality(COPQ) – The parameters which affect the COPQ are excess inventory, excess employee turnover, rework, idle time of machines, over processing and overproduction.

Pareto Chart was invented by Wilfred Pareto an Italian Economist it principles on the fact that 80% of the defects are caused by 20% of the defect types.

Defects per Million Opportunities (DPMO)

A very simple product may have only one CTQ characteristic. For Example, a paper clip may have only have one opportunity for defect whether it holds the paper or not. Complex products have more than one CTQ characteristic and therefore have more opportunities for defects. Let Us consider that a computer has 10 opportunities for defects bad drives, system crashes display flickers etc.

Example a Paper Clip has one opportunity for defect we find 7 clips defective in 1000 clips produced so the Defect per Opportunity and the DPMO can be calculated as follows

$$DPO = \text{Defects} / \text{Opportunities} * \text{total items produced}$$

$$\text{So DPO} = 7 / 1 * 1000 = 0.007$$

$$DPMO = DPO * 1 \text{ Million}(10^6) = 0.007 * 10^6 = 7000.$$

Example: - A process produces 40,000 pencils, three types of defect can occur the number of occurrences of each defect type are:

1] Blurred printing: 36

2] Wrong Dimension: 118

3] Rolled Ends: 11

Total number of defects is = 165 so DPU = Number of defects/ Number of Units = 165/40,000 = 0.004125

Defects per Million Opportunities (DPMO)

To calculate the number of opportunities it is necessary to find the number of ways the defect can occur for each item.

For this product blurred printing occurs in only one way so there are 40,000 opportunities for this defect to occur ; there are 3 independent places where the dimensions are checked so there are 3 * 40,000=120,000 opportunities for

dimensional defects ; rolled ends can occur at the top or bottom of the pencil so there are $2 * 40,000=80,000$ opportunities for this defect to occur thus the total number of opportunity for defects is

$$40,000 + 120,000 + 80,000 = 240,000$$

There Defects per million opportunities is calculated as

$$DPMO = (\text{Number of defects})(1,000,000)/\text{Total number of opportunities} = 165,000,000/240,000 = 687.5$$

$$\text{So DPMO} = 687.5$$

PPM (Parts per Million is defined simply as $PPM=DPU * 1,000,000$ from the above example of Pencils $PPM= 0.004125 * 1000,000$ so $PPM= 4125$

PPM is also used to refer to contaminants as seen in the following example

Example:- Suppose 0.23 Grams of Insect parts is found in 25 Kilograms of Product so $PPM=(0.23/25,000) * 1,000,000$

$$\text{So PPM} = 9.2$$

2.3.2 Throughput Yield

For a process, throughput is a measure of the productivity of the process in producing error free product or service. For a product, throughput is a measure of the quality of the product of the entire product as a function of the quality of its various features.

The throughput yield is calculated based on the defects per unit (DPU) for each process step

$$DPU = \text{\#errors} / \text{\# items}$$

$$\text{Throughput Yield} = 1 - DPU \text{ or } e^{-dpu} = 10^{-dpu}$$

For example if there are 74 errors in products and 600 products $DPU = 74/600 = 0.12$ and $Y = 1 - 0.12 = .88 = 88\%$

$$DPO = \text{Defects per Opportunity} = \text{total defects} / \text{total opportunity.}$$

$$DPMO = DPO * 1 \text{ Million}$$

Throughput (T) is the rate at which an organization generates money through sales.it is based on the total sales minus the variable cost, which includes the cost of material's used to produce the goods sold.

Inventory (I) Money invested in purchasing things intended for sales.

Operational Expense (OE) is Money Spent to turn inventory into throughput it includes salaries and wages paid to workers, rent, utilities etc.

$$\text{Net profit} = T - OE$$

$$\text{Return on Asset} = (T - OE) / I$$

$$\text{Productivity} = T / OE$$

$$\text{Asset Turn} = T / I$$

Overall Equipment efficiency is calculated based on availability * performance * Quantity

$$OEE = A * P * Q$$

A = actual operating time / planned time

P = Ideal Cycle time / Actual cycle time

Q = Acceptable output/ Total output

Staff Required on the Production Line = Total cycle time/Takt Time

2.3.3 Rolled Throughput Yield (RTY)

RTY summarises stepwise throughput yields for a multiple step process or multiple featured product. RTY is the measure of the overall quality level rolled throughput yield RTY for a N Step process is calculated using the formula

$$RTY = Y1 * Y2 * Y3 * \dots * Yn$$

In the Define Phase we also calculate the future value, present value and net present value of the project which the formulas are given in my earlier articles.

The First Pass Yield is also first time yield = Units Passing Without Rework/ Input Yield

Final Yield = Final Yield (Including Rework)/Input Yield.

2.3.4 Process Maps

A process map is used in the define stage to document top level process activities and their stake holder.

2.3.5 Assembling the team

As soon as possible the project manager and project sponsor should arrange a short informal meeting with managers of all stakeholders groups to inform them that a project is proposed in their functional area with the permission and direction of the senior executives, this meeting represents an invitation for the middle managers to participate in the project by contributing a member of their team or to challenge the scope, objective or business needs addressed by the project proposal.

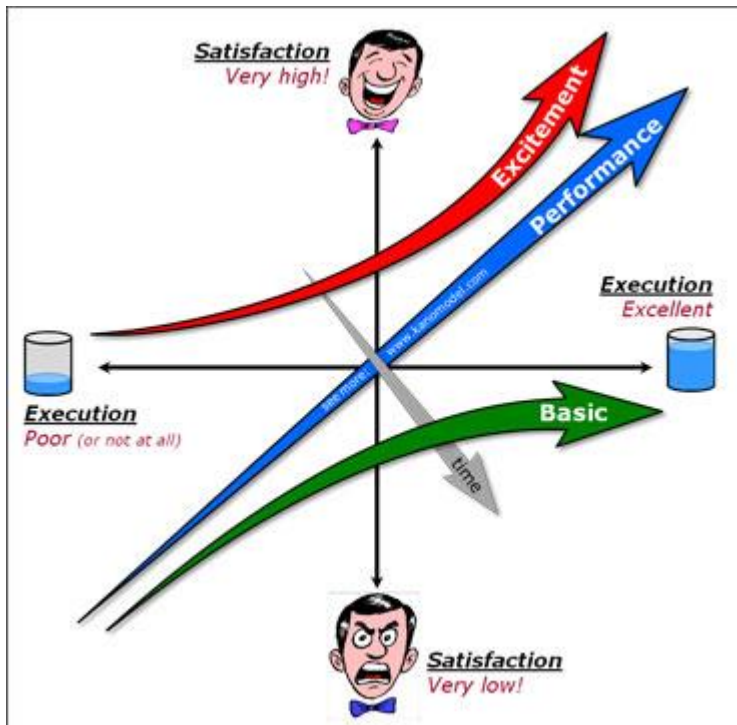
Quality Function Deployment

Quality function deployment (QFD) is a customer driven process for planning innovation in products or services. It starts with the voice of customer which becomes the basis of requirements. QFD Matrices are sometimes referred to as “House of Quality”. In its simplest form QFD involves a matrix that represents customer requirements as rows and product or service features as columns. The Cell where the row and column intersect shows the correlation between individual customer requirements and the product or service requirement. The matrix is sometimes called as the “requirement matrix”.

Engineering Characteristics
(Technical Requirements)

Customer Requirements	Customer Weight	Feature 1	Feature 2	Feature 3	Feature 4
Requirement 1	1	1			
Requirement 2	2		1		
Requirement 3	3			1	
Requirement 4	4				1
Raw Score		1	2	3	4
Relative Weigth		10%	20%	30%	40%

The main purpose of the Kano Model is Customer Satisfaction it is diagrammatically illustrated below:



Hoshin Kanri is known as shining metal in Japanese (also called Policy Deployment) is a method for ensuring that the strategic goals of a company drive progress and action at every level within that company. This eliminates the waste that comes from inconsistent direction and poor communication.

Muda is called waste in Japanese.

5S Tool Can Be Best Represented Using Companion By Minitab in Kaizen Option:-

In industries and organizations that have adopted lean approaches, the 5S tool is used extensively. Its associated five pillars—**sort, set in order, shine, standardize and sustain**—are particularly useful for achieving sustainable development when addressing customer satisfaction through improved quality and productivity, lowered costs and reduced delivery lead times. Waste elimination projects using 5S offer immediate gains because they provide a natural starting point for continuous improvement and prepare the organization for its more in-depth lean journey. Despite the tool's usefulness, however, many organizations have found that they can implement the first three pillars, but struggle to institutionalize the final two. Research was conducted to identify key factors affecting successful implementation of all 5S pillars. ASQ members from four Midwestern states who were associated with automotive industry suppliers were invited to participate in the study. This group represented not only a substantially sized industry, but also one that has used 5S tools for many years.

2.3.6 Formulas Used In Six Sigma-Define & Pre-Define Phase

In Previous articles I have discussed about the vital DMAIC phase of Six Sigma where I have explained the process in each phase that is Define, Measure, Analyse, Improve and Control (DMAIC), in this context we are going to learn about formulas used in Define phase of DMAIC which are critical to control variation in a process thereby making the process or the product defect free.

We will discuss the First Phase that is given below:

2.3.7 Define and Pre-Define Phase

Takt time = Available time for production/customer demand.

Takt is a German Word for Baton which is used by Music Directors to Control the flow of Music.

Available production time = Scheduled time – Break time – Meeting time.

Production time is the time required in producing a product so for example if available production time is 400 minutes and customer demand is 20 units then Takt time = $400/20 = 20$ minutes so the production has to make 1 unit every 20 Minutes.

Cycle time is split into four components:

- Auto time – Time spent by machines running unattended;

- Manual time – Time spent in loading, unloading, and adding components during the same process;
- Delay time – Time spent waiting for operator or other process to finish, or parts or raw material availability; and
- Machine time – Time spent by machine working on the part.

Process time + Delay time = Total Cycle time

Staff Required on the Production Line = Total cycle time/Takt Time

In an ideal production process, the cycle time must be as close to the takt time as possible.

Lead time is the time between customer order and delivery of the product.

For Example in Delivering a Burger of Mc Donald's to the customer the company takes 30 minutes so 30 minutes is the lead time.

Reducing lead time should be the target, but to do so, you should reduce cycle time, set up time, and wait time. Lead time will automatically get impacted.

Solution

Step 1 – Production time needed

Production time needed = No. of machines * cycle time per part * average daily demand

Time available for set up = Available production time – production time needed

Example: A product works on a takt of 100 minutes and has production cycle time of 400 minutes. What is the crew size?

Solution:

•Crew Size = $400/100 = 4$

Staff Required on the Production Line = Total cycle time/Takt Time

•4 staff are needed on the line to ensure takt target is met.

ROI = PROFITS DIVIDED BY INVESTMENTS * 100

3. COMMON FINANCIAL MEASURES

Revenue growth is the projected increase in income that will result from the project that is calculated as the increase in gross income minus the cost. Revenue growth may be stated in dollars per year or as percentage per year.

Conceptually margin refers to the difference between income and cost however there are multiple financial concepts of margin which is used to measure the efficiency and effectiveness of an organization's financial capabilities and finesse.

- **Percent profit margin :** This metric provides and overall sense of how efficiently an organization converts sales into profits it is computed as given below :-
- **Percent profit margin= (Sales of a product or service- Cost of a product or service/ Sales of a Product or Service) * (100)**
- = (Net Profit/Net Sales) * 100**
- **Percent Gross Margin:** This metric measures the efficiency of producing sales. It is computed as Follows
- Percent Gross Margin = (Gross profit/Net Sales) * 100**
- **Percent Operating Margin:** This Metric Measures how effectively an Organization Controls Cost not related to production and sales of a product or service

It is Computed as Percent Operating margin = (Operating profit/Net Sales) * 100

- **Gross Profit = Net Sales – Cost of Goods Sold**
- **Cost of Goods Sold = Cost of Raw Goods + Cost of Labour + Cost of Inventory**
- **Net Profit = Net Income = Net Earnings = Bottom Line = Total revenue – Total Expenses**
- **Operating Profit = Operating Income = Earnings = Gross Profit – Operating Expenses**

3.1 Cost Benefit Analysis

Top Management must approve funding for Projects that require significant financial resources, therefore before a proposed project is initiated, a cost-benefit analysis using projected revenues, costs and net cash flows is performed to determine the project's financial feasibility. The analysis is based on estimates of the project's benefits and costs and the timing thereof. The result of the analysis are a key factor in deciding which projects will be funded. Upon project completion accounting data are analysed to verify the projects financial impact on the organization. In other words the management must confirm whether the project added to or detracted from the company's financial wellbeing.

A cost-benefit analysis attempts to evaluate the benefits and costs associated with a project or initiative. Two types of benefits can be evaluated: direct or indirect. Direct benefits are easily measured and includes issues such as increased production, higher quality, increased sales, reduced delivery costs and lower warranty costs. Indirect benefits are more difficult to measure and include such items as improved quality of work life, increased internal customer satisfaction and better trained employees.

The Simple formula for computing benefit-cost ratio is:

$\sum \text{NPV of all Benefits anticipated} / \sum \text{NPV of all costs anticipated}$

Note: NPV Stands for Net present Value where NPV is calculated using the formula $P = F (1+i)^{-n}$ where P = Net Present Value, F = Amount to be received n years from now also called as Future value: and I = Annual interest rate expressed as decimal the formula assumes that the money is compounded annually and n stands for number of years.

3.1.1 Strategizing

SWOT

A *SWOT analysis* is an effective strategic planning tool applicable to an organization or project objective. SWOT stands for: *strengths, weaknesses, opportunities, and threats*. It is attributed to Albert Humphrey from Stanford University during the 1950s and 1960s.

Strengths and weaknesses are identified with respect to the internal capabilities of an organization. They provide an introspective view. Strengths and weaknesses should not be considered opposites. This will become apparent in the example below.

Strengths address what the organization does well. This might be: research and development, production, marketing, acquisitions, partnering, and so on. Strengths need to be assessed fairly and objectively to bring credibility to the SWOT analysis.

Weaknesses address the organization in terms of skills, capabilities, and competencies. For example, an organization continually fails to deliver its products to its customers on time. This might be due to a lack of skill, incapable processes, or poor training of employees. Note: A weakness should never be improved at the expense of a strength! On the other hand, *opportunities* and *threats* provide the organization with an external view. Quite simply, we are trying to identify opportunities for the organization to grow to new levels, and threats to the organization that will impede its progress. Threats are often barriers to an organization's growth and can represent a competitive disadvantage. When analysing a SWOT, the question to ask is, "How can the organization leverage its strengths and improve its weaknesses to take advantage of the opportunities while mitigating or eliminating the threats?"

PEST

An analysis similar to SWOT is the *PEST analysis*, which brings together four environmental scanning perspectives that serve as useful input into an organization's strategic planning process:

- *Political*. This perspective looks at political stability, threats of wars, regulatory issues, tax considerations, labour problems, laws, and so forth. For example, as this book is being written, we have:

Political stability. Mid-term congressional elections have shifted the dominating party from Democratic to Republican in the U.S. House of Representatives.

Threat of wars. The war in Afghanistan continues; arguably, the war in Iraq is over; an unstable ruler of North Korea recently died and leadership was assumed by his hardliner son; Iran is clamouring for its own nuclear weapons and attempting to threaten the world's oil supply; and terrorist bombings are rampant.

Regulatory issues. The recent financial meltdown has resulted in strengthening the power and authority of regulatory agencies.

Tax considerations. In an attempt to stimulate the economy, the federal government has proposed special depreciation considerations designed to motivate organizations to purchase capital equipment.

Labour problems. Eight and a half percent unemployment continues to strain our government welfare systems, and public pension legacy issues continue to mount.

Laws. Broad-sweeping healthcare and financial reform laws were passed that will have significant impact to both organizations and individuals.

Economic. This perspective looks at currency exchange rates, market conditions, unemployment rates, inflation factors, interest rates, and so forth. Current examples include: the U.S. dollar against other major international currencies, rates of unemployment in the United States and in other major countries of the world, almost zero percent inflation, low GDP

growth rates in the developed world, outsourcing to low-cost nations such as India, China, and those of Southeast Asia, and, in some cases, negative interest rates on bonds.

Social. This perspective looks at education considerations, demographics, health factors, cultural implications, and so forth. Current examples include: the widening educational gap between the United States and other first-world countries, and shifting demographics such that “baby boomers” are now the largest group, and consequently place the biggest strain on the U.S. healthcare system.

Technological. This perspective looks at new technology developments, rate of technological change, cost impact of technology, and so forth.

Aligning Training to Strategy: If training is to be positioned as supporting the needs of the organization senior leaders must be involved with and stand behind the training plans. Technically training plans are action plans that support the strategic plans of the organization, if the plans do not have strategic outcomes, then the organization should not be supporting the training so the following sequence should be followed 1]Strategic plan→Human Resource Strategic Plan→Training Plan→Training Needs Analysis.

Training Plans Are Not a Panacea: Often poor Performance is attributed to a skill or knowledge deficiency that can be corrected by training. Nonetheless many times the root cause of poor performance is not skill related at all. However training cannot remedy a situation caused by poor performance or other root causes that are not skill related. For example, to expect training can fix operating systems problems is unrealistic. Consider the following points below where training would not be beneficial.

Increased defects in a manufacturing environment caused by outdated or inadequate equipment 2] unsatisfactory help desk customer support resulting from poor morale and compensation issues 3] Mistakes caused by employees working overtime on a sustained basis.

5.DFSS

While DMAIC may be traditionally viewed as the foundation for Lean Six Sigma, its application is primarily limited to improving existing processes; it does little to Address the design of new products or processes. This concept is illustrated in the flowchart given in Figure 1.0

Fortunately, several additional structured methodologies exist. Several of the common methodologies are depicted in Figure 1.0. Each of these methodologies has its usefulness as long as the nuances of each are fully understood. Table 1.1 has been created to provide a loose comparison between the DMAIC and DFSS methodologies. These comparisons are not exact, but do provide a sense of how the methodologies align. This table may be beneficial for readers who may be required to use multiple methodologies or find themselves in need of selecting one suitable for a particular project.

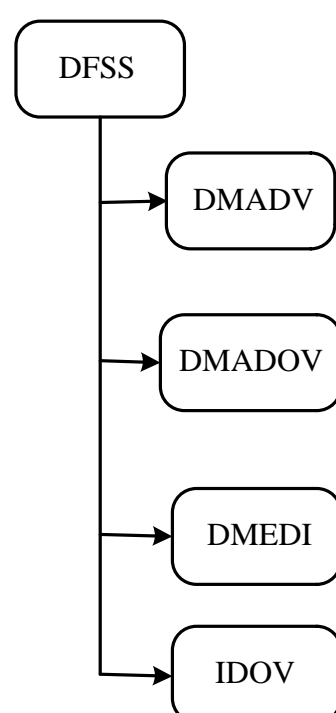


Figure 1.0 The DFSS family of methodologies.

Table 1.1 Comparing DMAIC and DFSS methodologies.

DMAIC	DFSS			IDOV
	DMADV	DMADOV	DMEDI	
Define	Define	Define	Define	Identify
Measure	Measure	Measure	Measure	
Analyse	Analyse	Analyse	Explore*	Design
Improve	Design	Design	Develop	
		Optimize		Optimize
	Verify**	Verify**	Implement	Validate***
Control				

DMADV is a well-recognized Design for Six Sigma (DFSS) methodology and an acronym for *define, measure, analyse, design, and verify*. Note that the ASQ Black Belt Body of Knowledge replaces “verify” with “validate.” The difference is likely because, although different, “verify” and “validate” are often used synonymously.

Design. Quite simply, this means carrying out the process of designing a product or process. Many organizations have well-established policies and procedures for their respective design processes. One valuable Lean Six Sigma technique that supports the design process is quality function deployment (QFD), the details of which were discussed in Chapter 15. Additional tools useful in this phase include pilot runs, simulations, prototypes, and models.

Verify. This phase is directed at ensuring that the design output meets the design requirements and specifications and is performed on the final product or process. Basically, *verification* speaks to the design meeting customer requirements and ensures that the design yields the correct product or process. By contrast, *validation* speaks to the effectiveness of the design process itself and is intended to ensure that it is capable of meeting the requirements of the final product or process.

Both verification and validation are necessary functions in any design process. As such, this suggests that DMADV might be more appropriately named DMADV.

5. SIX SIGMA (SARBANES OXLEY ACT-2002)

The Sarbanes–Oxley Act of 2002 changed many of the rules under which corporations and executives were required to provide information to investors and to others. In many respects, the Act defined a customer service approach that corporations would be required to adopt (in addition to attempting to prevent a repeat of many abuses among corporate executives, accounting firms, and analysts that came to light in 2001 and 2002). For example, the new law “requires steps to enhance the direct responsibility of senior corporate management for financial reporting and for the quality of financial disclosures made by public companies.”⁴ This “direct responsibility” means that the officers of companies are required to *personally* certify the accuracy of financial statements. In a very real sense, this represents a government-enforced form of customer service. If we recognize that the investor is a customer of the company whose stock he or she owns, then what are the customer requirements and expectations?

Where Lean Six Sigma meets Sarbanes-Oxley

A key requirement of Sarbanes-Oxley is that of internal control over financial reporting, which is covered under section 404 of the act further, the SEC has established rules that such internal controls satisfy the following three functions

Recordkeeping: This function addresses the processes associated with transactions and dispositions of a company’s assets. It requires that a sufficient level of detail should be kept that is accurate and detail.

Compliance: This function addresses the processes that ensure transactions are actually recorded and receipts and expenditures are accurate and made only when authorized by the management. Also financial statements must be prepared according to the generally accepted accounting principles (GAAP).

Prevention and detection this function addresses the processes that ensure the prevention and/or detection of unauthorized use or disposition of company assets.

From the above discussion, it should be clear that lean Six Sigma and Sarbanes-Oxley relate to each other through two central ideas

Processes: For Sarbanes- Oxley to be Successful, processes need to be inventoried, mapped and improved. Simplified process facilitate the compliance function

Control: Sustaining the gains is what improvement is all about consequently , tools like mistake proofing and control plans are beneficial and support prevention and detection of functions

6 TOOLS OF THE TRADE

6.1 Tools used In Define Phase

5 Whys	Data Collection Plan	Prioritization Matrix
Activity network Diagrams	Failure Mode and effect analysis	Process Decision Program Chart
Advance quality planning	Flowchart/process mapping as is	Project Charter
Auditing	Focus Groups	Project management
Benchmarking	Force Field Analysis	Project Scope
Brainstorming	Gantt Chart	Project Tracking
Cause and effect diagrams	Kano Model	Quality Function Deployment (QFD)
Check sheets	Matrix Diagrams	Run Charts
Communication plan	Meeting Minutes	Sampling
Control Charts	Multi voting	Stakeholder Analysis
Critical to Quality tree	Nominal Group Technique	SIPOC
Customer feedback	Pareto Charts	Tollgate review
Customer identification	PERT	Tree diagrams, $Y= F(x)$

6.2 Tools used in Measure Phase

Basic Statistics	Histograms	Regression
Brainstorming	Hypothesis Testing	Run Charts
Cause and effect diagram	Measurement System Analysis	Scatter Diagrams
Check Sheets	Meeting Minutes	Statistical process Controls (SPC)
Circle Diagrams	Operational Definitions	SIPOC
Correlation	Pareto charts	Taguchi loss Function
Data Collection	Probability	Toll gate review
Data collection Plan	Process capability analysis	Value Stream maps
Failure Mode and effect analysis	Process Flow metrics	
Flowcharts	Process maps	
Gage R&R	Project Management	
Graphical methods	Project Tracking	

6.3 Tools used in Analyse Phase

Affinity Diagrams	Non Parametric Tests	$Y=f(x)$
Anova	Parametric tests	
Basic Statistics	Process Capability Analysis	
Brainstorming	Project Management	
Cause and Effect diagrams	Project tracking	
Components of Variation	Regression	
Design of experiments	Root cause analysis	
FMEA	Run charts	
Gap Analysis	Scatter diagrams	
Histograms	Simulation	
Hypothesis Testing	SIPOC	
Meeting Minutes	Tollgate review	
Multi vari Studies	Tree diagram	
Multiple regression	Waste analysis	

6.4 Tools used In Improve Phase

Activity network Diagrams	PERT
Anova	Process Capability Analysis
Brainstorming	Project Management
Control Charts	Project tracking
Design of Experiments(DOE)	Risk Analysis
FMEA	Taguchi designs
Process Mapping (to be –Future)	Toll gate review
Gantt charts	Value Stream maps
Hypothesis Testing	Work breakdown Structure (WBS)
Measurement System Analysis (MSA)	$Y=f(x)$
Meeting Minutes	
Pareto charts	

6.5 Tools used in Control Phase

5S	Pre control
Basic Statistics	Project management
Communication Plan	Project tracking
Control Charts	Run Charts
Control Plan	Six Sigma Story board
Data Collection	Standard Operating procedures(SOP)
Data Collection plan	Statistical process control (SPC)
Kaizen	Tollgate review
Kanban	Total productive maintenance
Lessons Learned	Training Plan deployment
MSA	Work instructions
Meeting Minutes	
Mistake proofing / Poka Yoke	

7. MEASURE PHASE SOME TOPICS

As we have already discussed about DMAIC (Define-Measure-Analyse-Improve-Control) phase of Six Sigma in my previous articles in this paper we will discuss about the formulas and the deliverables of the measure phase of Six Sigma.

Before proceeding ahead a brief recap of the Measure phase. Every organization measures various performance parameters during the course of the day to access whether their plans are working and achieving the desired results. If there are no measurements it will be not possible to know whether the organization is meeting customer requirement and making profits. At the same time it is not economical to measure every activity and control all the parameters. When there is a specific problem in the organization to be eliminated the first step is to measure the extent of the problem. Measurement of key performance parameters of any process is a key to find solutions and eliminate the problem in the process. During measure phase the performance of the current process is measured to identify the problem area.

We ask the following questions given below in measure phase:

- ✓ What needs to be measured?
- ✓ How to measure it?
- ✓ What is the customer requirement?
- ✓ What is the variation between the customer requirement and the current level of performance?
- ✓ Is there any additional measurement required to precisely know the problem?
- ✓ What measurements are required after the process is improved?

A factor rarely discussed in the ranks of Six Sigma is the raw cost of collecting data. Unfortunately promising projects may be abandoned simply because of the high cost of acquiring the necessary data.

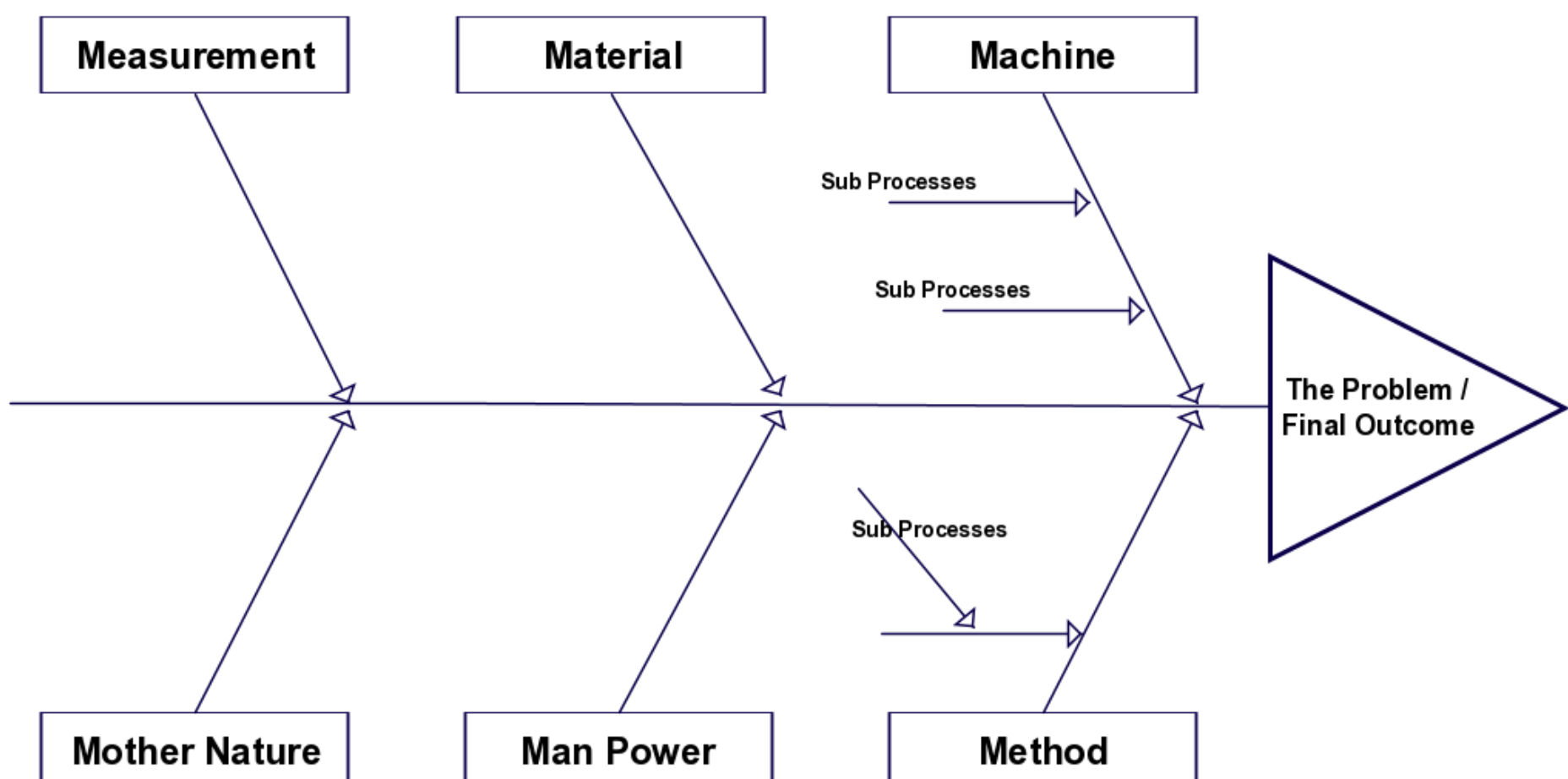
In Measure phase we will be faced with 2 types of Data that is 1] Continuous Data 2] Discrete Data.

Continuous data are the data which are measured to the finer elements and expressed in units of physical property such as length, height, diameter, temperature and weight and Discrete data is that data which cannot be measured these are data regarding characteristics like colour, odour, and count data such as pass/fail.

To measure data to improve the process of a part manufacturing factory we should also determine the sampling plan from the sampling types such as 1] Simple random Sample here the lot we collect is homogeneous that is from the same category. 2] Stratified Sample this is a refined method of random sampling where the lot is segregated based on certain criteria from a heterogeneous lot. 3] Systematic Sample in this samples are taken after every fixed interval of time when the machine produces the part 4] Rational Sample is done when the team finds there are large numbers of data already existing, it is possible to identify a critical area from the past records and collect samples only from specific area.

Dr Kaoru Ishikawa Diagram or Fishbone Diagram or Cause and Effect Diagram

5 why's is a tool used that is often used in conjunction with cause and effect diagram; the idea of 5 why's is to take potential causes of a problem and to dig deeper by asking "Why" upto 5 times. The goal is to get something that is actionable.



Central Limit Theorem: The Central Limit Theorem states that the sampling distribution of the sample means approaches a normal distribution as the sample size gets larger — *no matter what the shape of the population distribution*. This fact holds especially true for sample sizes over 30. *All this is saying is that as you take more samples, especially large ones, your graph of the sample means will look more like a normal distribution.*

Data are of 2 types Continuous or variable and Discrete or Attribute.

Nominal data examples are Go-No-Go-Reject-No Reject; ordinal Data examples are taste attractiveness; interval type examples are calendar, time, temp and ratio examples are elapsed time, distance, weight etc.

Approaches for Sampling:-

1] Systematic Sampling where samples are collected after a particular period of time.

2] Collecting sample in Groups

3] Random Sampling that is randomly pick up samples from the assembly line.

4] Stratified random sampling data collected in strata(Segment).

Formulas used in Measure Phase

Some Little Statistics

Central tendency is defined as the tendency for the values of a random variable to cluster round its mean, mode, or median.

Central Tendency	Population	Sample
Mean/Average	$\mu = \sum (Xi)^2/N$	$XBar = \sum (Xi)^2/n$
Median	Middle value of Data	
Mode	Most occurring value in the data	
Dispersion		
Variance	$\sigma^2 = \sum (X - \mu)^2 / N$	$S^2 = \sum (Xi - XBar)^2 / n-1$
Standard Deviation	$\sigma = \sqrt{\sum (Xi - \mu)^2 / N}$	$S = \sqrt{\sum (Xi - XBar)^2 / n-1}$
Range	Max-Min	

Standard deviation is a mathematical tool to help us assess how far the values are spread above and below the mean.

A high standard deviation shows that the data is widely spread (less reliable) and a low standard deviation shows that the data are clustered closely around the mean (more reliable).

The **standard deviation** is a measure of the spread of scores within a set of data. Usually, we are interested in the **standard deviation** of a **population**. However, as we are often presented with data from a **sample** only, we can estimate the **population standard deviation** from a **sample standard deviation**.

Variance

The square of standard deviation is known as Variance. Why did we need to define one more parameter, when standard deviation appears to be the right answer? It has some interesting and useful properties. Prominent one is the addition property for two independent data sets or observations. In other words, variance of the sum of independent data sets or observations (also referred random variables) is the sum of their variances.

Z is the Z score, or Sigma score. A low Z score means that a significant portion of the tail of the distribution is extending past the specification limit. A high Z score means that not much of the distribution is extending past the specification limit. The table below shows Z scores related to defects per million opportunities. Notice that the Sigma values we identified earlier are represented here.

You may also see the z score formula shown to the left. This is exactly the **same** formula as $z = x - \mu / \sigma$, except that \bar{x} (the sample **mean**) is used instead of μ (the population **mean**) and s (the sample **standard deviation**) is used instead of σ (the population **standard deviation**).

Gage Repeatability& Reproducibility (R&R) Study

Repeatability also Known as Equipment variation and Reproducibility also known as appraiser variation.

Attribute R&R study is for conducting study on discrete variables and not continuous variables which can be measured on a measuring scale.

The MSA Manual published by Automotive Industry Action Group (AIAG) specifies the following Norms:

- ✓ IF GRR is less than 10 percent the measurement system is acceptable
- ✓ If GRR is between 10 and 30 percent equipment may be accepted based upon the importance of application, cost of measurement, device, cost of repair etc.
- ✓ Measurement System is not acceptable for GRR beyond 30 percent.

We can use Minitab to execute GAGE R&R Studies by Clicking on Assistant option then → Click on Gage R&R.

Gage Attribute MSA is for Categorical data (Good, Bad) to do this in Minitab Go to Assistant Menu -> Click on MSA(Measurement System Analysis) and Select Attribute Agreement Worksheet then select number of operators , how many times to repeat this procedure , number of items , Standard of items whether good or bad then you get worksheet created by Minitab in that enter in the results column the 2 categories (Good, Bad) then click on Attribute Analysis and select the relevant columns and you get the thermometer graph which tells us accurately what the percentage accuracy of the Gage. Kindly note Gage R&R Variable MSA is For Continuous Data and Gage Attribute MSA is for Categorical Data or discrete data.

Reproducibility: Variation is average of measurement made by different operators using the same **gage** measuring the same object. ... **Repeatability:** The random variations in measurement when one operator uses the same **gage** to measure the same part at several times

In Gage Repeatability & Reproducibility for Variable/Continuous data you should have minimum 10 parts and 3 operators in the study to get a favourably precise output.

It is very important to know that if the process is found to be stable using Capability Analysis formulas and by Gage R&R Study we can directly move to the Improve or control Phase else we have to start again from Define Phase.

About Minitab: In Minitab Options of Probability Distributions are available in Calc Option and all the formulas are available by clicking on the options Help → Sample Data all the data sets of all tests available here you have to just download and do the respective analysis.

Graphical Analysis

1] Gage Run Chart

Example of Gage Run Chart

An engineer selects 10 parts that represent the expected range of the process variation. Three operators measure the 10 parts, three times per part, in a random order.

The engineer creates a gage run chart to assess the differences in measurements between different operators and parts.

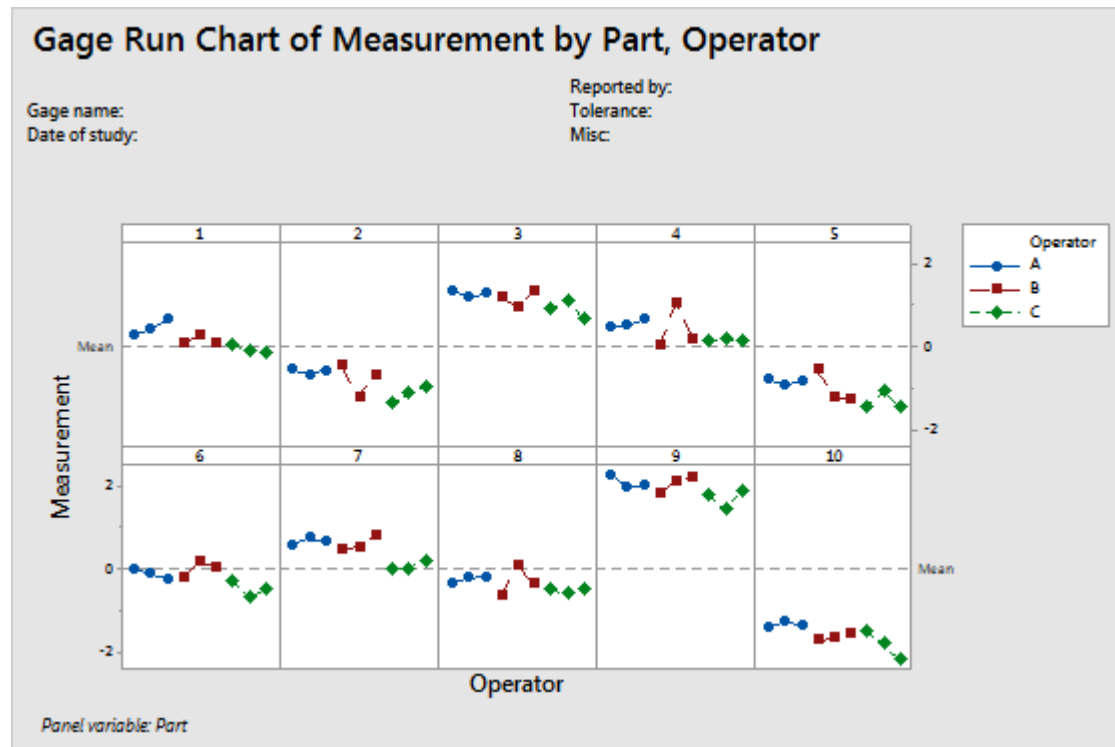
1. Open the sample data, [GageData.MTW](#).
2. Choose **Stat > Quality Tools > Gage Study > Gage Run Chart**.
3. In **Part numbers**, enter *Part*.
4. In **Operators**, enter *Operator*.
5. In **Measurement data**, enter *Measurement*.

6. Click **OK**.

Interpret the results

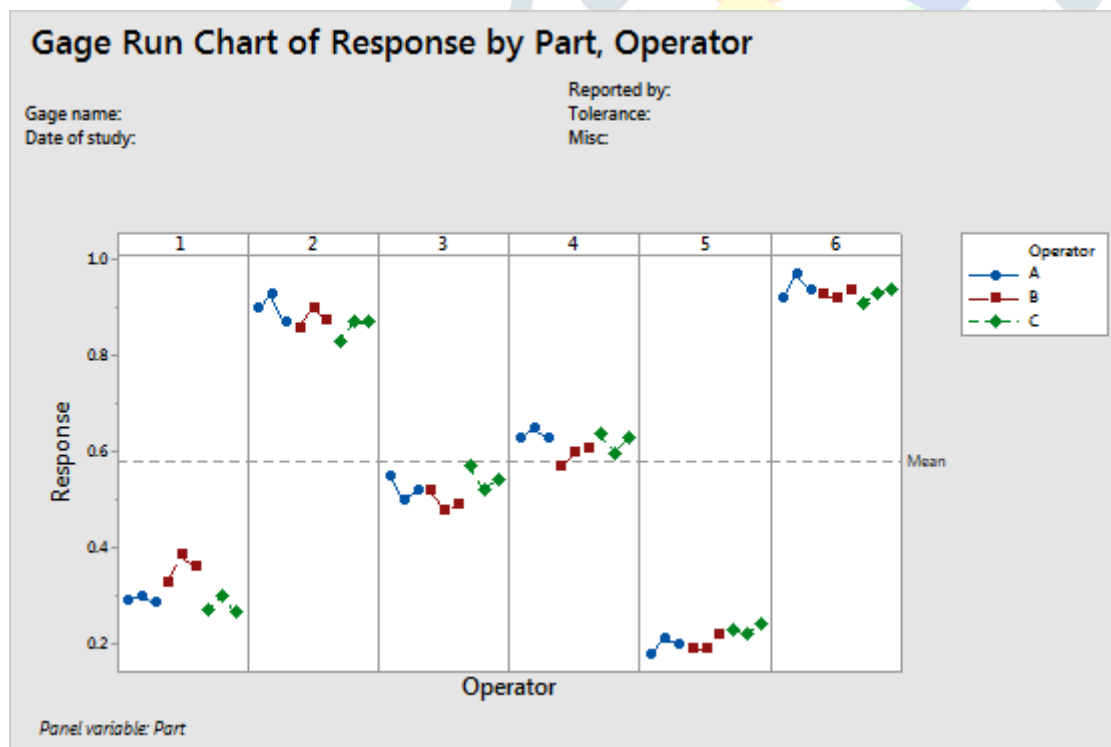
For each part, you can compare the variation between measurements that were made by each operator and examine the differences in measurements between operators. You can also view the measurements in relationship to the horizontal reference line. By default, the reference line is the mean of all the observations.

Most of the variation is due to differences between parts. Some smaller patterns also appear. For example, Operator B does not measure consistently, and Operator C usually measures lower than the other operators.



Use a gage run chart to assess the variation in the measured values for each measured part. The gage run chart helps you see how much measurement variability is due to operators or parts.

This gage run chart shows the measurements of 6 parts that are selected from a manufacturing process. The parts represent the normal variation of the process. 3 operators measure each part with the same gage 3 times in a random

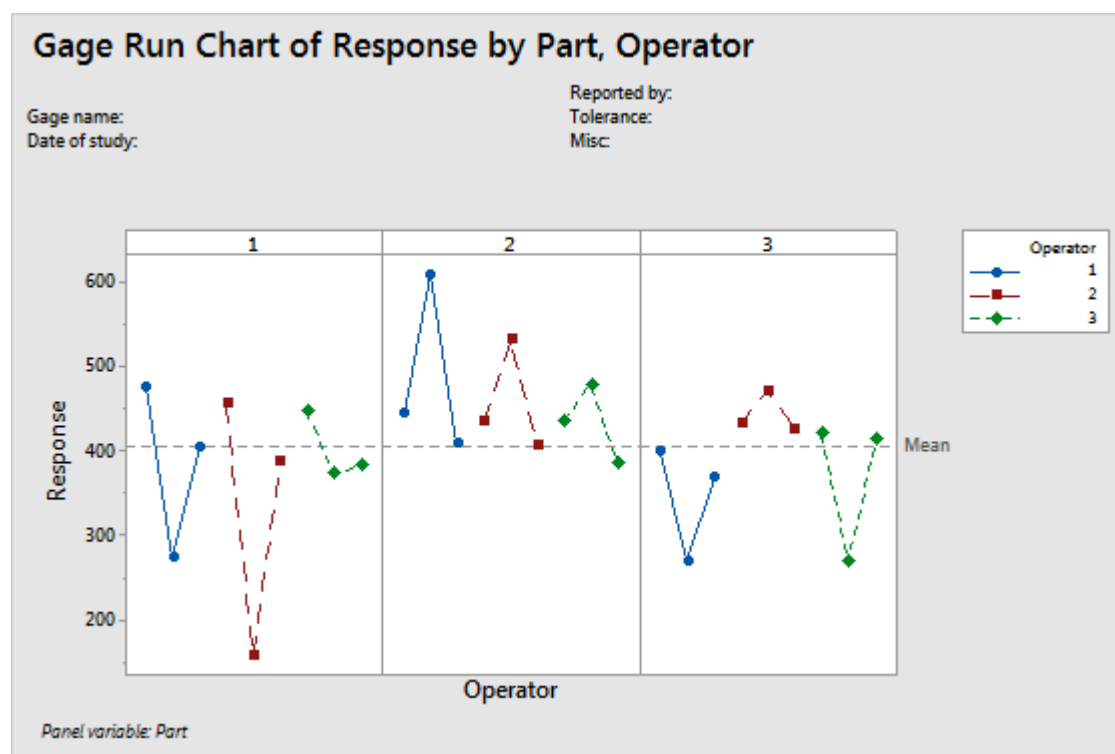


order.

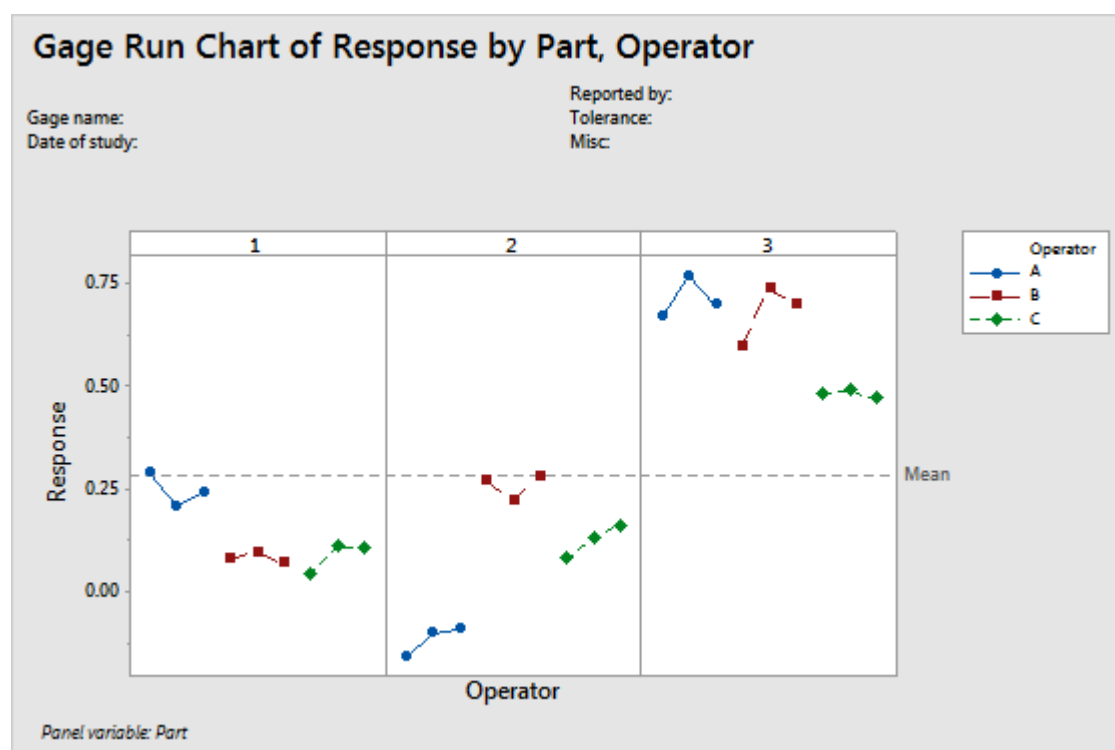
The part-to-part variation is quite noticeable in this chart—Part 2 and Part 6 have higher measurements than Part 1 and Part 5.

Step 2: Assess the variation between measurements from each operator

For each part, you can compare both the variation between measurements made by each operator, and differences in measurements between operators. You can also look at the measurements in relationship to the horizontal reference line. By default the reference line is the mean of all observations.



This gage run chart shows that the operator's measurements do not have good repeatability. Each operator measures each part 3 times (3 trials). When the same operator measures the same part, the differences between the measurements are large. Oscillations could indicate that the operators adjust how they measure between measurements.



This gage run chart shows that the operators measure more consistently for each part, but the differences in the measurements between operators for the same part are large

Example of Crossed Gage R&R Study

An engineer selects 10 parts that represent the expected range of the process variation. Three operators measure the 10 parts, three times per part, in a random order.

The engineer performs a crossed gage R&R study to assess the variability in measurements that may be from the measurement system.

1. Open the sample data, [GageData.MTW](#).
2. Choose **Stat > Quality Tools > Gage Study > Gage R&R Study (Crossed)**.
3. In **Part numbers**, enter *Part*.
4. In **Operators**, enter *Operator*.
5. In **Measurement data**, enter *Measurement*.
6. Under **Method of Analysis**, select **ANOVA**.
7. Click the **Options** button. Under **Process tolerance**, select **Upper spec - Lower spec**, and enter 8.
8. Click **OK** in each dialog box.

Interpret the results

The two-way ANOVA table includes terms for the part, the operator, and the part-operator interaction. If the p-value for the interaction is ≥ 0.05 , Minitab omits the interaction from the full model because it is not significant. In this example, the p-value is 0.974, so Minitab generates a second two-way ANOVA table that omits the interaction from the final model.

Use the variance components (VarComp) to compare the variation from each source of measurement error to the total variation. In these results, the %Contribution column in the Gage R&R table shows that the variation from Part-To-Part is 92.24%. This value is much larger than Total Gage R&R, which is 7.76%. Thus, much of the variation is due to differences between parts.

Use %Study Var to compare the measurement system variation to the total variation. The Total Gage R&R equals 27.86% of the study variation. The Total Gage R&R %Contribution might be acceptable depending on the application. For more information, go to [Is my measurement system acceptable?](#)

For this data, the number of distinct categories is 4. According to the AIAG, you need at least 5 distinct categories to have an adequate measuring system. For more information, go to [Using the number of distinct categories](#).

The graphs also provide the following information about the measurement system:

- In the Components of Variation graph, the %Contribution from Part-To-Part is larger than that of Total Gage R&R. Thus, much of the variation is due to differences between parts.
- The R Chart by Operator shows that Operator B measures parts inconsistently.
- In the Xbar Chart by Operator, most of the points are outside the control limits. Thus, much of the variation is due to differences between parts.
- The By Part graph shows that the differences between parts are large.
- In the By Operator graph, the differences between operators are smaller than the differences between parts, but are significant (p -value = 0.00). Operator C's measurements are slightly lower than the measurements of the other operators.
- In the Operator* Part Interaction graph, the lines are approximately parallel and the p -value for the Operator*Part interaction found in the table is 0.974. These results indicate that no significant interaction between each Part and Operator exists.

Gage R&R Study - ANOVA Method

Two-Way ANOVA Table With Interaction

Source	DF	SS	MS	F	P
Part	9	88.3619	9.81799	492.291	0.000
Operator	2	3.1673	1.58363	79.406	0.000
Part * Operator	18	0.3590	0.01994	0.434	0.974
Repeatability	60	2.7589	0.04598		
Total	89	94.6471			

α to remove interaction term = 0.05

Two-Way ANOVA Table Without Interaction

Source	DF	SS	MS	F	P
Part	9	88.3619	9.81799	245.614	0.000
Operator	2	3.1673	1.58363	39.617	0.000
Repeatability	78	3.1179	0.03997		
Total	89	94.6471			

Gage R&R

Variance Components

%Contribution

Source	VarComp	(of VarComp)
Total Gage R&R	0.09143	7.76
Repeatability	0.03997	3.39
Reproducibility	0.05146	4.37
Operator	0.05146	4.37
Part-To-Part	1.08645	92.24
Total Variation	1.17788	100.00

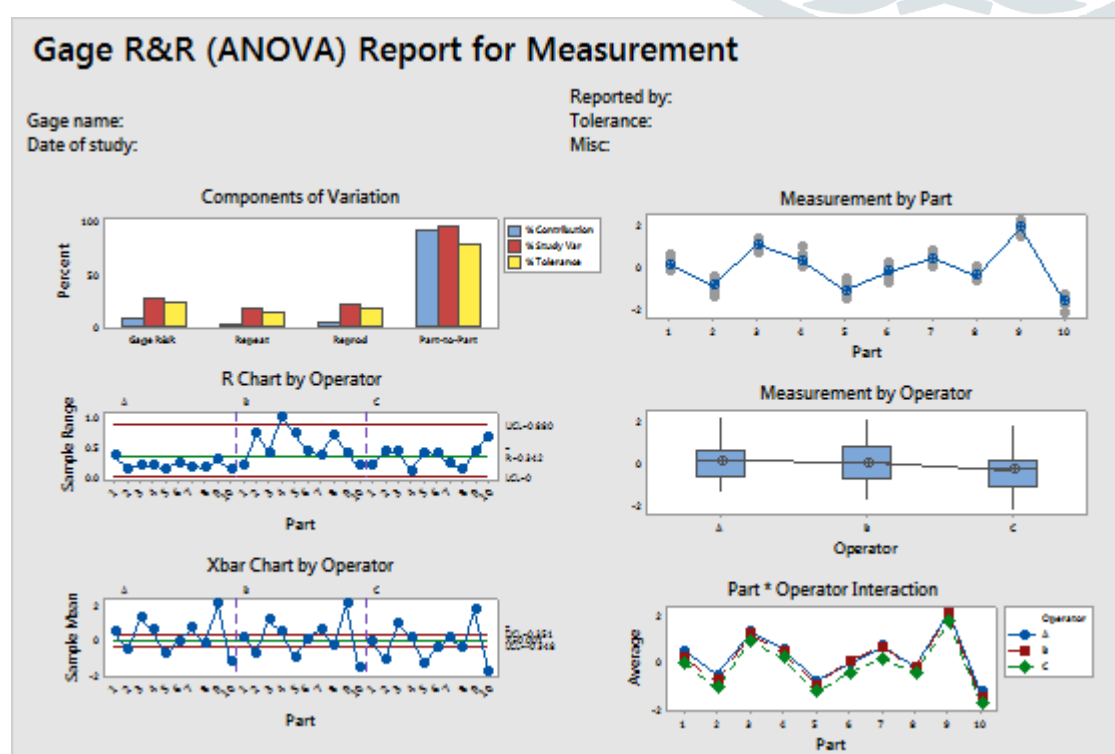
Process tolerance = 8

Gage Evaluation

Source	StdDev (SD)	Study Var (6 × SD)	%Study Var (%SV)	%Tolerance (SV/Toler)
Total Gage R&R	0.30237	1.81423	27.86	22.68
Repeatability	0.19993	1.19960	18.42	14.99
Reproducibility	0.22684	1.36103	20.90	17.01
Operator	0.22684	1.36103	20.90	17.01
Part-To-Part	1.04233	6.25396	96.04	78.17
Total Variation	1.08530	6.51180	100.00	81.40

Number of Distinct Categories = 4

Gage R&R for Measurement



2] Box Plot

A box plot sometimes called a box and whisker plot is an alternative to the histogram , it shows the location,spread , shape of the data it may also show outliers.

A box plot consists of 2 tails the length of the box describes the middle 50% of the data it is called as interquartile range , the two tails extend out to the expected range of measurements the middle line in the box is the median.

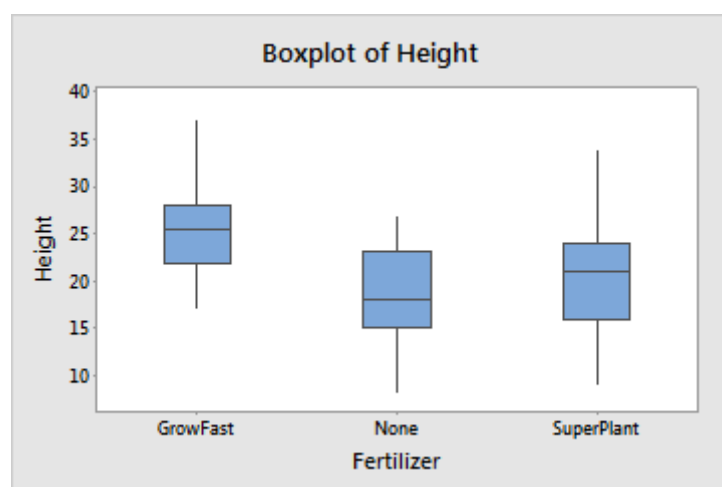
A plant fertilizer manufacturer wants to develop a formula of fertilizer that yields the most increase in the height of plants. To test fertilizer formulas, a scientist prepares three groups of 50 identical seedlings: a control group with no fertilizer, a group with the manufacturer's fertilizer, named GrowFast, and a group with fertilizer named SuperPlant from a competing manufacturer. After the plants are in a controlled greenhouse environment for three months, the scientist measures the plants' heights.

As part of the initial investigation, the scientist creates a boxplot of the plant heights from the three groups to evaluate the differences in plant growth between plants with no fertilizer, plants with the manufacturer's fertilizer, and plants with their competitor's fertilizer.

1. Open the sample data, [PlantGrowth.MTW](#).
2. Choose Graph > Boxplot > One Y > With Groups.
3. In Graph variables, enter Height.
4. In Categorical variables for grouping (1-4, outermost first), enter Fertilizer.
5. Click OK.

Interpret the results

GrowFast produces the tallest plants overall. SuperPlant also increases plant height, but its variability is greater, and SuperPlant does not have a positive effect on a large proportion of the seedlings. The graph shows that GrowFast causes a greater and more consistent increase in plant height.



Run charts are also known as trend charts or time series charts they are plot of data over time. The vertical axis is the data value and the horizontal axis represents time eg days, weeks. The run chart helps determine shifts, drifts, pattern in data over time. Run charts can show a stable process where the location, spread and shape of the data remain constant over time. When the process has change the run chart shows an upward trend.

what is the difference between standard normal distribution and normal distributions?

Both are **normal distributions** but **standard normal distribution** is a particular case of **Normal distribution** when the mean is 0 and **standard** deviation or variance is 1. This makes Y a “**Standard**” **distribution**. So basically X is Normally **distributed** and Y is “**Standard**” normally **distributed**.

what is the difference between z distribution and normal distribution?

The '**z**' **distribution** is a special case of the **normal**, whose mean is zero and standard deviation is one. ... It is a statistic of data generally known as **z**-statistic for **normal distribution** and the value of **z** is defined as $z = (x - \text{mean}) / \text{s.d.}$ that makes the mean and variance of given data to zero (0) and 1 respectively.

CAPABILITY ANALYSIS

Normal Capability Analysis requires the data to be normal you can check the normality of the data in Minitab 18 using **Anderson-Darling test** and then convert the data into normal form using **Box-Cox Transformation** or **Johnson Transformation**.

Calculate the Process Capability – Process Capability is measured by comparing the variation in the product to the upper and lower specification limits, assuming that the product mean value is exactly centred between the upper and lower specification limits

The process capability is calculated using the formula $CP = \frac{USL - LSL}{6 * \text{Standard Deviation}}$ where CP = process capability index, USL = Upper Specification Limit and LSL = Lower Specification Limit.

The Process capability measures indicates the following

1. Process is fully capable
2. Process could fail at any time
3. Process is not capable.

When the process is spread well within the customer specification the process is considered to be fully capable that means the CP is more than 2. In this case, the process standard deviation is so small that 6 times of the standard deviation with reference to the means is within the customer specification

Example: The Specified limits for the diameter of car tires are 15.6 for the upper limit and 15 for the lower limit with a process mean of 15.3 and a standard deviation of 0.09. Find Cp and Cr what can we say about Process Capabilities ?

$$Cp = \frac{USL - LSL}{6 * \text{Standard deviation}} = \frac{15.6 - 15}{6 * 0.09} = \frac{0.6}{0.54} = 1.111$$

$$Cp = 1.111$$

$$Cr = \frac{1}{1.111} = 0.9$$

Since Cp is greater than 1 and therefore Cr is less than 1; we can conclude that the process is potentially capable.

Long term Potential Performance

The long term process performance is calculated using the similar methods as Cp and Cpk with the difference being that the standard deviation used in this case applies to long term variations.

Formula is $Cp = \frac{USL - LSL}{6 * \text{Long term Standard Deviation}}$.

$Cpk = \frac{USL - \bar{X}}{3 * \text{Standard deviation}}$ or $Cpk = \frac{\bar{X} - LSL}{3 * \text{standard deviation}}$.

The process is said to be stable and centred when $Cp = Cpk$

It is perfectly possible for Cpk to take a negative value if the centre of the process is outside one of the specification limits this would represent over 50% of nonconforming products.

Formulas for Cp Cpk and Pp Ppk are Similar

People often get confused about the difference between Pp/Ppk and Cp/Cpk.

Cp, Cpk Formula & Calculations

$$Cp = \frac{(USL - LSL)}{6\sigma}$$

$$CpU = \frac{(USL - \bar{X})}{3\sigma}$$

$$CpL = \frac{(\bar{X} - LSL)}{3\sigma}$$

$$Cpk = \text{Min}(CpU, CpL)$$

(Cpk > 1.33 is desirable)

$$\hat{\sigma}$$

Pp, Ppk Formula & Calculations

$$Pp = \frac{(USL - LSL)}{6\sigma}$$

$$PpU = \frac{(USL - \bar{X})}{3\sigma}$$

$$PpL = \frac{(\bar{X} - LSL)}{3\sigma}$$

$$Ppk = \text{Min}(PpU, PpL)$$

(Ppk > 1.33 is desirable)

C_{pk} measures how close you are to your target and how consistent you are to around your average performance. A person may be performing with minimum variation, but he can be away from his target towards one of the specification limit, which indicates lower C_{pk} , whereas C_p will be high. On the other hand, a person may be on average exactly at the target, but the variation in performance is high (but still lower than the tolerance band (i.e., specification interval)).

In such case also C_{pk} will be lower, but C_p will be high. C_{pk} will be higher only when you r meeting the target consistently with minimum variation.”

The rationale for considering 1.5 sigma shift is based on the Motorola experience that the mean of the process does not remain constant but varies depending on factors such as tool wear , temperature changes, material variation, drift in measuring instruments, changes in chemical concentration.

Difference between Cp & Cpk :

C_p is the potential capability indicating how well the process would be if it were centred on the target; this is not necessarily it's actual performance because it does not consider the location of the process, only the spread it doesn't take into account closeness of the estimated process mean to the specification limits.

Another metric used in conjunction with C_p is C_{pk} the formula for $C_{pk} = \min[(USL - \bar{X}) / 3 \text{ standard deviation} , (\bar{X} - LSL) / 3 \text{ Standard Deviation}]$.

C_{pk} does take into account the location of the data by considering the closeness of the mean to the specification limits.

If $C_{pk} > 1.5$ then it is Motorola 6 Sigma Capability and as per Motorola university C_p, C_{pk}, Ppk, Pp are interpreted in the same manner.

You must have a C_{pk} of 1.33 [4 sigma] or higher to satisfy most customers.

In the Six Sigma quality methodology, process performance is reported to the organization as a sigma level. The higher the sigma level, the better the process is performing.

The process capability index is a dimensionless number (Number without unit of measure) that is used to represent the ability to meet customer specification limits. The index compares the variability of a characteristic to the specification limit. two basic process capability indices are C_p, C_{pk} .

$C_p = (USL - LSL) / 6 * \text{standard deviation}$ or $C_p = \text{Voice of Customer} / \text{Voice of the process} = \text{VOC} / \text{VOP} = \text{Tolerance} / \text{Natural process variation}$

However the C_p index is limited in its use since it does not address the centering of the process relative to the specification limits for that reason C_{pk} was developed; it is defined as $C_{pk} = \text{Min}(C_{pku}, C_{pkL})$

Where C_{pku} is the upper process capability index and given by $C_{pku} = (USL - \text{Mean}) / 3 * \text{Standard Deviation}$

C_{pkL} is the lower process capability index and given by $C_{pkL} = (\text{Mean} - LSL) / 3 * \text{Standard Deviation}$ where Mean is the process average.

The C_{pk} can be easily Converted into a sigma level using Sigma Level = 3 * C_{pk}

In C_{pk} the K is called the k-factor; it measures the level of variation of the process mean from the specified target.

$C_{pk} = (1 - k) C_p$

Where $k = (USL + LSL) / 2 - \bar{X} / ((USL - LSL) / 2)$

If K=0 it means that the process is perfectly centred and therefore Cpk = Cp

Cp and Cpk represent long term capability while Pp and Ppk represent short term capability. The AIAG suggests that using Ppk for less than 30 day production run(that is short term) and Cpk for everything thereafter (That is long term).

Long term variability = short term variability + 1.5 sigma Shift.

If the data is Normal and you have multiple set of variables you can do a Capability Analysis using the option Stat→Quality Tools→Capability Analysis→Multiple Variables Normal.

If the data is Non-Normal and you have multiple set of variables you can do a Capability Analysis using the option Stat→Quality Tools→Capability Analysis→Multiple Variables Non-Normal.

Non Destructive Testing (Ultrasonic Testing)

The application of high frequency vibration to the testing of materials is a widely used and important non-destructive testing method. Ultrasonic waves are generated in a transducer and transmitted through a material which may contain a defect. A portion of the waves will strike any defect present and be reflected or “echoed” back to a receiving unit, which converts them into a “spike” or “blip” on a screen. Ultrasonic inspection has also been used in the measurement of dimensional thickness. One useful application is the inspection of hollow wall castings, where mechanical measurement would be difficult because of part interference. The ultrasonic testing technique is similar to sonar. Sonic energy is transmitted by waves containing alternate, regularly spaced compressions and refractions. Audible human sound is in the 20 to 20,000 Hertz range. For non-destructive testing purposes, the vibration range is from 200,000 to 25,000,000 Hertz. (Where 1 Hertz = 1 cycle per second).

Process Performance Indices

The Process Performance Index is a dimensionless number that is used to represent the ability to meet specification limits on a characteristic of interest.

The Pp is Computed as $Pp = \frac{USL - LSL}{6 * \text{Standard deviation}}$.

Similar to the Cpk the Ppk is Computed as $Ppk = \text{Min}(Ppku, Ppkl)$

Where Ppku is the upper process capability index and given by $Ppku = \frac{USL - \bar{X}}{3 * \text{Standard Deviation}}$.

Ppkl is the lower Process capability and given by $Ppkl = \frac{\bar{X} - LSL}{3 * \text{Standard deviation}}$.

If the data is Non-Normal you can do a Non-Normal Capability Analysis using the option Stat→Quality Tools→Capability Analysis→Non Normal Capability Analysis.

Long Term and Short Term Capability

Cp and Cpk Represents long term capability while Pp and Ppk represent short term capability. The AIAG suggests using Ppk for less than a 30 day production (that is short term) and Cpk for everything thereafter is long Term.

Example of Normal Capability Sixpack

An engine manufacturer uses a forging process to make piston rings. The quality engineers want to assess the process capability. They collect 25 subgroups of five piston rings and measure the diameters. The specification limits for piston ring diameter are 74.0 mm ± 0.05 mm.

The engineers perform normal capability sixpack to check the assumptions for normal capability analysis and to evaluate how well the diameters of the piston rings meet the requirements.

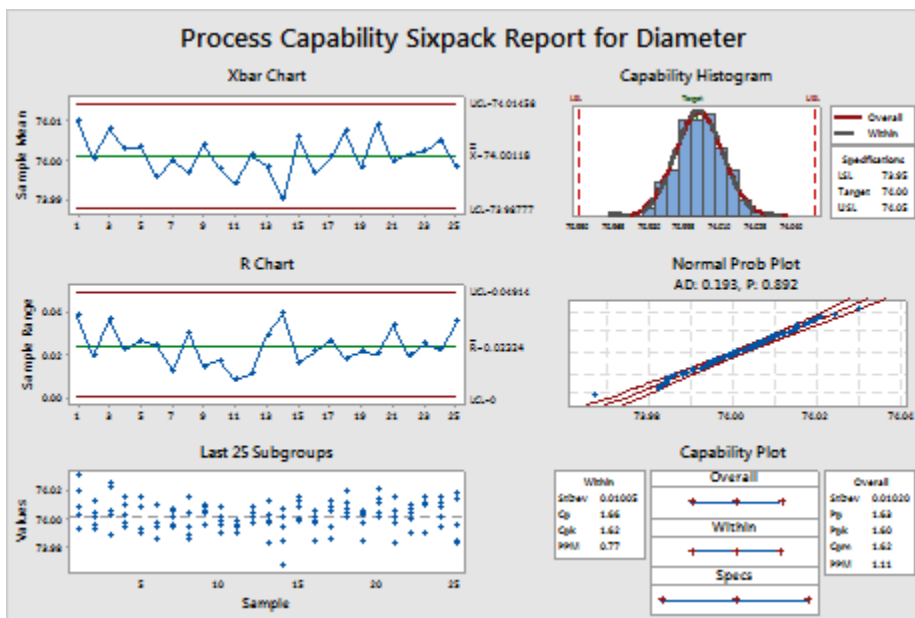
1. Open the sample data, PistonRingDiameter.MTW.
2. Choose **Stat > Quality Tools > Capability Sixpack > Normal**.
3. In **Single column**, enter *Diameter*.
4. In **Subgroup size**, enter 5.

5. In **Lower spec**, enter 73.95.
6. In **Upper spec**, enter 74.05.
7. Click **Options**.
8. In **Target (adds Cpm to table)**, enter 74.
9. Click **OK** in each dialog box.

Interpret the results

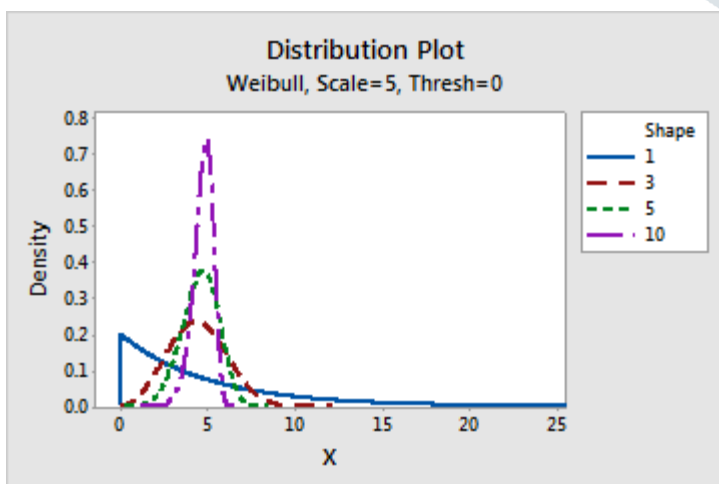
The Xbar and R charts indicate that the process is stable, with no points beyond the control limits. The Last 25 Subgroups plot indicates the data are randomly and symmetrically distributed around the process mean. The normal probability plot indicates that data are normally distributed. Therefore, the assumptions for normal capability analysis are satisfied and the capability of the process can be analyzed.

The histogram and capability indices indicate that the process is approximately centered on the target and the measurements are within the specification limits. The capability indices Cpk, Ppk, and Cpm are all greater than 1.33, which is a generally accepted minimum value for a capable process. Therefore, the engineers conclude that the forging process satisfies customer requirements for the diameters of the piston rings.

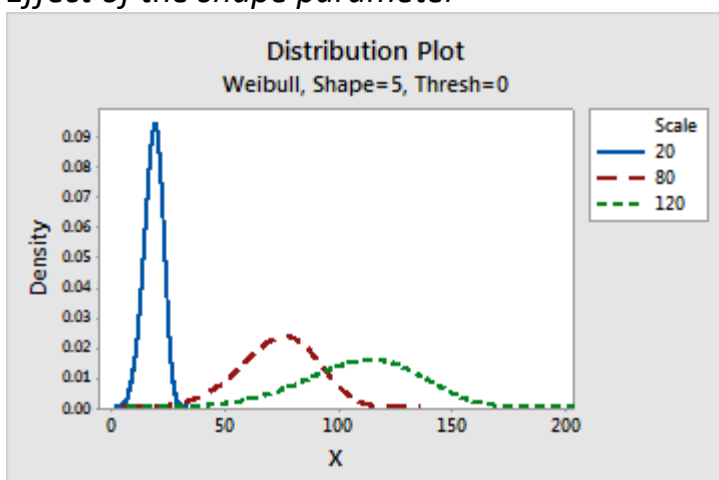


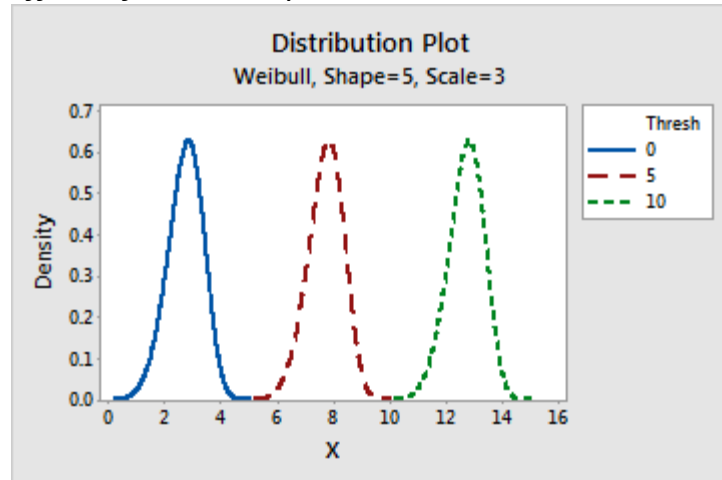
Why is Weibull the default distribution for nonnormal capability analysis?

The Weibull distribution is one of the most commonly used distributions to model nonnormal data. The Weibull distribution is described by the shape, scale, and threshold parameters. Depending on the values of these parameters, the Weibull distribution can take various shapes.



Effect of the shape parameter



Effect of the scale parameter*Effect of the threshold parameter*

One of the main advantages of the Weibull distribution is that it can be similar to the characteristics of other types of distributions, making it very flexible in fitting different types of data. Examples of uses of the Weibull distribution in place of other distributions:

- The exponential distribution is a special case of Weibull distribution frequently used to study the scattering of radiation or wind speed.
- The Weibull distribution is an alternative to the normal distribution in the case of skewed data.

Other distributions for nonnormal capability analysis

Minitab also provides the following nonnormal distributions to model data for nonnormal capability analysis:

- Lognormal
- 3-parameter lognormal
- Exponential
- 2-parameter exponential
- Gamma
- 3-parameter gamma
- Logistic
- Loglogistic
- 3-parameter loglogistic
- Smallest extreme value
- Largest extreme value



To determine which distribution best fits your data, use your process knowledge and the results of a distribution fit test, such as Individual Distribution Identification available in Stat → Quality Tools → Individual Distribution identification.

Before and After Capability Analysis Using Minitab

Open Minitab-.> Select Assistant->Select baseline Process data & Enter the before column values and in improved Process data -> Select the column where the improved data is there -> enter -> specify the lower specification limit and upper specification limit and the target value which you want to achieve and then click on OK.

How to Determine Sample Size, Determining Sample Size

In order to prove that a process has been improved, you must measure the process capability before and after improvements are implemented. This allows you to quantify the process improvement (e.g., defect reduction or productivity increase) and translate the effects into an estimated financial result – something business leaders can understand and appreciate. If data is not readily available for the process, how many members of the population should be selected to ensure that the population is properly represented? If data has been collected, how do you determine if you have enough data?

Determining sample size is a very important issue because samples that are too large may waste time, resources and money, while samples that are too small may lead to inaccurate results. In many cases, we can easily determine the minimum sample size needed to estimate a process parameter, such as the population mean μ .

When sample data is collected and the sample mean \bar{x} is calculated, that sample mean is typically different from the population mean μ . This difference between the sample and population means can be thought of as an error.

Sample Size Calculation Example

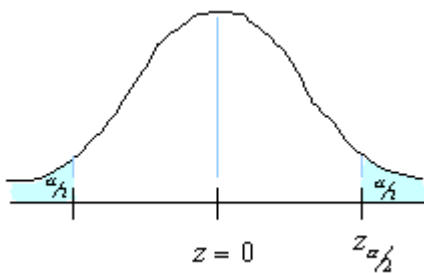
Problem

A production manager at a call centre wants to know the average time an employee should spend on the phone with a customer. She wants to be within 2 minutes of the actual length of time and standard deviation of the average time spent is 3 minutes. What sample Size of calls should she consider if she wants to be 95% confident of her result?

Solution

We are solving for the sample size n .

A 95% degree confidence corresponds to $\alpha = 0.05$. Each of the shaded tails in the following figure has an area of $\alpha/2 = 0.025$. The region to the left of $z_{\alpha/2}$ and to the right of $z = 0$ is $0.5 - 0.025$, or 0.475 . In the table of the standard normal (z) distribution, an area of 0.475 corresponds to a z value of 1.96 . The critical value is therefore $z_{\alpha/2} = 1.96$.



$$n = \left[\frac{z_{\alpha/2} \sigma}{E} \right]^2$$

The margin of error $E = 2$ and the standard deviation $\sigma = 3$, $z_{\alpha/2} = 1.96$ Using the formula for sample size :

$$n = (z_{\alpha/2} * \sigma / E)^2 = (1.96 * 3 / 2)^2 = 34.574 / 4 = 8.643$$

So we will need to sample at least 9 calls.

Minitab Navigation Stat→Power and Sample Size→Sample Size for Estimation and Enter the known standard deviation earlier and the confidence interval.

Minitab’s power and sample size capabilities allow you to examine how different test properties effect each other. For example with a 2 Sample T test you can calculate.

- 1] Sample Sizes.
- 2] Differences (Effects) the minimum difference between the mean for one population and the mean for other you can detect.
- 3] Power-The probability of detecting a significant difference when one truly exists.

If you enter values for any 2 Minitab will calculate the third.

The Deliverables of the Measure are given in the table below:

Project Stage	Deliverables
Measure	Detail Process Map with points at which to gather data
	Data collection forms
	Measurement tools, include gauge R&R studies
	Data on the Process
	Process capability and Six Sigma Calculation
	Cost of Quality
	Project Status Form

Probability is the chance or likelihood that something will happen. In statistics the words chance and likelihood are seldom used to describe the possibilities for the event to take place; instead the word probability is used along with some other basic concepts. An event is an outcome of an experiment. Determining the number of defects out of a sample of 100 is an experiment and there are many possible events the possible outcomes can be anywhere between 0 and 100.

Discrete probability distributions

A distribution is said to be discrete if it is built on discrete random variables. The discrete probability distributions are Binomial and Poisson

Binomial distribution is one of the simplest probability distributions. It assumes an experiment with n identical trials with each trial having only 2 possible outcomes considered as success or failure. Each trial is independent of the previous one.

Problem: An operations manager has 16 production lines that produce the exact same type of product. The Production lines are supposed to be operational at 6:00 AM every day. The Probability for a machine on a line to go down is 0.15. The operational manager wants to know how many lines will be operational at the start of the shift. He also wants to have the distribution for the probability of the lines not operating.

Solution :- Open Minitab -> Click on Calc then Click on Probability Distribution and then on Binomial and in column C1 enter values 1 to 16 when the binomial Distribution box appears in number of trials enter 16 and probability of success enter 0.15 and in input column enter C1 you get the probability as shown in the table below.

Binomial with $n = 16$ and $p = 0.15$

x	$P(X \leq x)$
1	0.28390
2	0.56138
3	0.78989
4	0.92095
5	0.97646
6	0.99441
7	0.99894
8	0.99984
9	0.99998
10	1.00000
11	1.00000
12	1.00000
13	1.00000
14	1.00000
15	1.00000
16	1.00000



Poisson distribution: The binomial are used to calculate the probability for one outcome out of two. Not all situations exhibit only 2 alternatives. For instance an engineer might want to know the number of defects in a engine these examples do not state a number of trials nor do they state the number of alternative outcomes they only describe the occurrence of an event within a time frame.

Problem: For Example the employee attrition rate at a company follows a Poisson distribution with a mean of 4 a month; the HR Director wants to know the probability that between five and seven employees would leave the company next month.

We will use SigmaXL tool to do the Poisson Distribution : Open SigmaXL->Templates & calculators->Poisson Distribution calculator->Poisson and enter the values in the macro enable sheet and you will find the probability of five employees leaving is = 0.156.

Exponential Distribution: The exponential distribution is close to the poisson distribution but the poisson distribution is built on discrete random variables while the exponential distribution is continuous variables.

Solution: - Open Minitab -> Click on Calc then Click on Probability Distribution and then on Exponential and in column C1 enter values from the file Acid1.

Output as Shown Below:-

Results for: Acid1.MTW

Cumulative Distribution Function

Exponential with mean = 1

x P(X ≤ x)
 0.123 0.115736
 0.109 0.103270
 0.110 0.104166
 0.109 0.103270
 0.112 0.105956
 0.109 0.103270
 0.110 0.104166
 0.110 0.104166
 0.110 0.104166
 0.112 0.105956
 0.110 0.104166
 0.101 0.096067
 0.110 0.104166
 0.110 0.104166
 0.110 0.104166

Equivalence Tests are used to determine whether the mean of the sample is equivalent to a target or reference value examples are 1 Sample Test available at Stat→Equivalence Tests→ 1 Sample.

8.PARAMETRIC HYPOTHESIS TESTING (ANALYSE PHASE)

We often talk about fact-based decision making. In reality however there are a number of factors that influence or decision some of these are

Experience

Judgement

Assumption

Data base decision can help you overcome such influences and lead you in the right direction.

In hypothesis tests some commonly used terms are given below

Null Hypothesis written as Ho the hypothesis is the assumption about a population parameter such as mean standard deviation etc.

Alternate Hypothesis written as H1 or Ha is the opposite of null hypothesis.

Inequality sign indicates 2 tails while < or > indicates one tail.

Examples of Null Hypothesis and Alternate Hypothesis

1. A cement plant has found that the historical mean strength of cement is 25 units. The Company wants to assess whether the mean strength continues to be the same. In Null hypothesis we will assume that the mean strength (25 Units) has not changed. Therefore the null and alternate hypothesis will be written as :

$$H_0 : \mu = 25$$

$$H_1 : \mu \neq 25$$

The Number of tails is 2 as we want to assess whether the mean strength has changed

2. We want to evaluate whether a new incentive scheme has increased the mean daily production of the company. The historical mean is μ_0 . In Null hypothesis we will assume that the mean production level has not changed. Therefore the null and alternate hypothesis would be written as

$$H_0 : \mu = \mu_0$$

$$H_1 : \mu > \mu_0$$

The number of tails =1 (right tail) as we want to assess whether the mean production has increased.

3. A company has appointed a new courier service. They wish to assess whether the package are delivered faster than before. In Null hypothesis we will assume that the mean delivery time μ_0 has not changed; the null and alternate hypothesis will therefore be written as

$$H_0 : \mu = \mu_0$$

$$H_1 : \mu < \mu_0$$

The number of tails =1 (Left tail) as we want to assess whether the mean service time has reduced.

What is Confidence Interval?

The Practical Meaning of 95 Percent Confidence Interval can be states as "The Probability that the value of the population mean μ will lie between the two confidence interval bounds is 0.95". For example if take 1000 samples from a population we will have 1000 sample means and therefore 1000 confidence intervals ; we can expect that 95 percent or approximately 950 intervals will contain population mean μ .

In Statistics confidence intervals are of 3 types first is 95% Confidence Interval, second is 90% CI and Third is 99% CI ; by default it is always 95% CI but you can have 90 and 99% CI also. The statistical Term alpha is derived as α is = 1-0.95 or 1-0.99 or 1-0.90. We can calculate the Confidence Interval using formulas given in statistics.

For Example :- Suppose an Estimate is needed for the average coating thickness for a population of 1000 circuit boards received from a supplier. Rather than measure the coating thickness on all 1000 boards one might randomly pick up 36 boards for measurement. Suppose the average coating thickness of these 36 boards is 0.003 and the standard deviation of the 36 coating measurements is 0.0005. The standard deviation is assumed known from past experience. Determine the 95% confidence interval for the true mean.

From the Z table as the sample size is greater than 30 we use Z Table available in goggle search we know that $Z_{\alpha/2} = 1.96$ also we have

$$\alpha = 0.05, \bar{X} = 0.003, \sigma = 0.0005, n=36$$

We will use the statistical formula to calculate Confidence Interval which is given below:-

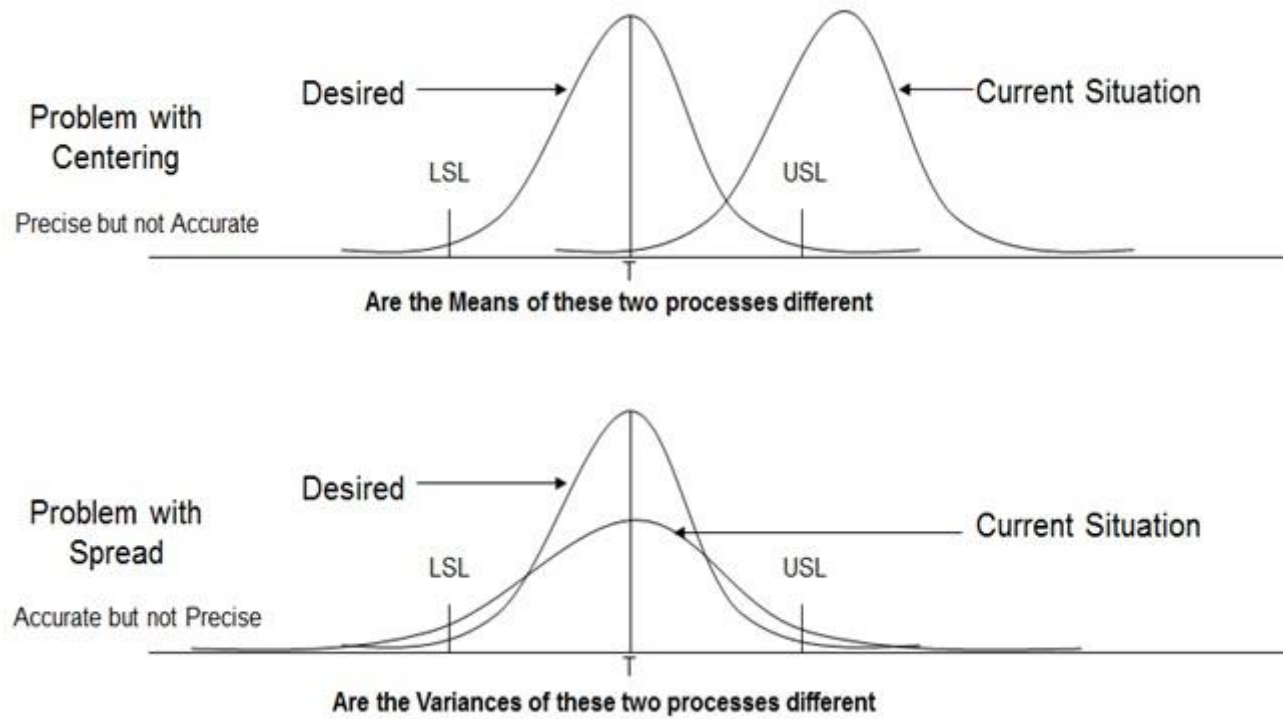
$$\bar{X} - Z_{\alpha/2} * \sigma / \text{sqrt of } n \leq \mu \leq \bar{X} + Z_{\alpha/2} * \sigma / \text{Sqrt of } n$$

Substituting the values in the formula we obtain

$$0.003 - (1.96) * 0.0005 / \text{Sqrt of } 36 \leq \mu \leq 0.003 + 1.96 * 0.0005 / \text{Sqrt of } 36$$

$$0.00284 \leq \mu \leq 0.00316 \text{ thus the 95\% confidence interval for the mean is } (0.00284, 0.00316)$$

In a process, we may face Problem with Centering and/or Problem with Spread. Below diagram will allow us to understand these two problems in detail:



Practical Six Sigma Problems that require Hypothesis Testing

Hypothesis testing tells us whether there exists statistically significant difference between the data sets for us to consider that they represent different distributions.

What is the difference that can be detected using Hypothesis Testing?

For Continuous data, hypothesis testing can detect Difference in Average and Difference in Variance. For Discrete data, hypothesis testing can detect Difference in Proportion Defective.

Steps in Hypothesis Testing:

- **Step 1:** Determine appropriate Hypothesis test
- **Step 2:** State the Null Hypothesis H_0 and Alternate Hypothesis H_a
- **Step 3:** Calculate Test Statistics / P-value against table value of test statistic
- **Step 4:** Interpret results – Accept or reject H_0

Mechanism:

- **H_0 = Null Hypothesis** – There is No statistically significant difference between the two groups
- **H_a = Alternate Hypothesis** – There is statistically significant difference between the two groups

Hypothesis Testing Errors:

Type I Error – $P(\text{Reject } H_0 \text{ when } H_0 \text{ is true}) = \alpha$

In type I Error, we reject the Null Hypothesis when it is true. It is also called as Alpha error or Producer’s Risk.

Type II Error - $P(\text{Accept } H_0 \text{ when } H_0 \text{ is false}) = \beta$

Similarly, in type II Error, we accept Null Hypothesis when it is false. It is also called as Beta error or Consumer’s Risk.

		JUDGMENT	
		Not Guilty H_0	Guilty H_a
TRUTH	Not Guilty H_0	Correct Decision	Type I Error (α)
	Guilty H_a	Type II Error (β)	Correct Decision

Six Sigma Hypothesis Testing Errors

P Value – Also known as Probability value, it is a statistical measure which indicates the probability of making an α error. The value ranges between 0 and 1. We normally work with 5% alpha risk, a p value lower than 0.05 means that we reject the Null hypothesis and accept alternate hypothesis.

Types of Hypothesis Testing:

We use the following grid to select the appropriate hypothesis test depending on the data types:

Select the appropriate Test		Output (Dependant Variable)Y	
		Continuous	Discrete
Input (Independent Variable) X's	Continuous	Simple Linear Regression Correlation	Logistic Regression
	Discrete	<u>Normal Data</u> T Test (1,2 Sample & Paired) ANOVA, F test , HOV <u>Non-Normal Data</u> Moods Median , HOV	Chi Square

Types of Six Sigma Hypothesis Testing

Situation	Statistical Test
Comparing a population mean against a given standard For example: Is the mean TAT of thread ≤ 15 minutes.	1-sample t-Test
Comparing means of two different populations For example: Is the mean of morning shift = mean of night shift.	2-sample t-Test
Comparing the means of more than two populations For example: Is the mean of staff A = mean of staff B = mean of staff C.	ANOVA
Comparing the variance of two or more than two populations For example: Is the variation of staff A = variation of staff B = variation of staff C.	Homogeneity Of Variance

Normal Continuous Y and Discrete X

Situation	Statistical Test
Comparing the variance of two or more than two populations For example: Is the variation of staff A = variation of staff B = variation of staff C.	Homogeneity Of Variance
Comparing the medians of two or more than two populations For example: Is the median of staff A = median of staff B = median of staff C.	Mood's Median Test

Non-Normal Continuous Y and Discrete X

Situation	Statistical Test
To see how output (Y) changes as the input (X) changes For example: If we need to find out how staff A's accuracy is related to his number of years spent in the process.	Simple Linear Regression

Continuous Y and Continuous X

Situation	Statistical Test
To see how output counts (Y) from two or more sub-groups (X) differ For example: If we want to find out whether defects from morning shift are significantly different from defects in the evening shift.	Chi-square Test of Independence

Discrete Y and Discrete X

Test	Null	Alternate
Normality	Data is Normal	Data is not Normal
Run Tests	No Special Causes	Special Causes Exist
t-test (1-sample)	$\mu = \text{constant or } T$	$\mu \neq \text{constant or } T$
t-test (2-sample, 2-tailed)	$\mu_1 = \mu_2$	$\mu_1 \neq \mu_2$
t-test (2-sample, 1-tailed)	$\mu_1 \leq \mu_2$	$\mu_1 > \mu_2$
Chi-square	$\chi^2 = 0$	$\chi^2 \neq 0$
HOV (3-sample)	$\sigma^2_1 = \sigma^2_2 = \sigma^2_3 = \dots = \sigma^2_n$	$\sigma^2_1 \neq \sigma^2_2 \neq \sigma^2_3 \neq \dots \neq \sigma^2_n$
ANOVA(3-sample)	$\mu_1 = \mu_2 = \mu_3 = \dots = \mu_n$	Atleast one is different
Mood's Median (3-sample)	$\text{Median}_1 = \text{Median}_2 = \dots = \text{Median}_n$	$\text{Median}_1 \neq \text{Median}_2 \neq \dots \neq \text{Median}_n$

A hypothesis is a value judgment made about a circumstance, a statement made about a population. Based on experience an engineer can for instance assume that the amount of carbon monoxide emitted by a certain engine is twice the maximum allowed legally. However his assertions can only be ascertained by conducting a test to compare the carbon monoxide generated by the engine with the legal requirements.

Number of Tails: The number of tails refers to the distribution used for testing the hypothesis. There are left and right tails for distribution. Depending on the objective of the test the left, right or both tails are considered in a hypothesis test and the number of tails decided. The Number can either 2 or 1 inequality sign \neq indicates 2 tail and less than or greater than indicates one tail. ($<$ $>$).

In statistical [significance testing](#), a **one-tailed test** and a **two-tailed test** are alternative ways of computing the [statistical significance](#) of a [parameter](#) inferred from a data set, in terms of a [test statistic](#). A two-tailed test is used if deviations of the estimated parameter in either direction from some benchmark value are considered theoretically possible; in contrast, a one-tailed test is used if only deviations in one direction are considered possible.

If the data used to make the comparison are parametric data that is data that can be used to derive the mean and the standard deviation, the population from which the data are taken are normally distributed they have equal variances. A standard error based hypothesis testing using the t-test can be used to test the validity of the hypothesis made about the population. There are at least five steps to follow when conducting hypothesis.

Null Hypothesis: The first step consists of stating the null hypothesis which is the hypothesis being tested. In the case of the engineer making a statement about the level of carbon monoxide generated by the engine, the null hypothesis is H_0 : the level of carbon monoxide generated by the engine is twice as great as the legally required amount. The Null hypothesis is denoted by H_0

Alternate hypothesis: the alternate (or alternative) hypothesis is the opposite of null hypothesis. It is assumed valid when the null hypothesis is rejected after testing. In the case of the engineer testing the carbon monoxide the alternative hypothesis would be

H_1 : The level of carbon monoxide generated by the engine is not twice as great as the legally required amount.

Testing the hypothesis: the objective of the test is to generate a sample test statistic that can be used to reject or fail to reject the null hypothesis. The test statistic is derived from Z formula if the samples are greater than 30.

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

If the sample are less than 30, then the t-test is used

$$T = \frac{\bar{X} - \mu}{s / \sqrt{n}}$$

Level of Risk: The level of risk addresses the kinds of errors that can be made while making an inference based on the test statistics obtained from the testing. Two types of errors can be made. The experimenter can falsely reject a hypothesis that is true. In that case we say that he made a type 1 error or α error. If he fails to reject a hypothesis that is actually false he makes a type II error or β error. In the case of the engineer testing the level of carbon dioxide is in fact twice as great as the prescribed level and he rejected the null hypothesis he would have made a type 1 error. If the carbon monoxide generated by the engine is less than the legally prescribed level and the experimenter fails to reject the null hypothesis he would have made a type ii error he would have failed to reject a null hypothesis that happened to be false.

Decision Rule: Only two decisions are considered rejecting the hypothesis or failing to reject it. The decision rule determines the conditions under which the null hypothesis is rejected or failed to be rejected. The Decision to reject the null hypothesis is based on the alpha level. Before conducting the test the experimenter must set the confidence level for

the test. He can decide to test the hypothesis with a 95% confidence level. That means he would be 95% sure that the decision to reject or fail to reject the null hypothesis is correct. However 95% confidence level means that 5% chance that an error will be made.

Example: A machine used to average a production rate of 245 units per hour before it went for repair. After it came back from repair over a period of 25h. It produced an average of 249 units with a standard deviation of 8. Determine if there is a statistically significant difference between the machines productivity before and after repair at a confidence level of 95%?

Solution:-

Since the sample is smaller than 30 we will use the t-test

The null hypothesis is the following

H₀: The productivity before repair = Productivity after repair

The alternate hypothesis should be the opposite

H_A: The productivity before repair ≠ the productivity after repair.

N=25, s=8, XBar=249, μ= 245

Since the Confidence level is set at 95%, $\alpha = 1 - 0.95 = 0.05$, since the null hypothesis is stated as equality we have a 2 tailed curve with each tail covering the one half of alpha.. With degree of freedom $df = n - 1 = 25 - 1 = 24$ the t critical can be obtained from the t table which is = 2.064.

If the calculated t-statistics falls within the interval $[-2.064, + 2.064]$ we would fail to reject the null hypothesis; otherwise the null hypothesis would be rejected.

Let us find the calculated t-statistic:

$$T = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{249 - 245}{8/\sqrt{25}} = 2.5$$

The calculated t statistic is 2.5 since it falls outside the interval $[-2.064, + 2.064]$ we will have to reject the null hypothesis and conclude that there is a statistically significant difference between the productivity of the machine prior to repair and after repair.

We can also Use the confidence interval method:

We can use the formula for confidence interval

$$\bar{X} - t_{\alpha/2, n-1} s/\sqrt{n} \leq \mu \leq \bar{X} + t_{\alpha/2, n-1} s/\sqrt{n}$$

There fore

$$249 - 2.064 \cdot 8/\sqrt{25} \leq \mu \leq 249 + 2.064 \cdot 8/\sqrt{25}$$

$$245.698 \leq \mu \leq 252.302$$

The null hypothesis is rejected because the mean μ (245) does not fall within the interval $[245.698, 252.302]$

1-Sample t Test (Mean v/s Target) this test is used to compare the mean of a process with a target value such as an ideal goal to determine whether they differ it is often used to determine whether a process is off centre

1 Sample Standard Deviation This test is used to compare the standard deviation of the process with a target value such as a benchmark whether they differ often used to evaluate how consistent a process is

2 Sample T (Comparing 2 Means) Two sets of different items are measured each under a different condition there the measurements of one sample are independent of the measurements of other sample.

Example of 2 sample T Test is is two populations two samples from this test we can find the average expenditure of the male customer if it is equal to the average expenditure of the female customer.

Paired T The same set of items are measured under 2 different conditions therefore the 2 measurements of the same item are dependant or related to each other.

2-Sample Standard This test is used when comparing 2 standard deviations

Standard Deviation test This Test is used when comparing more than 2 standard deviations of samples to be compared.

Generally, z-tests are used when we have large sample sizes ($n > 30$), whereas t-tests are most helpful with a smaller sample size ($n < 30$). Both methods assume a normal distribution of the data, but the z-tests are most useful when the standard deviation is known.

A T test is usually done to compare the means of two treatments for instance if we want to compare to compare the performance of a machine before some adjustments are performed on it and the performance after the adjustments are performed , the mean of one sample of products taken prior to adjustments can be compared to the mean of another sample taken after adjustment. In that case a t-test can be useful.

The hypothesis testing performed based on t-test is conducted using the degree of freedom and the confidence level, but when two sample means are being compared there is always a room for making error. If alpha = 0.05 there would be 5% chance of rejecting a null hypothesis that happens to be true. If for instance three sample means A,B,C are being compared using the t-test with a confidence interval of 95% two factors are compared at a time .A is compared with B , then A with C and then b with C. Every time two factors are being compared there are 0.05 probabilities for rejecting a true null hypothesis . Therefore when are three factors are compared using the t-test the type of making Type I error is inflated. In order to limit the chances of making a Type I Error inflation , we can use analysis of variance (ANOVA). ANOVA is a hypothesis test when more than two factor means are being compared.

P-Value Method: In the previous example we rejected the null hypothesis because the value of the calculated t-statistic was outside the interval $[-2.064, + 2.064]$ had been within that interval we would have failed to reject the null hypothesis the reason why $[-2.064, + 2.064]$ was chosen because the confidence level was set at 95% which translates into $\alpha = 0.05$. If alpha were set at another level the interval would have been different. The results obtained do not allow a comparison with a single value to make an assessment. Any value of the calculated t-statistic that falls within that interval would lead to a no rejection of the null hypothesis.

The use of p-value method enables us not have pre-set the value of alpha , the null hypothesis is assumed to be true ; the p-value sets the smallest value of alpha for which the null hypothesis has to be rejected. For instance in the example above the p-value generated from Minitab was 0.020 and alpha = 0.05 there alpha is greater than the p-value and we have to reject the null hypothesis.

- a) So if p value is less than alpha we have to reject the null hypothesis and accept the alternate hypothesis where alpha is pre-set at 0.05.
- b) If p value is greater than alpha we fail to reject the null hypothesis and reject the alternate hypothesis.

Two Errors in Hypothesis Testing:-

Type I Error: - Error occurs when you reject the null hypothesis although null hypothesis is true

α risk : Maximum probability of Committing a type I Error

Type II Error: - Error that you accept Null Hypothesis although Null Hypothesis is False

β risk : Probability of Committing a Type II Error.

Power of test : power of test is the probability of correctly rejecting H_0 ; the power of test is also called as $(1-\beta)$ thus if $\beta = 0.1$ the power is $(1-0.1)$ or 0.9.

Z Table. When a set of data follows a normal distribution pattern, the mean and standard deviation can be used to calculate the percentage of data falling inside a given range.

Example: The diameter of steel ring is critical to quality the production process for manufacturing the steel rings has yielded a mean of 5cm with a standard deviation of 0.05 cm. The process mean of 5 cms meets engineered standards ; after oil was changed on the machine that produces the steel rings the new process mean has been 5.05 cm with the standard deviation unchanged. A sample of five steel rings are taken.

What is the probability that the supervisor would not be alerted that the process is going out of control

Ans:- This is an example of Type II error B risk. A false null hypothesis error that you accept the false null hypothesis although it false based on the power probability $1-\beta = 1-0.777=0.223$ in other words the probability that the supervisor would not be alerted that the process is going out of control when in fact it is is $\beta = 0.777$.

T-test and Z test are used in the analyze phase in Hypothesis tests; T Tests are for Sample Size less than 30 it is also called Students Distribution and Z test for Sample Size greater than 30.

If $\alpha=0.05$ there would be 5% chance of rejecting a null hypothesis that happens to be true.

Problem: The mean flow of a fuel injection component is a critical characteristic. The manufacturing process is known to produce parts with a mean flow of 100 kg/ minute with a standard deviation of 5 kg the machine undergoes preventive maintenance and is reset by the operator; after starting the machine again the first five component measure 97,99,101,96 and 100 should the operator continue production; assume the confidence level of 95%.

$$\mu_0 = 100, \sigma = 5, n = 5, \bar{X} = (97+99+101+96+100)/5 = 98.6$$

Null hypothesis $H_0 = \mu = 100$

Alternate Hypothesis $H_1 = \mu \neq 100$

Since we are concerned about the difference this is a test with 2 tails thus $\alpha = 0.05$ should be distributed on both tails $Z = \bar{X} - \mu_0 / \sigma / \sqrt{n} = 98.6 - 100 / 5 / \sqrt{5} = -0.626$

From Z table we get $Z_{\alpha/2} = Z_{0.025} = -1.96$ and $+1.96$ if we find the formula calculated value between -1.96 and $+1.96$ we should conclude that the average has not changed; the calculated z value is -0.626 thus we cannot reject the null hypothesis so the mean which was 100 kg/minutes has not changed.

Problem

The mean weekly sales of a company for a specific consumer product are Rs 20 Million with standard deviation of 2 million. The Company has launched a new advertisement with a superstar. Data of subsequent 6 weeks show average weekly sales of 24 million. Have the sales increased significantly with 95 per cent confidence?

Mean weekly sales $\mu_0 = 20, \sigma = 2, \alpha_{\text{risk}} = 5\% = 0.05$;

$\bar{X} = 24, n = 6$;

Null Hypothesis $H_0 = \mu = 20$; Alternate Hypothesis $H_a: \mu > 20$ (Right tailed Test);

Test Statistic $Z = \bar{X} - \mu_0 / \sigma / \sqrt{n} = 24 - 20 / 2 / \sqrt{6} = 4.89$;

Critical value of Z is Z_{α} As $\alpha = 0.05$ this value is $Z_{0.05} = 1.64$ this can be determined from z tables or excel functions

$Z_{0.05} = \text{NORMSINV}(0.95)$

As the calculated Z statistic is 4.89 which is > 1.64 we must reject H_0 ; thus we conclude that the sales have changed significantly

The use of Z Test in Practice is difficult as we often do not Know population standard deviation; in such cases we need to use students t-test.

Problem

A software company has set a target of 75 days to complete a new software project of a particular type after initiating many actions , the team finds that the average of 16 completed projects in 72 days with standard deviation of 12 days . Is the evidence enough to recognize the team for achieving the target at 99 percent confidence interval?

From the problem we have $\mu_0 = 75, n = 16, H_0: \mu = 75, H_1: \mu < 75, \bar{X} = 72, \alpha = 0.01$ distributed on left tail.

$$T = \bar{X} - \mu_0 / s / \sqrt{n} = -1$$

T find the critical value of T Distribution in T table or using the Excel Function $\text{TINV}(2*0.01, 15) = -2.6025$.

Since T Statistic is -1 and T Critical is -2.6025 that is T Statistic is greater than T Critical so in that case cannot reject the Null Hypothesis.

Paired T Test is used to compare population means when data is in pairs and not independent of each other examples are performance of participant's before and after training

Problem Paired T Test: Blood samples of five persons taken at the same time were sent to two laboratories to check blood sugar . value of sugar were found to be 122/136, 148/153 , 93/98,234/236 and 105/112 is there a significance difference between the labs at 95 per cent confidence?

$T = \frac{\bar{d}}{s_d/\sqrt{n}}$ where \bar{d} is the average difference between the 2 samples , s_d is standard deviation of differences and n is the number of pairs. here in this example n is $5-1 = 4$

Lab 1 Lab 2 Diff

122	136	14
148	153	5
93	98	5
234	236	2
105	112	7

So calculate the Diff column Standard deviation which is = 4.51 and $\bar{d} = 6.60$, so $T_{calc}/statistic = 3.276$ and $T_{critical}$ obtained from the T table = 2.776 ; since T_{calc} 3.276 is beyond $t_{critical}$ 2.776 we can conclude that there is significant difference between the laboratories.

Problem

The monthly electricity bills for a company averages \$500 the company decides to cut down on electricity consumption by encouraging employees to open their window shades instead of using lights. Twelve months later the electricity bills were as shown in the table below. Can we say that there has been an improvement in the electricity consumption with a confidence level of 99%?

Jan	Feb	March	April	May	June	july	August	Sept	oct	Nov	Dec
479	509	501	470	489	494	445	502	467	489	508	500

The Null hypothesis is

H_0 : The average electricity bill before changes = the average electricity bill after changes

And the alternate hypothesis

H_a : The average electricity bill before changes not equal to the average electricity bill after changes.

Where $s = 19.264$, $\bar{X} = 487.75$, $n = 12$

Because the confidence interval is 99% we have $\alpha = 1 - 0.99 = 0.01$

We can determine the confidence interval using the formula

$\bar{X} - t_{\alpha/2, n-1} s/\sqrt{n} < \mu < \bar{X} + t_{\alpha/2, n-1} s/\sqrt{n}$

Substituting the values in the formula we get $470.477 < \mu < 505.023$

The mean \$500 falls well within the confidence interval; therefore we cannot reject the null hypothesis and have to conclude that there has not been a statistically significant change in the level of electricity consumption because the employee opening the window shades instead of using their lights.

Comparing Variances F-Test:-

Example: A new Supplier has supplied samples of valves. Five Samples of these and six samples of Current Source were tested for 1000 hours to compare wear of the former with the current source. Wear on the five valves tested from new source was 0.054, 0.062, 0.068, 0.058 and 0.04 mm. wear on the six valves from current source was 0.047, 0.059, 0.068, 0.082, 0.070 and 0.056. is the variation in new source comparable to the current source ? assume 95 percent confidence level.

$H_0: \sigma_1^2 = \sigma_2^2$ and $H_1: \sigma_1^2 \neq \sigma_2^2$.

Standard deviation for Current from formula = $F_{\text{Current}} = 0.0123$ and $S_{\text{new}} = 0.0105$

$$F = S_1^2/S_2^2 = 0.0123^2/0.0105^2 = 1.3598.$$

Critical value from F table $F_{0.025,5,4} = 9.36$

Now $F_{\text{cal}} = 1.3598$ and $F_{\text{critical}} = 9.36$ as $F_{\text{cal}} < F_{\text{critical}}$ we cannot reject H_0 so the variances are therefore not significantly equal.

Test of two proportions:-

A company has 2 manufacturing plants, both manufacture the same model of car. Plant 1 has produced 2500 cars, 75 of which required rework of transmissions. Plant 2 has manufactured 1800 cars and 63 of these were reworked. Is there a significant difference in quality? Assume 95% confidence level.

$H_0: P_1 = P_2$ $H_1: P_1 \text{ Not equal to } P_2$, $X_1=75$, $n_1= 2500$, $X_2=63$, $n_2=1800$

The formula for 2 proportion is $Z = \frac{X_1/n_1 - X_2/n_2}{\sqrt{p(1-p)(\frac{1}{n_1} + \frac{1}{n_2})}}$

Where $P = \frac{X_1 + X_2}{n_1 + n_2}$

$$P = \frac{75 + 63}{2500 + 1800} = \frac{138}{4300} = 0.0321$$

$$Z = \frac{75/2500 - 63/1800}{\sqrt{0.0321(1 - 0.0321)(\frac{1}{2500} + \frac{1}{1800})}} = -0.92$$

From the Z table we get the Z critical as $Z = 1.96$

If $Z_{\text{cal}} < Z_{\text{critical}}$ or table we cannot reject the null hypothesis; in this case $Z_{\text{cal}} = -0.92$ and $Z_{\text{critical}} = 1.96$ so Z_{cal} is less than Z_{critical} so we cannot reject the null hypothesis and conclude that there is no significant differences in the 2 plants at 95 percent confidence level.

Anova is a Parametric Test

ANOVA TABLE MANUAL CALCULATION

Sources of Variation	Sum of Squares	Degree of Freedom	Mean Square	F-Statistic
Between Treatments	$SST = \sum nj(\bar{X}_j - \bar{X})^2$	$Df_{\text{treatments}} = K - 1$	$MST = SST / Df_{\text{treatment}}$	$F = MST / MSE$
Within Treatments	$\sum_{j=1}^k \sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2$	$Df_{\text{error}} = n - k$	$MSE = SSE / Df_{\text{error}}$	
Total	$SST + SSE$	$Df_{\text{treatments}} + Df_{\text{error}}$		

If for instance 3 sample means A, B, C are being compared using the t-test is cumbersome for this we can use analysis of variance ANOVA can be used instead of multiple t-tests.

ANOVA is a Hypothesis test used when more than 2 means are being compared.

If K Samples are being tested the null hypothesis will be in the form given below

$$H_0: \mu_1 = \mu_2 = \dots = \mu_k$$

And the alternate hypothesis will be

H1: At least one sample mean is different from the others

Anova are One-way, 2-way, 3 way etc. one way annova has one independent factor and 2 way annova has 2 Independent factors.

The Sum of Squares between samples (or Treatments) is noted as SST and is obtained from the formula $SST = \sum n_j (\bar{X}_j - \bar{X})^2$.

Example: - An operations Manager is Comparing the Productivity per Hour of Four Machines. He takes sample of units produced from each Machine and tabulates them in the table given below. Can we say that the production of the four machines are equal with a confidence level of 95%?

Table A:

Sample 1	Sample 2	Sample 3	Sample 4
23	25	25	23
24	22	25	23
25	22	25	23
24	23	24	23
24	24	24	24
25	22	23	24
26	23	26	25

Solution: - The Grand Mean \bar{X} is = 23.893

The null hypothesis will be

H0: mean of sample 1= mean of sample 2 = mean of sample 3 and the alternate hypothesis HA: At least one sample mean is different from others.

Table B: Sum of Square for treatments

Sample 1	Sample 2	Sample 3	Sample 4
23	25	25	23
24	22	25	23
25	22	25	23
24	23	24	23
24	24	24	24
25	22	23	24
26	23	26	25
\bar{X}_j =23.893	23.00	24.571	23.571
$(\bar{X}_j - \bar{X})^2$ =0.287	0.797	0.460	0.103

$$\sum (\bar{X}_j - \bar{X})^2 = 0.287 + 0.797 + 0.460 + 0.103 = 1.648.$$

$$SST = \sum n_j (\bar{X}_j - \bar{X})^2 = 7 * 1.648 = 11.536$$

Table C: Sum of Squares for Error

Sample 1	Sample 2	Sample 3	Sample 4
(23-24.429) ² =2.041 By Applying the formula (Xij-Xbar J) ²	4.000	0.184	0.327
0.184	1.000	0.184	0.327
0.327	1.000	0.184	0.327
0.184	0.000	0.327	0.327
0.184	1.000	0.327	0.184
0.327	1.000	2.469	0.184
2.469	0.000	2.041	2.041
5.714	8.000	5.714	3.714

So SSE = 5.714 + 8.0000 + 5.714 + 3.714 =23.143

So TSS = SST + SSE = 11.536 + 23.143 = 34.679.

Degree of Freedom

Degree of freedom between treatments: Since there are four treatments (Samples),the degree of freedom=DF=k-1= 4-1 =3.

Degree of Freedom within treatments: Since there are 28 observations and 4 treatments the degree of freedom will be =dferror=n-k = 28-4 = 24.

Mean Squares MST = SST/ DFtreatment = 11.536 / 3 = 3.845.

MSE = SSE / DF error = 23.143 / 24 = 0.963.

Calculated F Statistic F = MST / MSE = 3.845 / 0.963 = 3.993.

The Computed Statistics can be summarize in the ANOVA table

Sources of Variation	Sum of Squares	Degree of Freedom	Mean Square	F-Statistic
Between Treatments	11.536	3	3.845	
Within Treatments	23.143	24	0.963	3.993
Total	34.679	27		

We can find the F-Statistics in the F-Table. The Critical F Statistic is based on the degree of freedom of the treatment, the degree of freedom for error based on the table.

Based on the table the Critical F-Statistic is equal to 3.01 which we get from the F-Anova table; while the calculated F-Statistic is = 3.993 the calculated F statistic is larger than the critical F-Statistic ; therefore we have to reject the null hypothesis and conclude that at least one sample mean is different from others.

Problem of 2-way Anova: Consider an experiment on aluminium castings .Customer requires hardness to be controlled. Hardness is an important to quality characteristic (CTQ) we therefore want to evaluate the effect of two factors on hardness (y).The 2 potential contributors are percentage of copper and magnesium. We have Controlled Copper percentage at 3.5 and magnesium at 1.2 and 1.8 percent data is available in the file hardness.xls

Solution: Open the XLS file Anova Hardness and the Minitab window opens click on stat->Anova-> and select 2 way a window pops up in response select hardness and row factor select copper and in column factor select magnesium at 95% confidence intervals you will get the output as shown below

Two-way ANOVA: Hardness versus Copper_1, Magnesium

Source	DF	SS	MS	F	P
Copper_1	1	1.125	1.125	1.29	0.320
Magnesium	1	21.125	21.125	24.14	0.008
Interaction	1	15.125	15.125	17.29	0.014
Error	4	3.500	0.875		
Total	7	40.875			

S = 0.9354 R-Sq = 91.44% R-Sq(adj) = 85.02%

The p value of copper is > 0.05 which is not significant so copper alone does not contribute to the hardness; P value of Magnesium is significant at 0.008 so it contributes to the hardness and the interaction between copper and magnesium is also 0.014 which is statistically significant as it is less than 0.05 so magnesium and the interaction between magnesium and copper contribute to the hardness.

Example of General MANOVA

An automotive parts supplier assesses the usability and quality of the door locks that they provide. The locks are manufactured using two different methods at three plants. The production manager wants to determine whether the production method and the plant affect the final product. The production manager collects data on locks from each plant, produced by each method.

The manager collects data on the quality and usability of samples of locks. To assess how method and plant affect both response variables at the same time, the manager does a general MANOVA. The manager decides to use a significance level of 0.10 to decide which effects to examine in more detail.

1. Open the sample data, CarLockRatings.MTW.
2. Choose **Stat > ANOVA > General MANOVA**.
3. In **Responses**, enter 'Usability Rating' 'Quality Rating'.
4. In **Model**, enter *Method Plant Method*Plant*.
5. Click **OK**.

Interpret the results

The p-values for the production method are statistically significant at the 0.10 significance level. The p-values for the manufacturing plant are not significant at the 0.10 significance level for any of the tests. The p-values for the interaction between plant and method are statistically significant at the 0.10 significance level. Because the interaction is statistically significant, the effect of the method depends on the plant.

General Linear Model: Usability Ra, Quality Rati versus Method, Plant

MANOVA Tests for Method

Criterion	Test	DF			P
	Statistic	F	Num	Denom	
Wilks'	0.63099	16.082	2	55	0.000
Lawley-Hotelling	0.58482	16.082	2	55	0.000
Pillai's	0.36901	16.082	2	55	0.000
Roy's	0.58482				

$s = 1$ $m = 0.0$ $n = 26.5$

MANOVA Tests for Plant

Criterion	Test	DF			P
	Statistic	F	Num	Denom	
Wilks'	0.89178	1.621	4	110	0.174
Lawley-Hotelling	0.11972	1.616	4	108	0.175
Pillai's	0.10967	1.625	4	112	0.173
Roy's	0.10400				

$s = 2$ $m = -0.5$ $n = 26.5$

MANOVA Tests for Method*Plant

Criterion	Test	DF			P
	Statistic	F	Num	Denom	
Wilks'	0.85826	2.184	4	110	0.075
Lawley-Hotelling	0.16439	2.219	4	108	0.072
Pillai's	0.14239	2.146	4	112	0.080
Roy's	0.15966				

$s = 2$ $m = -0.5$ $n = 26.5$

8.1 Non parametric hypothesis testing:-

Non Parametric tests should be used when all the following conditions are true

- **Data are Non-Normal.**
- **Data cannot be readily transformed to Normality**
- **Sample size is small, say less than 30.**

In the previous examples we used means and standard deviation to determine if there were statistically significant differences between samples. What happens if the data cannot yield arithmetic means and standard deviations? What happens if the data are nominal or ordinal?

When we deal with categorical, nominal, or ordinal data non parametric statistics are used to conduct hypothesis testing. The Chi-square test and the Mann-Whitney U test, Kruskal Wallis, Moods Median are examples of nonparametric tests.

Chi-Square test:-

Chi Square goodness of fit test:-

Fouta electronics and Touba Inc are computer manufacturers that use the same third party call centre to handle their customer services. ToubaInc conducted a survey to evaluate how satisfied its customers were with the services that they receive from the call centre, the results of the survey are given in the table below.

Table A

Categories	Rating %
Excellent	10
Very good	45
Good	15
Fair	5
Poor	10
Very poor	15

After having the results of the survey, Fouta electronics decided to find out whether they apply to its customers, so it interviewed 80 randomly selected customers and obtained the results as shown in table below.

Categories	Rating(absolute value)
Excellent	8
Very good	37
Good	11
Fair	7
Poor	9
Very poor	8

To analyse the results the quality engineer at Fouta electronics conducts a hypothesis testing. However in this case, because he is faced with categorical data he cannot use a t-test since a t-test relies on the standard deviation and the mean which we cannot obtain from either table. We cannot deduct a mean satisfaction or a standard deviation satisfaction. Therefore another type of test will be needed to conduct the hypothesis testing. The test that applies to this situation is the Chi-square test, which is a non-parametric test.

Step 1: State the hypothesis

The null hypothesis will be

H0: The results of Toubainc Survey = the same as the results of Fouta electronics survey

And the alternate hypothesis will be

H1: The results of Toubalnc Survey \neq the same as the results of Fouta electronics survey.

Step 2: Test Statistic to be used: The test statistic to conduct hypothesis testing is based on the calculated

$\chi^2 = \sum (f_0 - f_e)^2 / f_e$ where f_e represents the expected frequencies and f_0 represents the observed frequencies

Step 3: Calculating the χ^2 test statistic:-

We will use the same table as given above if a sample of 80 customers were surveyed the data in the table

Table A

Categories	Rating %	Expected frequencies f_e
Excellent	10	$0.10 * 80 = 8$
Very good	45	$0.45 * 80 = 36$
Good	15	$0.15 * 80 = 12$
Fair	5	$0.05 * 80 = 4$
Poor	10	$0.10 * 80 = 8$
Very poor	15	$0.15 * 80 = 12$
Total	100	80

We can summarize the observed frequencies and the expected frequencies in the table given below:

Categories	Observed Frequencies f_o	Expected Frequencies f_e	$(f_o - f_e)^2 / f_e$
Excellent	8	8	0
Very good	37	36	0.028
Good	11	12	0.083
Fair	7	4	2.25
Poor	9	8	0.125
Very poor	8	12	1.33
Total	80	80	3.816

Therefore we conclude $\chi^2 = \sum (f_o - f_e)^2 / f_e = 3.816$.

Now that we have found the calculated χ^2 we can find the critical χ^2 from the table. The Critical χ^2 is based on the degree of freedom and the confidence level since the number of categories is 6 the degree of freedom $df = 6 - 1 = 5$, if the confidence level is set at 95%, $\alpha = 0.05$; therefore the critical χ^2 is equal to 11.070 which we get from Chi-Square Table.

Since the critical $\chi^2_{0.05,5} = 11.070$ is much greater than the calculated $\chi^2 = 3.816$ we fail to reject the null hypothesis and we have to conclude that the surveys done by Touba Inc and Fouta Electronics gave statistically similar results.

In Minitab go to assistant click on hypothesis test and select chi-square goodness of fit test under more than 2 samples and take the sample data as umbrella from Help.

Chi-Square using Tools: Click on SigmaXL->Select Chi-Square-Goodness test->Enter the values as shown in the table above you will get the same results. As the excel sheet is macro enabled.

Mood's Median Test

This test is used to determine whether there is sufficient evidence to conclude that samples are drawn from populations with different medians

Ho: The Population from which the samples are drawn have the same median

Ha: At least one of the populations has a different median

The procedure is best illustrated with an example.

Example: - Three machines produce plastic parts with a critical location dimension samples are collected from each machine with each sample size at least 10. The Dimension is measured on each part. Does Mood’s median test permit rejection at the 90% significance level of the hypothesis that the median dimensions of the three machines are the same?

The Data:

Machine # 1: 6.48, 6.42, 6.47, 6.49, 6.47, 6.48, 6.46, 6.45, 6.46

N= 10

Machine # 2 : 6.42,6.46,6.41,6.47,6.45,6.46,6.42,6.46,6.46,6.48,6.47,6.43,6.48,6.47

N = 14

Machine # 3: 6.46,6.48,6.45,6.41,6.47,6.44,6.47,6.42,6.43,6.47,6.41

N = 11

Procedure:

1. Find the overall median of all 35 readings , overall median = 6.46
2. Construct a table with one column showing number of readings above the overall median and another showing number of medians below the overall category median or each category. Half of the readings that are equal to the median should be counted in the “above” column and half in the “below” column.

Observed Values			
	Number above the overall Median	Number below the overall median	
Machine #1	7	3	
Machine #2	7	7	
Machine #3	4.5	6.5	

If the three Populations have the same median, each should have half their readings in the “above” column and half “below”. Therefore the expected value for number of readings would be half the sample size in each case, so the expected values would be as shown in the table.

Expected Values		
	Number above overall median	Number below overall median
Machine # 1	5	5
Machine # 2	7	7
Machine # 3	5.5	5.5

The next step is to calculate $(\text{observed} - \text{expected})^2 / \text{expected}$. For each cell in the table The sum of these values in these cells =1.96 this is χ^2 test statistic with degrees of freedom k-2 where k is the number of machines. Since the critical value for χ^2 1, 0.95 = 3.84 the data do not justify rejecting the null hypothesis.

(Observed-Expected)²/ expected		
	Number above overall median	Number below overall median
Machine # 1	$2^2/5 = 0.8$	$2^2/5 = 0.8$
Machine # 2	0	0
Machine # 3	$1/5.5=0.18$	$1/5.5 =0.18$

To find the P-value for 1.96 using MS Excel, choose the statistical function CHIDIST and enter 1.96 for x^2 and 1 for the degrees of freedom. You get the value 0.161513.

Mann-Whitney Test (Non-Parametric Test)

Mann-Whitney test is better explained thru an example.

Mann-Whitney test is used in Minitab by selecting the option Stat → Non Parametric → Mann Whitney Test select sample 1 row and sample 2 row and check whether their medians are equal at 95% Confidence interval; the null hypothesis is that the medians are equal and the alternate hypothesis is the medians are not equal check the output and see whether the medians are equal or not.

Example: An operations manager wants to compare the number of inventory discrepancies found in two operating shifts. The inventory discrepancies are not normally distributed. The manager takes a sample of discrepancies found over 7 days for the first shift and 5 days for the second shift tabulates the data as shown below

First Shift	Second Shift
15	17
24	23
19	10
9	11
12	18
13	
16	

We can make several observations from this table. First the sample sizes are small and we only have 2 samples so the first thing that comes to mind would be to use t-test, however the t-test assumes that the population from which the samples are taken should be normally distributed which is not the case therefore the t-test cannot be used, instead the Mann-Whitney U Test should be used. The Mann-Whitney U test assumes that samples are independent and from dissimilar populations

Step 1: Define the Null Hypothesis: Just as the case of T-Test the Mann-Whitney U Test is a hypothesis test the null and alternate hypothesis are

H₀: The number of Discrepancies in the first shift is the same as the one in the second shift

H₁: The number of Discrepancies in the first shift is different from the ones in the second shift

The result of the test will lead to the rejection of the null hypothesis or a failure to reject the null hypothesis.

Step 2: Analyse the data: The first step in the analysis of the data consists of naming the groups. In our case they are already named first and second shift. The Next Step consists of grouping the two columns in one and sorting the observations in ascending order ranked from 1 to n. Each observation is paired with the name of the original group to which it belonged

The table below illustrates the fact:

Observations	Group	Ranks
9	First Shift	1
10	Second Shift	2
11	Second Shift	3
12	First Shift	4
13	First Shift	5
15	First Shift	6
16	First Shift	7
17	Second Shift	8
18	Second Shift	9
19	First Shift	10
23	Second Shift	11
24	First Shift	12

We will call v_1 the sum of the ranks of the observations of the first group called as first shift and v_2 the sum of the ranks of the observations for group second shift

$$V_1 = 1+4+5+6+7+10+12 = 45$$

$$V_2 = 2+3+8+9+11 = 33$$

Step 3: Determining the values of U Statistic by the formula given below

$$U_1 = n_1 * n_2 + n_1 * (n_1 + 1) / 2 - v_1$$

$$U_2 = n_1 * n_2 + n_2 * (n_2 + 1) / 2 - v_2$$

If any or both of the sample size are greater than 10, then U will be approximately normally distributed and we could use Z transformation but in our case both sample sizes are less than 10 therefore

$$U_1 = 7 * 5 + 7(7+1)/2 - 45 = 35 + 28 - 45 = 18$$

$$U_2 = 7 * 5 + 5(5+1) / 2 - 33 = 35 + 15 - 33 = 17$$

Since the calculated test statistic should be the smallest of the two we will use $U_2 = 17$ and $n_2 = 7$ and $n_1 = 5$ from the Mann Whitney table we find that p value is 5 therefore we fail to reject the null hypothesis and have to conclude that the number of discrepancies in the first shift are the same as the ones in the second shift.

Kruskal-Wallis Test is done in Minitab using the sample data Exh Stat and select growth and treatment and keep the growth as the response variable and Treatment = x factor. It is a Non-Parametric Test. For the Higher rank of treatment 3 we can conclude that treatment 3 significantly contributes to the growth. This test can be done in Minitab by going to Stat→Non-Parametrics→Kruskal Wallis Test.

9. NORMALIZING DATA

- **Non Normal data** Outliers can cause your data to become skewed. The mean is especially sensitive to outliers. Try removing any extreme high or low values and testing your data again. Capability analysis and other tests require the data to be normal.

Minitab Navigation-Stat→Basic Statistics→Normality→Tests→Anderson darling tests tells us whether the data is normal or not or in other words tells us whether outliers are present or not.

When the data being analysed are not normal and the tools required for analysis require their normality then one option would be to normalize them. Normalizing the data means transforming them from non-normal to normal. This can be done using Box-Cox transformation or the Johnson transformation.

From the menu bar of Minitab click on stat then click on quality tools and from the drop down list select Johnson transformation the graph shows the probability plot before and after transformation now the p value becomes greater than the standard (0.05) the data are therefore concluded to be normally distributed.

Multi-Vari analysis is used when you have multiple discreet Xs (like work shift, employee, location) and Y is continuous (like part length or cycle time).

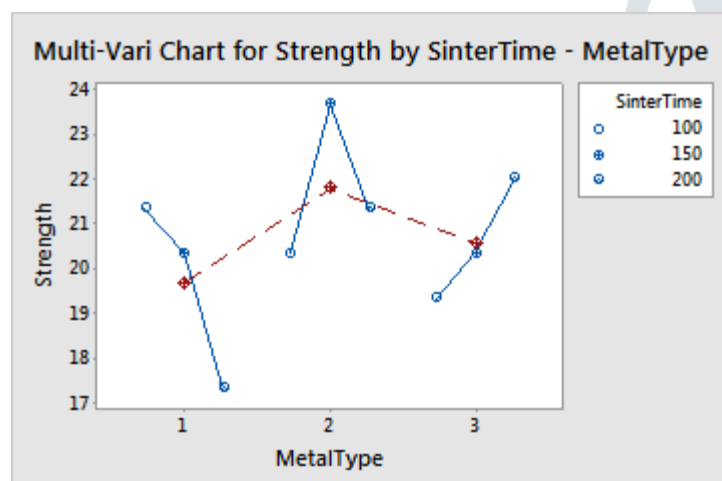
An engineer wants to assess the effect of sintering time on the compressive strength of three different metals. The engineer measures the compressive strength of five specimens of each metal type at each sintering time: 100 minutes, 150 minutes, and 200 minutes.

The engineer creates a multi-vari chart to look for possible trends and interactions in the data.

1. Open the sample data, *SinteringTime.MTW*.
2. Choose **Stat > Quality Tools > Multi-Vari Chart**.
3. In **Response**, enter *Strength*.
4. In **Factor 1**, enter *SinterTime*.
5. In **Factor 2**, enter *MetalType*.
6. Click **OK**.

Interpret the results

The multi-vari chart indicates a possible interaction between the type of metal and the length of sintering time. The greatest compressive strength for Metal Type 1 is obtained by sintering for 100 minutes, for Metal Type 2 by sintering for 150 minutes, and for Metal Type 3 by sintering for 200 minutes.



The data show that the strength varies differently across sintering times for different metal types, indicating an interaction.

To quantify this interaction and to determine whether it is statistically significant, the engineer could further analyse this data with an ANOVA (analysis of variance) or a general linear model.

R-squared is a statistical measure of how close the data are to the fitted regression line. It is also known as the coefficient of determination, or the coefficient of multiple determination for multiple regression. ... 100% **indicates** that the model explains all the variability of the response data around its **mean**.

Variance inflation factor is a measure of the amount of **multicollinearity** in a set of multiple regression variables. ... **Variance inflation factor** measures how much the behavior (variance) of an independent variable is influenced, or inflated, by its interaction/correlation with the other independent variables.

Multicollinearity generally occurs when there are high correlations between two or more predictor variables. ... **Examples** of correlated predictor variables (also called multicollinear predictors) are: a person's height and weight, age and sales price of a car, or years of education and annual income.

Multicollinearity in regression is viewed as more of disadvantage, as it practically inflates unnecessarily the standard errors of coefficients in regression. Having studied Variance Inflation Factor (VIF) we know that a VIF of 5 and above is not good for regression model because it might render other significant variables redundant.

The **adjusted R-squared** is a modified version of **R-squared** that has been **adjusted** for the number of predictors in the model. The **adjusted R-squared** increases only if the new term improves the model more than would be expected by chance. It decreases when a predictor improves the model by less than expected by chance.

The Variation is given by Sum of Squares / Degree of Freedom

Each MS(mean Square) value represents variance due to the source mentioned in the row we can conclude that magnesium an interaction between magnesium and copper have significant effect on the hardness of aluminium castings.

In ANOVA if the calculations are done manually using formulas we apply the F-Statistic table and find the end result.

MULTIVARIATE ANALYSIS IS USED IN ANALYSE PHASE –

Overview for Principal Components Analysis

Use Principal Components Analysis to identify a smaller number of uncorrelated variables, called "principal components", from a large set of data. With this analysis, you create new variables (principal components) that are linear combinations of the observed variables. The goal of principal components analysis is to explain the maximum amount of variance with the fewest number of principal components.

For example, an analyst uses a principal components analysis to analyze customer responses to several characteristics of a new shampoo. The analyst wants to determine whether they can form a smaller number of uncorrelated variables that are easier to interpret and analyze than the observed variables that they measured.

Principal components analysis is commonly used as one step in a series of analyses. For example, you can use principal components before you perform a regression analysis, in order to avoid multi-collinearity or to reduce the number of predictors relative to the number of observations.

Example: - Using Census Data Collected in the table below perform a principal components analysis using Minitab. Determine the factors which contribute the greatest to the variability.

Principal Components, Factor Analysis, Discriminant Analysis, Multiple Analysis of Variance are used to investigate the sources of variation and are done in the analyze phase.

Principal Components have wide applicability in the social sciences and market research.

Population	School	Employ	Health	Home
5.935	14.2	2.265	2.27	2.91
1.523	13.1	0.587	0.75	2.62
2.599	12.7	1.237	1.11	1.72
4.009	15.2	1.649	0.81	3.02
4.687	14.7	2.312	2.50	2.22
8.044	15.6	3.641	4.51	2.36

2.766	13.3	1.244	1.03	1.97
6.538	17.0	2.618	2.39	1.85
6.451	12.9	3.147	5.52	2.01
3.314	12.2	1.606	2.18	1.82
3.777	13	2.119	2.83	1.80
1.530	13.8	0.798	0.84	4.25
2.768	13.6	1.336	1.75	2.64
6.585	14.9	2.763	1.91	3.17

Choose Stat → Multivariate → Principal Components → Select all five fields for variables and select correlation for type of Matrix.

Principal Component Analysis: Population, School, ... y, Health, Home Eigen analysis of the Correlation Matrix

Eigenvalue	3.0287	1.2911	0.5725	0.0955	0.0123
Proportion	0.606	0.258	0.114	0.019	0.002
Cumulative	0.606	0.864	0.978	0.998	1.000

Eigenvectors

Variable	PC1	PC2	PC3	PC4	PC5
Population	0.558	0.131	-0.008	-0.550	0.607
School	0.313	0.629	0.549	0.453	-0.007
Employ	0.568	0.004	-0.117	-0.269	-0.769
Health	0.487	-0.310	-0.455	0.648	0.200
Home	-0.174	0.701	-0.691	-0.015	-0.014

The Eigen value row in the top table provides the variance of each component ; for instance population has variance of 3.028 and we also find the Eigen values to be low for health which is 0.0954 and home 0.0121 so from this we can interpret that the first 2 factors population and school account for 86.4% of the variability.

The Screen Plot Below shows with Eigen value on the Y-Axis and Principal Components on the X axis from here we can say that the factors which contribute most to the variability are Population, school & Employment.



Eigen Values: A High Eigen Value indicates that the respective factor contributes to a high variation.

Factor analysis

Use Factor Analysis to assess the structure of your data by evaluating the correlations between variables. Factor analysis summarizes data into a few dimensions by condensing a large number of variables into a smaller set of latent factors that you do not directly measure or observe, but which may be easier to interpret. Using this analysis, you can model each original variable as a linear function of these underlying factors. Factor analysis is commonly used in the social sciences, market research, and other industries that use large data sets.

For example, a credit card company uses factor analysis to ensure that a customer satisfaction survey address three factors before sending the survey to a large number of customers. If the survey does not adequately measure the three factors, then the company should reevaluate the questions and retest the survey before sending it to customers.

10. REGRESSION

The Simple regression analysis is that it involves only 2 variables the independent and the dependant variable so in the equation $y=f(x)$ y is the dependant variable and x is the independent variable.

Linear Regression: - In Linear regression the value of y increases proportionately to the independent variable x. Where it is necessary to calculate the value of y at a given x value the following important factors are to be borne in mind.

1. The intercept or the point at which the slope starts when the value of x is zero.
2. What exactly is the slope which represents the value of y with any changes in x?
3. Whether there would be any deviation in the value of y from the expected value of x?

We can explain the intercept in a simple equation $y = a + bx$. Where b is called the regression coefficient of the slope.

A fitted value is a statistical model's prediction of the mean response value when you input the values of the predictors, factor levels, or components into the model. Suppose you have the following regression equation: $y = 3X + 5$. If you enter a value of 5 for the predictor, the fitted value is 20. Fitted values are also called predicted values.

After a regression line has been computed for a group of data, a point which lies far from the line (and thus has a large residual value) is known as an *outlier*. Such points may represent erroneous data, or may indicate a poorly fitting regression line. If a point lies far from the other data in the horizontal direction, it is known as an *influential observation*. The reason for this distinction is that these points have may have a significant impact on the slope of the regression line. Notice, in the above example, the effect of removing the observation in the upper right corner of the plot:

Linear Regression is when all the variables are continuous and non linear regression is when the Y(KPOV) is discrete and the X's are Continuous, you can use the Binary Logistic Regression option in Minitab under stat→Regression→Binary Logistic Regression, when the response variable is discrete.

Example: - A Six Sigma Project team in a chemical manufacturing company was analysing the customer complaints regarding the variation in specific gravity of the chemical being supplied. The team was interested to know whether there was any relationship between one specific chemical and the specific gravity of the solution they collected data of specific gravity at varying amounts of Chemical as given in the table below

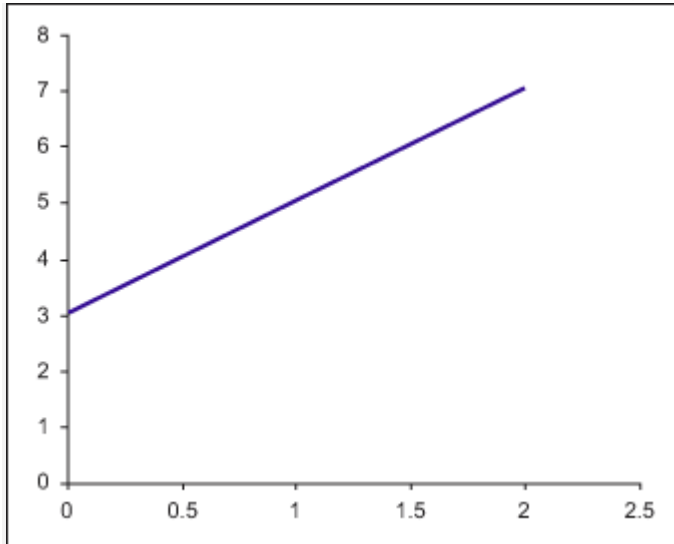
Quantity of a Chemical in gm	Specific gravity of the Solution
5	1
6	1.04
7	1.07
8	1.12
9	1.15
10	1.19
11	1.23
12	1.27
13	1.31
14	1.35

Everyone is taught in school the equation of a straight line:

$$Y = a + bX$$

Where a is the Y-intercept and b is the slope of the line. Using this equation and given any value of X , anyone can compute the corresponding Y .

Figure 1: Charting the Formula for a Straight Line



In Figure 1, $Y = 3 + 2X$. It is easy to see visually that a is 3. For the slope b , any two points on the line need to be chosen, say $X_1 = 1, Y_1 = 5$ and $X_2 = 2, Y_2 = 7$ and apply the following formula:

:

$$b = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{7 - 5}{2 - 1} = \frac{2}{1} = 2$$

The relationship between y (Specific gravity of the Solution) and the quantity of the chemical was established and shows that the presence of chemical increases the specific gravity of the solution increases proportionately.

Coefficient of Correlation The statistic used to determine the measure of relatedness between factors is the coefficient of correlation generally noted as R it is a number between -1 and $+1$. When the coefficient of correlation is equal to -1 there is a perfect negative correlation between the factors; in other words an increase in the magnitude of the independent factor will necessarily lead to a decrease in the dependent factor in the exact same proportion and a decrease in the independent factor will lead to an increase in the dependent factor in the exact same proportion.

If the coefficient of correlation is equal to 0 there is absolutely no relationship between the 2 factors and when it is equal to $+1$ there is a positive correlation between the factors and increase in the independent factor leads to an increase in the dependent factor in the same proportions.

Coefficient of Determination : While the coefficient of correlation measures the strength of the relation between the 2 factors, the coefficient of determination shows what proportion in the variations of the dependent factor (y) is due to the variations in the independent factor (x). The coefficient of determination noted r^2 is the square of coefficient of relation

Multiple regression analysis: When more than one independent factor is used to explain the dependent factor a multiple regression analysis is used. The principle followed when conducting a multiple regression analysis is the same as when conducting a simple regression with the difference that more input factors (x 's) are used.

Residual Analysis. Residuals are estimates of experimental **error** obtained by subtracting the observed response from the predicted response. ... Residuals can be thought of as elements of variation unexplained by the fitted model.

FMEA is one of the special tools used during analyse/improve phase to verify whether the experiment being planned will have its effect and what are the chances of failure. Failure mode effect analysis is a tool for facilitating the process of predicting failures, planning preventive measures, estimating the cost of failure, and planning redundant systems or system responses to failures. The primary objective is to identify ways in which the process input variables x can fail and determine what effect that has on process outputs.

Types of FMEA:-

Design FMEA and the process FMEA are the two most commonly used tools by Six Sigma teams. A design FMEA is called DFMEA, analyses the design for potential failures that could arise as a result of weakness in the design of the product rather than due to its processes or manufacture. If both FMEAs are used the design FMEA is usually done first before production begins. The Process FMEA called as PFMEA is used to identify the failures caused by weakness in processes such as transaction, assembly or production processes. During the process FMEA it is presumed that the product meets the intent of the design.

RELIABILITY ANALYSIS:

In Minitab go to Stat → Reliability/Survival → Distribution Analysis → Parametric Distribution Analysis or Non parametric Distribution Analysis

What is censored data?

In reliability analysis, failure data frequently contain individual times to failure. For example, you might collect times to failure for units operating at a particular temperature. You might also collect samples of times to failure under different temperatures, or under different combinations of stress variables.

Sometimes you record exact times to failure. Other times, the exact times to failure of some test units are unknown. In this case, the data are called censored. Failure data are often censored in some way. Therefore, you may have any of the following types of observations:

- Exact times to failure
- Right-censored data
- Interval-censored data
- Left-censored data

Exact failure time data

The exact time that each item failed is known. For example, an engineer tests electric fans and records the exact time to failure of each fan.

Right-censored data

Failures are seen only if they occur before a particular time. A unit surviving longer than that time is considered a right-censored observation. Right-censored data are sometimes time-censored or failure-censored. Time censoring means that you perform the study for a specified period of time. All units still operating at the end of the study are time-censored. Time censoring is also known as Type I censoring on the right. Failure censoring means that you conduct the study until you observe a specified number of failures. Failure censoring is also known as Type II censoring on the right.

For example, suppose that an engineer tests five fan belts. Three fan belts fail in 67 hours, 76 hours, and 104 hours. The remaining two fan belts are still operating when the engineer stops the test at 110 hours. These last two fan belts are right-censored at 110 hours.

Right-censored data can be:

Singly-censored

All the test units operate for the same amount of time. Units surviving at the end of the study are considered censored data. Failed units are considered exact failures. Singly-censored data are more common in controlled studies.

Suppose you collect failure data until 12 units fail:

Item	Unit	Failure Time
1	Failed	18.5
2	Failed	20.5
3	Failed	22.0
4	Failed	23.5
5	Failed	24.3
6	Failed	25.0
7	Failed	25.6
8	Failed	26.3
9	Failed	27.0
10	Failed	29.0
11	Failed	32.0
12	Failed	33.0
13	Censored	33.0
14	Censored	33.0
15	Censored	33.0



Minitab interprets this data set as single censoring because the failure times of the censored items (units 13 - 15) are the same as the failure time of the 12th unit.

11. IMPROVE PHASE:

In many projects when data is analysed using the six sigma tools there are good chances that a solution will emerge in the analyse phase itself. This Solution may pertain to anything from say finding a new supplier, change of a process parameter, modification in a layout or to provide a new software application, a new poka-yoke gadget; however if the solution is known implementing the process to get the desired result requires a great deal of effort. Take for instance the change in the measuring equipment; it may take a few weeks or even a month or two to procure or develop a new system. Similarly if we want to switch to a new supplier, it may involve negotiation, capacity creation etc. in Some Projects we may not be lucky enough to arrive at the final solution at this stage. In this case we need to use some more tools such as the design of experiments (DOE).

The customer satisfaction index (CSI) is used to measure how pleased customers are about the services that they receive from the bank. The Quality engineer at the bank wants to know what factors have been causing the index to decrease recently to an unprecedented level. He isolated four factors as being potentially significant.

- Customers being kept on hold over the phone for too long.
- Long lines at the bank.
- Too much paper work.
- High Interest rates on short term loans

The quality engineer knows that some of these factors or their interactions have been detrimental to the CSI but he does not know which ones are. The experiment that can help him determine the significant factors is expensive and time consuming it involves changes in the interest rates offered by the bank , changes in the volume of paperwork, the opening of special fast lines at the cashiers and the hiring of new customer service personnel to speed up calls.

To limit the cost involved in the experiment, the engineer chooses to use DOE.

The Cpk index measures how capable a process is at meeting or exceeding customers' expectations; lately he computed Cpk has proved to be unacceptably low. The operations manager has isolated the four factors could contribute to poor performance.

- Raw material from different suppliers
- Machine operator
- Temperature in storage after processing
- Temperature in the trailers that transport the finished product to customers.

He wants to test these factors to determine which one of them or what combination is contributing to the lowering of the Cpk.He wants to isolate the significant factors to take corrective action. He chooses DOE to limit the cost involved in the testing.

Treatments :- For Example a level of temp at 65 degrees and level of time at 45 mts describes a treatment as it is related to the output of the yield.

Block :- A block is usually used when uncontrollable factors cannot be avoided during the DOE.

An Illustration of DOE:-

To Understand DOE let us begin with a simple example of Cooking rice. Consider the time taken to cook rice called response Y, is to be minimised. Time Y may depend on many factors such as the method of heating , the type of utensil used , the altitude of the location, the atmospheric temperature and humidity, the amount of water used , quality of rice grains etc. These are called factors and are usually denoted by X1,X2,.....Xn. some of these factors can be controlled during the experiment whereas others such as atmospheric temperature, altitude etc our beyond our control; therefore these factors are called as uncontrollable factors. X's are denoted as KPIV's and Y's are denoted as KPOV's . X's are classified as controllable and uncontrollable and are coded as C and U respectively. Uncontrollable factors are sometimes called as noise factors or common cause of variation.

	<u>KPIV's (X's)</u>	<u>C/U</u>	<u>Process</u>	<u>KPOV's (Y's)</u>	
<u>X1</u>	<u>Cooking Method</u>	<u>C</u>	<u>Rice Cooking</u>	<u>Cooking time</u>	<u>Y1</u>
<u>X2</u>	<u>Heating Method</u>	<u>C</u>			
<u>X3</u>	<u>Amount of Water</u>	<u>C</u>			
<u>X4</u>	<u>Atmospheric Temperature</u>	<u>U</u>			
<u>X5</u>	<u>Atomospheric Humidity</u>	<u>U</u>			
<u>X6</u>	<u>Altitude</u>	<u>U</u>			

We select open utensil as low level and pressure cooker as high level ; thus there are 2 factors and each factor has 2 level ; Experiments are denoted as L^F where L is the number of Levels and F is the number of Factors; therefore the experiment is 2² experiment totally the possible number of Combinations of factors and levels is four when all possible combinations are run the experiment is called as full factorial experiment.

DOE (Design of Experiments)

Design of experiments (DOE) is a key tool in the Six Sigma methodology. DOEs help improve processes in a quantum fashion, and is an approach for effectively and efficiently exploring the cause and effect relationship between numerous process variables (Xs) and the output or process performance variable (Y).

DOE helps in the following ways:

- The DOE helps in identifying the vital few sources of variation.
- DOEs quantify the effects of the important Xs including their interactions.
- Performing DOE produces an equation that quantifies the relationship between the process Xs and the process output Y, thus enabling the transition to the $Y=f(x)$ philosophy of Six Sigma.

There are many types of DOEs which can be applied to a particular problem based on various planning factors and the outcome desired at the end of the analysis. This article will try and explain the analysis strategy that a Black Belt can undertake for Resolution III and IV Design of Experiments.

Though a full factorial design is the most desirable design wherein one could gather information on all the main effects, two way interactions, three way interactions and other higher order interactions are very unpractical to run due to the prohibitive size of the experiments. For a design of seven factors at two levels one would have to complete 128 runs.

Fractional factorial designs are good alternatives to a full factorial design, especially in the initial screening stage of a project. The same seven factors could be tested in either 8 runs or 16 runs or 32 runs with the loss of certain information.

Let us consider the example of an experimenter who is trying to optimize the production of organic foods. After screening to determine the factors that are significant for his experiment he narrows the main factors that affect the production of fruits to "light" and "water". He wants to optimize the time that it takes to produce the fruits. He defines optimum as the minimum time necessary to yield comestible fruits.

To conduct his experiment he runs several tests combining the two factors (water and light) at different levels. To minimize the cost of experiments he decides to use only 2 levels of the factors: high and low. In this case we will have two factors and two levels therefore the number of runs will be $2^2=4$. After conducting observations he obtains the results tabulated in the table below.

Factors	Response
Water –High Light High	10 days
Water high – Light low	20 days
Water low – Light high	15 days
Water low – Light low	25 days

The coded form of the table above is given below:

Water	Light	Response in Days
+	+	10
+	-	20
-	+	15
-	-	25

The reading we make of the tables is that when the level of light is high and the level of water is high, it takes the fruits 10 days of time to become comestible and when the level of water is low and the level of light is low it takes 25 days for the fruit to become comestible. So the factors water and light has a significant effect on the response variable which is days in this experiment. You can also illustrate this experiment using Minitab 17.

Example: In Minitab open file Yield.mtw and take the 3 levels time, temperature, catalyst which will affect the yield of the chemical. Go to Stat-> DOE->Factorial->Create Factorial Design Select 2 level factorial and number of factors as 2 and if you want to repeat the experiment 2 times choose the number of replicates as 2 or 3 etc. After Designing the factorial select the option Analyse Factorial Design to find out which factor time, temperature or catalyst gives the maximum yield of the chemical and in response column give yield as the response factor and select pareto graph the graph will show which factors contribute to high yield. By selecting terms option you can remove the non-contributing factors by double clicking on them and get the result of the contributing factors which give the maximum yield.

Design of Experiments using Factorial Method

How to Plan a Six Sigma 2^k Factorial Experiment

Like in most other endeavours, time spent planning for Six Sigma is rewarded with better results in a shorter period of time. Planning 2^k factorial experiments follows a simple pattern: choosing the factors you want to experiment with, establishing the high and low levels for those factors, and creating the coded design matrix.

Select the experiment factors

The first thing to do is identify the input variables, the X s that you'll include in your experimental investigation. The factors you include should be potential contributors to the output Y you're investigating and should be ones that are critical. How many factors you want in your experiment guides you in choosing the right experimental design. 2^k factorial experiments work best when you have between two and five X s.

If you have over five X s in your experiment, full 2^k factorial experiments become relatively inefficient and can be replaced with pared down versions called *fractional factorials*, or with other screening designs. One good strategy is to include all potential X s in a first screening experiment — even the ones you're sceptical about.

You then use the analysis of the experiment results to tell you objectively, without any guessing, which variables to keep pursuing and which ones to set aside. Remember, in Six Sigma, you let the data do the talking.

Plackett-Burman experiment designs are an advanced method you may hear about for efficiently screening dozens of potential X s. Although they don't reveal all the detailed knowledge provided by a 2^k factorial design, Plackett-Burman experiments quickly identify which experimental variables are *active* in your system or process. You then follow these screening studies up with more detailed characterization experiments.

Set the factor levels

2^k factorial experiments all have one thing in common: They use only two levels for each input factor. For each X in your experiment, you select a high and a low value that bound the scope of your investigation.

For example, suppose you're working to improve an ice cream carton filling process. Each filled half-gallon carton needs to weigh between 1,235 and 1,290 grams. Your Six Sigma work up to this point has identified ice cream flavour, the time setting on the filling machine, and the pressure setting on the filling machine as possible critical X s to the Y output of weight.

For each of these three factors, you need to select a high and a low value for your experiment. With only two values for each factor, you want to select high and low values that bracket the expected operating range for each variable. For the

ice cream flavour variable, for example, you may select vanilla and strawberry to book-end the range of possible ice cream consistencies.

Variable	Symbol	Low Setting	High Setting
Ice cream flavour	X_1	Vanilla	Strawberry
Fill time (seconds)	X_2	0.5	1.1
Pressure (psi)	X_3	120	140

2^k experiments are intended to provide knowledge only *within* the bounds of your chosen variable settings. Be careful not to put too much credence on information inferred outside these original boundaries.

Explore experimental codes and the design matrix

With the experiment variables selected and their low and high levels set, you're ready to outline the plan for the runs of your experiment. For 2^k factorial experiments, you have 2^k number of unique runs, where k is the number of variables included in your experiment.

For the ice cream carton filler example, then, you have $2^3 = 2 \times 2 \times 2 = 8$ runs in the experiment because you have three input variables. For an experiment with two variables, you have $2^2 = 2 \times 2 = 4$ runs, and so on.

Each of these 2^k experimental runs corresponds to a unique combination of the variable settings. In a full 2^k factorial experiment, you conduct a run or cycle of your experiment at each of these unique combinations of factor settings. In a two-factor, two-level experiment, the four unique setting combinations are with

- Both factors at their low setting
- The first factor at its high setting and the second factor at its low setting
- The first factor at its low setting and the second factor at its high setting
- Both factors at their high setting

These groupings are the only ways that these two factors can combine. For a three-factor experiment, eight such unique variable setting combinations exist.

A quick way to create a complete table of an experiment's run combinations is to create a table called the *coded design matrix*. Make a column for each of the experiment variables and a row for each of the 2^k runs. Using -1 s as code for the low variable settings and $+1$ s as code for the high settings, fill in the left-most variable column cells with alternating -1 s and $+1$ s.

Repeat the process with the next column to the right, this time with alternating *pairs* of -1 s and $+1$ s. Fill in the next column to the right with alternating *quadruplets* of -1 s and $+1$ s, and so on, repeating this process from left to right until, in the right-most column, you have the first half of the runs marked as -1 s and the bottom half listed as $+1$ s.

Run	X_1	X_2	X_3
1	-1	-1	-1
2	+1	-1	-1
3	-1	+1	-1
4	+1	+1	-1

5	-1	-1	+1
6	+1	-1	+1
7	-1	+1	+1
8	+1	+1	+1

Remember that these three factors are coded values; when you see a -1 under the X_1 column, it really represents a discrete value, such as “vanilla” in the ice cream experiment; a +1 really represents the other value, like “strawberry.”

12. CONTROL PHASE

Deliverables are Control Charts, Control Plan, Poka-Yoke, Project Closing.

Many Six Sigma belts and teams do a good job in the analyse and improve phase ; however the improvements do not sustain over a long period as certain important requirements of the control phase are not met. Typical causes of the failure to sustain improvements are as follows.

- Inadequate training to operators on the changed process or equipment.
- Validation of improvement for a short period of time or sample sizes.
- Inadequate control plan or audits
- Process improvement is qualified without consideration to ‘ noise factors’ which eventually effect the process adversely.

Maintaining Gains

If your project has successfully entered the Monitor and control phase you have to make sure that you maintain this stability, here are some suggested ways to protect hard won gains.

- Policy Changes – Which corporate policies should be changed as a result of the project – have some policies become obsolete are new policies needed
- New Standards- Did the project bring the organization into compliance with a standard. If so having the company adopt the standard might prevent backsliding, there are standards like ISO, ANSI, ASTM etc. from which you can take your pick.
- Modify procedures-procedures describe the way things are supposed to be done , since the project produced better results we can find the best suited procedure which has resulted in better modification or change.
- Change Manufacturing Planning-An organizations manufacturing plan describe in detail how a product is to be processed and produced. Often the Six Sigma team will discover better way of doing things.
- Modify training-personnel need to become familiar with the new way of doing things. Be sure all employees are retrained. Evaluate the existing training materials and revise them as necessary.

12.1 Tools and techniques useful for Control Planning

- Project Planning-Many of the six sigma tools and techniques used during the define, measure, analyse and improve phases can also be used to develop a control plan.
- Force Field diagram- A force Field diagram can be very useful at this point. Show the Forces that will push to undo the changes and create counterforces that will maintain them.

12.2 Preparing the Process Control Plan

Process Control plan should be prepared for each key process. The plans should be prepared by teams of people who understand the process. The team should begin with creating a flow chart of the process using the process elements. The flow chart will show how the process elements relate to each other which will help in the selection of control point. It will also show the point of delivery to the customer which is usually an important control point.

Process Audits

The requirement for all processes should be documented. A process audit check list should be prepared and used to determine the condition of the Process prior to production.

Example :- A production process is supposed to generate pistons that weight 10 kg each. Samples of 5 pistons are taken at time intervals of 45 min for inspection and the mean weights of the samples are plotted on the control chart from the output we see that every sample mean has fallen within the control limits until sample 33 was plotted. Since the mean of the sample is way outside control limits something special must have happened to cause that sample way out of control. The production process needs to be stopped so that the special causes of variation can be investigated and

corrections made. The cause of that variation can be anything from a defective or unattended machine to an operators mistake. Once the correction are made the process should be back in control.

12.3 SPC (Statistical Process Control)

Control Charts

Variable Charts

The Xbar and R Chart is the flagship of the control charts. It is called a variables chart because the data to be plotted result from measurement on a variable on continuous scale. This type of measurement occurs when for each pair of values there are infinite number of values between them. On an inch scale for instance there are infinite number of values between 1.250 and 1.251, values such as 1.2503, 1.2508 and so on. The Xbar and S chart is another variable control chart with this chart sample standard deviation s is used to indicate dispersion instead of the range. The Standard deviation is a better measure of spread especially if the sample size is large so this chart is somewhat more precise than the xbar and R chart. The Xbar and S chart should be used when calculation of the standard deviation is feasible. Users who are comfortable with the standard deviation function on a handheld calculator will be able to calculate S almost as easily as R . The Xbar and S chart is also the preferred chart when the chart is constructed using a software package.

XBar and R Control Charts:

To get Control charts in Minitab please do the following open Minitab → Click on Assistant? → Click on Control Charts → You will get flow chart of various types of Control Charts → Select the data from the sheet column → and then specify estimate control limits from the data → you will get the different types of control charts which can determine whether the process is in control.

Control Limits for c and u charts respectively

Control Limit for c chart

$$UCL = \bar{c} + 3 * \sqrt{\bar{c}}$$

Where \bar{c} = centre line

$$LCL = \bar{c} - 3 * \sqrt{\bar{c}}$$

Control Limits for u chart

$$UCL = \bar{\mu} + 3 \sqrt{\bar{\mu}/n}$$

Where centre line = $\bar{\mu}$

$$LCL = \bar{\mu} - 3 \sqrt{\bar{\mu}/n}$$

Attribute Charts

Attribute Charts are used for Count Data

In attribute Control Charts every item would be either in one or two categories such as “good” or “bad”.

P control charts

The P Chart is used to chart Binary Data where each item is in one or two categories. This chart would be the appropriate chart for plotting number of defectives.

$$UCL \text{ for P Chart} = \bar{p} + 3 \sqrt{\bar{p}(1-\bar{p})/n}$$

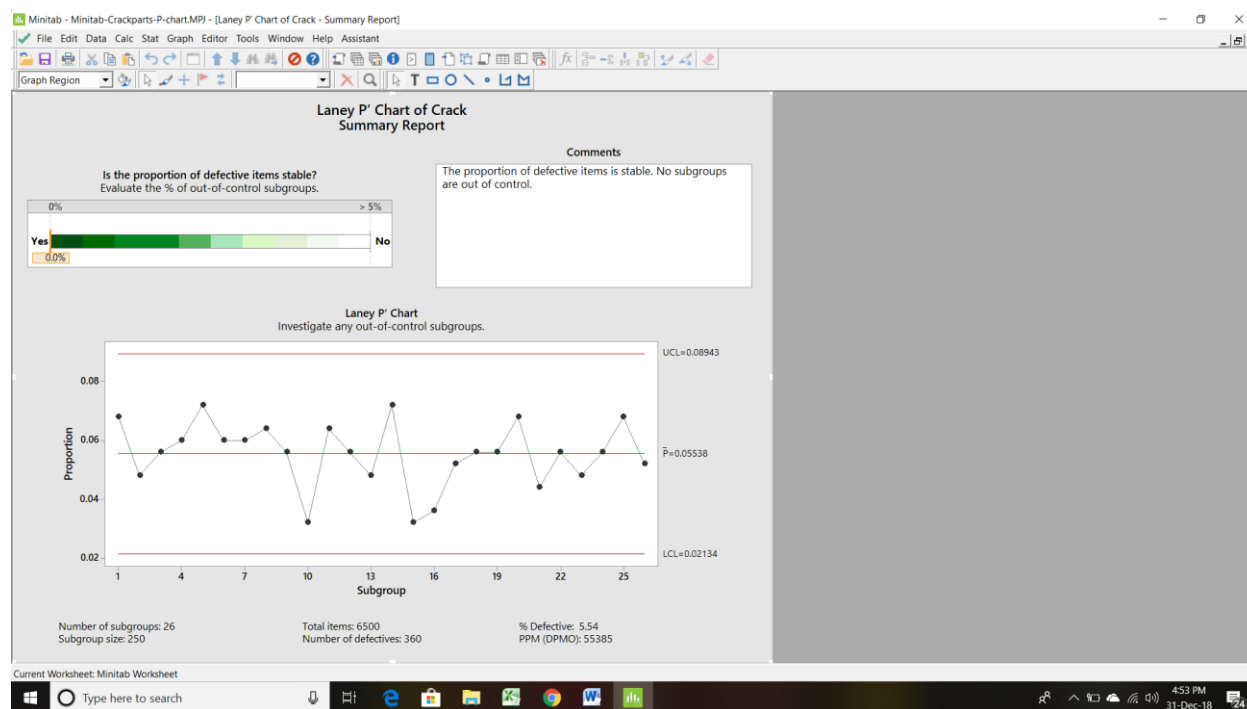
$$LCL \text{ for P Chart} = \bar{p} - 3 \sqrt{\bar{p}(1-\bar{p})/n}$$

Where n = Sum of Sample Sizes / Number of samples.

And $\bar{p} = \text{Sum of Discrepancies} / \text{Sum of sample Sizes}$

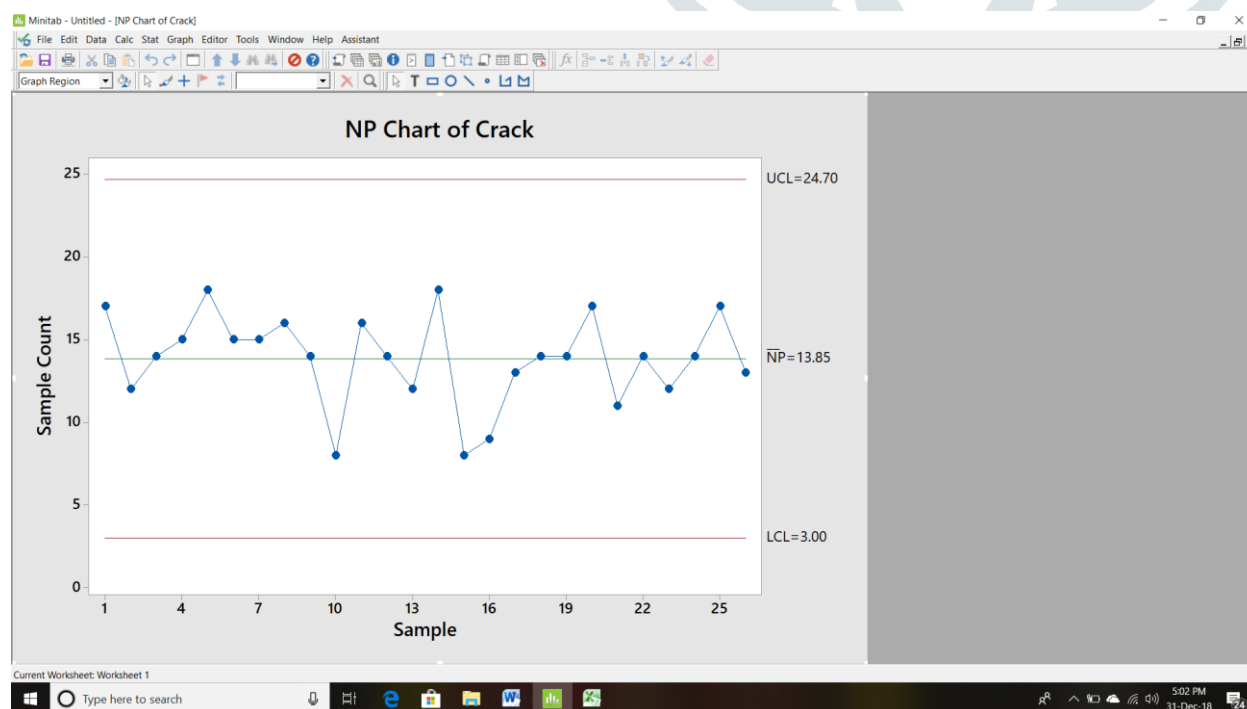
Analyse the P Chart for Process Control: - If all of the plotted data on the p-Chart are within control limits and randomly distributed around the centre line The defective rate for the process is stable and predictable.

In statistical quality control, the p-chart is a type of control chart used to monitor the proportion of nonconforming units in a sample, where the sample proportion nonconforming is defined as the ratio of the number of nonconforming units to the sample size, n. Sometimes you may see the control limits varying that is because the subgroup sizes are varying in the output below show the subgroup sizes are constant so the control limit are in a straight line. In the Graph below the proportion of defective items are stable but the average proportion of cracked gears is 0.0554 or 5.54 percent however 5.54 percent is very high rejection and requires immediate corrective action, hence a stable process does not mean that it is a good process.



Np Control Charts

If defectives are being counted and the sample size remains Constant the np chart can be used the np chart also has definite formulas to Calculate the upper and lower control limits.



U Control Chart Charts: The u Chart is appropriate when defects rather than Defectives are being counted for examples while inspecting panes of glass in which defects include bubbles, scratches, chips, waves etc.

There are also standard formulas to calculate the upper and lower control limit of U control Charts.

C Control Charts

When defects are counted and the sample size is constant the c chart maybe used used instead of U CHART.

Here also there are standard formulas to calculate the upper and lower control limit of C chart.

A Process is said to be out of Control if

- 1] One or more data points fall outside the control limit.
- 2] Seven Consecutive data points increasing or decreasing.
- 3] Eight consecutive data points are on one side of average.
- 4] Fourteen Consecutive data points alternating up and down.

EWMA Control Charts in Many cases individual control chart (I Chart) will give results comparable to the EWMA control chart when this is the case it is better to opt for the simpler I Chart.

CUSUM chart

A type of time-weighted control chart that displays the cumulative sums (CUSUMs) of the deviations of each sample value from the target value. Because it is cumulative, even minor drifting in the process mean will lead to steadily increasing (or decreasing) cumulative deviation values. Therefore, this chart is especially useful in detecting processes that are slowly slipping away from the target value due to machine wear, calibration issues, etc. The points on the CUSUM chart should fluctuate randomly around zero. If a trend develops upwards or downwards, it should be considered as evidence that the process mean has shifted, and you should look for special causes.

Minitab generates two kinds of CUSUMs:

- ☑ Two one-sided CUSUMs (the default). The upper CUSUM detects upward shifts in the level of the process and the lower CUSUM detects downward shifts. This chart uses control limits (UCL and LCL) to determine when an out-of-control situation has occurred.
- ☑ One two-sided CUSUM. This chart uses a V-mask, rather than control limits, to determine when an out-of-control situation has occurred. The V-mask standardizes the deviations from the target value, and plots the deviations from this value.

For example, a manufacturer of centrifuge rotors wants to track the diameter of all rotors produced during a week. The diameters must be close to the target because even small shifts cause problems. The following CUSUM chart (two one-sided CUSUMs) resulted.

Minitab Navigation:- Stat→Control Charts→Time Weighted Charts→Cusum Chart this Chart is to detect small Shifts which cause variation.

Natural variation or common cause variation cannot be eliminated. No 2 snowflakes are identical. The best we can hope for is to reduce it to a level where we can no longer measure it. This is probably not practical. Almost every book on process control will make a statement like this, "In any production process, regardless of how well designed or carefully maintained it is, a certain amount of inherent or natural variability will always exist.

If we really work to reduce it, we might use some of the traditional quality tools, such as a fishbone chart. Then use a Pareto chart to determine the influence of different causes and attack them one by one. However, when we have eliminated them all, time itself may be one cause we cannot eliminate.

Variation can be classified as common cause variation or special cause variation. Common cause variation is due to natural variation of the process; that is variation due to the way process was designed. An Example of common cause variation is the variation that might be seen by having several people working in the process. Each Person might do things slightly differently. Common cause of variation are expected, predicatable and random.

Special cause variation that is due to assignable causes. An example of special cause variation is the variation that might result if someone untrained allowed to work in the process. Special cause variation is variation that can be assigned a reason. A Control chart like I-MR,X-bar etc can detect special cause variation by points falling below the lower specification limit or higher specification limit. Special cause of variation are also unexpected, unpredictable, not normal, not random.

12.4 Poka Yoke are Mistake proofing devices which can be used to reduce variation in a process for example introduction of speed bumper in a junction which has lots of accidents or fitting a sophisticated device in the factory to inform of gas leaks in a factory etc.

The Deliverables of the Control Phase are:

- **Solution Implementation Plan.**
- **Successful Solution**
- **Process Control Plan.**

The four techniques used for process control plans are:

- Standardization
- Documentation
- Monitoring Plan
- Response Plan

Standardization:

Standardization of the “should-be” process steps is required to ensure all responsible for execution have the same understanding. Standardizing the should-be process helps answer queries like:

- What are the steps in the process?
- Who does these steps in the process and when?
- Where more detailed work instructions can be found?

Documentation:

Documentation is a necessary step to insure that the learning gained via improvement is institutionalized and shared across the team by having it documented with proper work procedures. Often the live processes have a tendency to evolve in an ad-hoc manner. How to accomplish each process activity is usually left up to the individuals and thus, much of the organizational knowledge resides only in the minds of all those responsible for execution. Procedure: A procedure is the documented sequence of steps & other instructions necessary to carry out an activity for a process.

Monitoring Plan:

Monitoring: It helps detect changes as and when they occur in the process and assure that improvements continue to hold for us to be able to meet customer requirements over a period of time. While observing a process a monitoring plan helps define:

- Key process and output measures for ongoing measurement of the improved process
- When data is to be collected and at how often
- Define the method for gathering, recording, and reporting data on the measures

Response Plan:

Response plan helps identify the next steps on what needs to be done if one detects a change in the process while monitoring. For each of the measure in the monitoring plan, the response plan helps define:

- What actions will be taken for an out-of-control event occurrence with a timeframe for the action
- Who takes action based on the monitoring data
- Where to find trouble-shooting procedures to fix problem

The key elements of a full scale implementation plan are:

- Clear Objectives
- Pilot Learning's Incorporated
- Implementation Milestones
- Resource Needs
- Influence Strategy
- Implementation Budget
- Process Control Plan
- Process Documentation

A Solution Plan Should include the following Components:

1. Potential Problem Analysis
2. Solution Implementation Schedule.
3. Training Plan.
4. Communication Plan.
5. Costs and Benefits.
6. Transfer to Owner Plan

Signature Six Sigma Application Table

Common/ Advanced	Signature Six Sigma Applications	Applicable to all Projects	Applicable to Most projects	Usually Not done in kaizen
	Process mapping	NO		
	Root cause analysis	NO		
	Measurement System Analysis	Yes	Yes	NO
	Capability Analysis	Yes		Done in kaizen
	Control Chart	Yes	Yes	Done in Kaizen
	Hypothesis Testing		Yes	Not done in kaizen
	Design of Experiments	Yes	Yes	Not Done in Kaizen
	Failure Mode & Effect Analysis	NO	Yes	Not Done in Kaizen
	Voice of the Customer	Yes	Yes	Not done in kaizen
	Ideation	Yes	Yes	Yes
	Regression	Yes	Yes	Not Done in Kaizen
	Advanced Statistics	Yes	Yes	Not Done in Kaizen

Kaizen Event Action List :-

Business Unit -----

Plant Name ----- Area/item# :- ----- Date ----- Page ----- of -----

Item	Problem to be Resolved	Action Needed	Responsibility	Due Date	Percentage Complete	TIMWOOD TYPE
1						
2						
3						
4						
5						
6						
7						
8						
9						

10									
TIMWOOD = Transportation, Inventory, Motion, Waiting, Overproduction, Overprocessing, Defects									

Waste Observation and Improvement Data Sheet :-

Gemba Waste Observations and Improvement Suggestions					
Date:-		Team Work Area :			
Item	Waste, Defect or Flow Disruption	Suggested Improvement Area	TIMWOOD type	Assigned To	Due Date
TIMWOOD = Transportation, Inventory, Motion, Waiting, Overproduction, Overprocessing, Defects					

Define Phase Questions – Management Perspective

Pre-Project Launch

1. Who is the customer and how will this project benefit the customer?
2. What is the impact on the business?
3. What is your problem or opportunity statement- What are you trying to fix or avoid?
4. How will the project help the business?
5. Has the project been narrowed to complete in an appropriate time frame ?
6. Are there sufficient resources in place to ensure project success?
7. Taking into account the opportunity statement, goals and scope outlined what is the teams charter?

Post Project Launch

8. Describe what you learned from the as-is process mapping.
9. What are the critical to customer requirements?
 - a. How did you determine them?
 - b. How are you measuring them?
 - c. How good is the measurement system?
 - d. Have you verified them with the customer?
 - e. Do the current specifications reflect them?
10. What are the critical to business requirements?
11. What are the quick wins?

12. Has the team put together a communication plan to let relevant people in the organization know what is happening and to regularly solicit inputs?
13. What are the next steps?

Measure Phase Questions – Management perspective

1. What are the key goals for this project?
2. What is the impact on the business? What strategic objectives will be impacted and how?

Questions about Metrics and Measurement

3. What critical to quality metric has been identified and how do they know the customer is impacted by them?
4. What critical to process metrics are important to the business?
5. Which ctq's / ctp's does this project focus on ? why?
6. How do you know the data collected are representative of the process?
7. Is the team rigorously following the appropriate methodology?
8. Do they adequate resources to complete the project?

Questions relevant to six sigma teams

9. What have they learned about the source of the variation from the initial data gathered? Is the process in control?
10. What is the current process sigma?

Questions for all teams

11. Did the team re-scope the project as a result of the measurement phase?
12. What are the next steps?

Questions in the Analyse Phase

1. What is the statement of the problem in terms of cause and effect?
2. What are the vital few factors causing variation in the output?
3. What analysis exist to verify the root cause?
4. Are you being open minded and creative in your team thinking about causes?

Questions mostly relevant to six sigma teams

5. How much of the problem is explained with the vital X's ? How much unexplained variation exists?
6. Are the vital few X's statistically significant ? are the effects of practical significance?

Questions for all teams

7. Are any of the current learnings transferrable across the business?
8. Do you have adequate resources to complete the project?
9. What are the next steps?

Improve phase – Management Perspective

Questions for all teams

1. What is the alternative solutions did the team consider for solving the problem?
2. How was the current solution selected?

3. What are the expected benefits from the solution? What are costs?
4. How will you communicate the solution to the organization?
5. Will the team make use of pilot project teams or do they have a complete fix?
6. Do you have adequate resources to complete the project?
7. What are the next steps?

Control Phase – Management Perspective

Questions for all teams

1. Did the team conduct a pilot of the new process? What was learned and changed as a result of the pilot?
2. What is the new process result(Sigma)? Has the project achieved its goals ? has the team demonstrated the new result with the data?
3. Is the learning transferrable across the business? What is the action plan for spreading the best practice? What lessons did the team learn?
4. What are the variables being monitored to assure that the process performance improvement continues?
5. What training has been conducted to assure the new process runs as expected? How are new habits or behaviours being learned?
6. What are the financial results of the project? How they have been calculated and documented? What change is the customer seeing?
7. What are next steps?

13.SIX SIGMA PROJECT EVALUATIONS

Project name	Date of Assessment		
Black belt	Master Black Belt		
Weighted overall project score	Project Number		
Criteria	Score	Weight	Weighted Score
Sponsorship			
Benefits (Specify Main Beneficiary)			
External Customer			
Shareholder			
Employee or Internal Customer			
Other (e.g. Supplier, Environment)			
Availability of resources other than team			
Scope in terms of black belt effort			
Deliverable			
Time to Complete			
Team membership			
Project Charter			
Value of Six Sigma Approach			
TOTAL (Sum of Weighted Score Column)			
Note any criterion scores of zero must be addressed before the project is approved			
Weighted Score = Project Score for each criterion times the weight			
1.0 Sponsor Ship			
Score			
9	Director-level Sponsor Identified , duties specified, Sufficient time committed and scheduled		
3	Director-Level sponsor identified duties specified and sufficient time committed but not scheduled		
1	Willing Director-Level sponsor who has accepted charter statement		
0	Director-level sponsor not identified or sponsor has not accepted the charter		
2.0 Stakeholder Benefits			
2.1 Stakeholder: External Customer			
2.1.1 Customer Satisfaction			
Score	Interpretation		
9	Substantial and statistically significant increase in overall		

	customer satisfaction or loyalty
3	Substantial and statistically Significant increase in major subcategory of customer satisfaction
1	Substantial and statistically Significant increase in focused area of customer satisfaction
0	Unclear or non-existent customer satisfaction impact
2.1.2 Quality Improvement (CTQ)	
Score	Interpretation
9	10 x or Greater is critical to quality (CTQ) Metric
5	5 x to 10 x improvement in CTQ Metric
3	2x to 5 x improvement in CTQ Metric
1	Statistically significant improvement in CTQ metric but less than 2 x magnitude
0	Project's impact on CTQ metrics undefined or unclear
2.2 Stakeholder Shareholder	
Score	Interpretation
9	Hard Net Savings (Budget or Bid Model Change greater than \$ 500 K Excellent ROI
5	Hard Net Savings Between \$ 150 K and \$ 500 K Excellent ROI
3	Hard net savings between \$ 50 K and \$ 150 K or cost avoidance greater than \$ 500 K Good ROI
1	Hard savings of at least \$ 50 K or cost avoidance between \$ 150 K and \$ 500 K Acceptable ROI
0	Project claims a financial benefit but has hard savings less than \$ 50K cost avoidance less than \$ 150 K it becomes unclear financial benefits achieved
2.2.2 Cycle Time Reduction	
Score	Interpretation
9	Cycle Time reduction that improves revenue , bid model or budget by more than \$ 500 K Excellent ROI
5	Cycle Time reduction that improves revenue , bid model or budget by \$ 150 K to \$ 500 K Excellent ROI
3	Cycle Time reduction that improves revenue , bid model or budget by \$ 50 K to \$ 150 K Creates a cost avoidance of more than \$ 500 K . Good ROI

1	Cycle time reduction that results in cost avoidance between \$ 150 K and \$ 500 K acceptable ROI
0	Project Claims a cycle time improvement but has hard savings less than \$ 50 K , cost avoidance less than \$ 150 K or unclear financial benefits from the improvement in cycle time
2.2.3 Revenue Enhancement	
Score	Interpretation
9	Significant increase in Revenue , Excellent ROI
3	Moderate Increase in Revenue , Good ROI
1	Increase in revenue with acceptable ROI
0	Unclear or non-existent revenue impact
2.3 Stake Holder , Employee or Internal Customer	
2.3.1 Employee Satisfaction	
Score	Interpretation
9	Substantial and statistically significant increase in overall employee satisfaction
3	Substantial and statistically significant increase in major element of employee satisfaction
1	Substantial and statistically significant increase in focused area of employee satisfaction
0	Unclear or non-existent employee satisfaction impact
2.4 Stakeholder : Other	
2.4.1 Specify Stakeholder	
Benefits	
Score	Interpretation
9	
5	
3	
1	
0	Unclear or Non-Existent Benefits
3.0 Availability of resources other than team	
Score	Interpretation
9	Needed Resources available when needed

3	Limited or low priority access to resources
0	Resources not available or excessive restrictions on access to resources
4.0 Scope in terms of Black Belt Effort	
Score	Interpretation
9	Projected returns substantially exceed required returns
3	Projected return exceeds required return
1	Projected Return approximately equals required return
0	Projected return not commensurate with required return
5.0 Deliverable (Scope)	
Score	Interpretation
9	New or improved process, product or service to be created is clearly and completely defined
3	New or improved process, product or service to be created is defined
0	Deliverable is poorly or incorrectly defined for example a "deliverable" that is really a tool such as process map
6.0 Time to Complete	
Score	Interpretation
9	Results realized in less than 3 months
3	Results realized in between 3 and 6 months
1	Results realized in 7 to 12 months
0	Results take more than 12 months to be realized
7.0 Team membership	
Score	Interpretation
9	Correct Team members recruited and time commitments scheduled
3	Correct team members recruited time committed but not scheduled
1	Correct team members recruited
0	Team members not recruited or not available
8.0 Project Charter	
Score	Interpretation
9	All elements of the project charter are complete and acceptable. Linkage between project activities and deliverable is clear
3	Project charter acceptable with minor modifications
0	Project charter requires major revisions

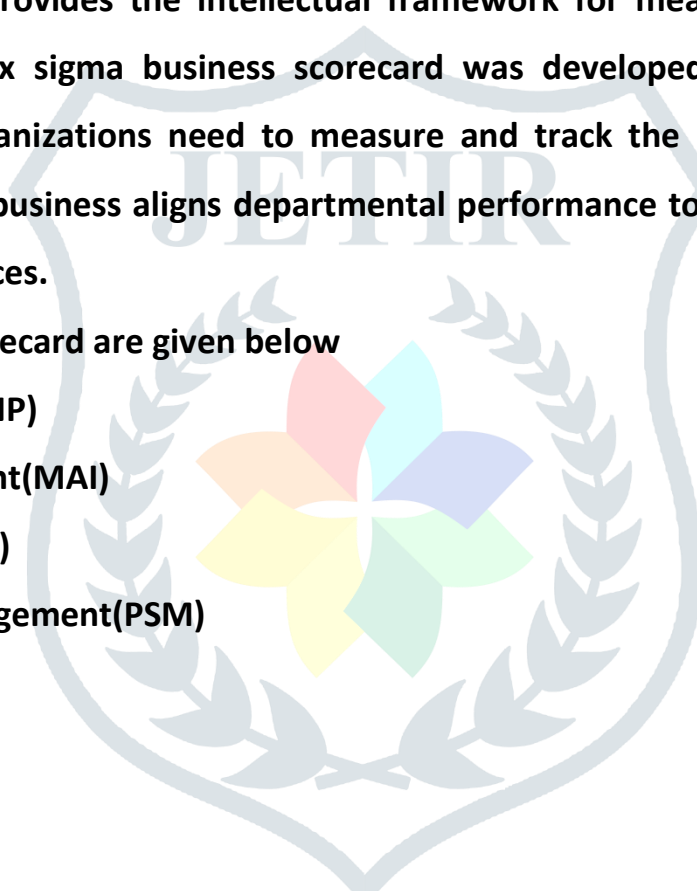
9.0 Value of Six Sigma Approach (DMAIC or Equivalent)	
9	Six Sigma approach essential to success of the project. Black Belt/Green Belt skill set required for success
3	Six Sigma approach helpful but not essential. Black Belt/Green Belt skill set can be applied
0	Usefulness of Six Sigma Approach not apparent. Specific Black Belt or Green Belt Skills are not necessary.

14.SIX SIGMA BUSINESS SCORECARD

The six sigma business scorecard provides the intellectual framework for measuring a comprehensive view of the organizational performance. The six sigma business scorecard was developed by Motorola University Consultant Praveen Gupta, in response to organizations need to measure and track the progress of all aspects of a business employing six sigma. The six sigma business aligns departmental performance to the customer expectations of better, faster and cheaper products or services.

The seven elements of six sigma scorecard are given below

- Leadership and profitability(LNP)
- Management and Improvement(MAI)
- Employees and innovation(EAI)
- Purchasing and Supplier management(PSM)
- Operational Execution(OPE)
- Sales and Distribution(SND)
- Service and growth(SAG)



Category	Objective	Areas of Measurement Metric
Leadership and Profitability(LNP)	Lead Company to wellness and Profitability	Communication Inspiration Planning Accuracy Community Perception Employee Perception Employee Recognition Compensation Asset utilization Return on investment Debt to Equity Ratio Profitability Shareholder’s Value Growth

Management and Improvement(MAI)	Drive Dramatic Improvement	Goal Setting Rate of Improvement Planning for Improvement
Employees and Innovation(EAI)	Involve Employees Intellectually	Innovative Recommendations per employee Investment per employee Number of patents or publications per employee
Purchasing and Supplier Management(PSM)	Reduce cost of goods or services	Material Acceptance Total spent/sale Supplier Defect rates (Sigma) Cost of Goods/Services Sold
Operational Execution	Achieve Performance Excellence	Operational Cycle Time Process Defect Rate (Sigma) Customer Defects, Total Defects
Sales and Distribution(SND)	Manager Customer relations and Generate revenue	Number of Inquiries, New Business Dollars/Total Sales Profit margin(\$)/Sales
Service and growth(SAG)	Gain Competitive advantage and grow	Customer satisfaction Customer retention Repeat business (\$)/Total Sales New Product or services patents or trademark

15. KEY Performance Indicators (KPI's):

Are measurements defined to show the state of the strategic goals or targets the organization has set for itself.

What is Metrics: Combination of two or more measures used to compare s/w processes, projects and products. Set of Metrics derived from data collected from past projects at the organization level. These baselines are used as a basis to arrive at project specific quality goals for KPI's.

Effort Variation

This metric is the difference between Estimated and Actual effort as compared against the Estimated Effort.

Objective: The objective of this metric is to study the distribution of workload by Stage and to reduce the deviation of the actual effort expended as against the estimated effort.

When it should be measured: It should be measured at overall project level, Stage level and Task level (Process level and Sub process level for SDLC stages)

Input/Measure	Formula	Example
Actual Effort Estimated Effort	$(\text{Actual Effort} - \text{Estimated Effort}) / (\text{Estimated Effort}) * 100$	Estimated Effort (in person days) = 5 Actual Effort (in person days) = 7 Effort Variation% = $=(7-5)/5 * 100 = 40\%$

Schedule Variation

This metric is the ratio of difference between the Actual End Date and Planned End Date Vs difference between Planned End Date and Planned Start Date for the project.

Objective: The objective of this metric is to reduce the schedule variation by tracking it from beginning stage of the project through the end of the project, thereby reducing time overruns. Schedule Variation metric is mainly used as an indicator for capability to meet milestones

When it should be measured: It should be measured at overall project level, Stage level and Task level (Process level and Sub process level for SDLC stages). Schedule variation need to be calculated only when the stage is completed.

Input/Measure	Formula	Example
Actual End Date Planned Start Date Planned End Date	$\frac{((\text{Actual End date} - \text{Planned End date}) / (\text{Planned End date} - \text{Planned Start date})) * 100}$	Planned Start Date = 1-Jan-13 Planned End Date = 31-Jan-13 Actual Start Date = 2-Jan-13 Actual End Date = 1-Feb-13 Schedule Variation % = $(1 / 31) * 100 = 3.22\%$

Duration Variation

This metric is the difference between Total Planned Vs Actual duration for the project

Objective: The objective of this metric is same as schedule variation metrics i.e. to reduce the duration variation by tracking it from beginning stage of the project through the end of the project, thereby reducing time overruns. Why we may need both Duration Variation and Schedule Variation is that at times the task/stage may be finished within Planned Duration (Delayed start compensated with delayed finish to the same extent as days it started late.) whereas it might have exceeded the committed deadline, which is ultimately the schedule slippage.

When it should be measured: It should be measured at overall project level, Stage level and Task level (Process level and Sub process level for SDLC stages). Duration variation need to be calculated only when the stage is completed.

Input/Measure	Formula	Example
Actual End Date Actual Start Date Planned Start Date Planned End Date	$\frac{((\text{Actual End Date} - \text{Actual Start Date}) - (\text{Planned End Date} - \text{Planned Start Date})) / (\text{Planned End Date} - \text{Planned Start Date}) * 100}$	Planned Start Date - 1-Jan-13 Planned End Date - 31-Jan-13 Actual Start Date - 2-Jan-13 Actual End Date - 1-Feb-13 Duration Variation % = $((31 - 31) / 31) * 100 = 0\%$

Work Efficiency Index

Load Work Efficiency Index is the ratio of percentage of work completed to percentage of effort expended.

Objective: This metric is particularly important and mandatory for fixed bid/fixed duration projects, and close monitoring is required to prevent effort overruns.

When it should be measured: It should be measured at overall project level.

Input/Measure	Formula	Example
Percentage of Work completed - (Actual Effort) / (Actual Effort + Effort required to complete remaining work) * 100 Percentage of effort expended - (Actual Effort/ Estimated Effort) * 100	$\frac{(\text{Percentage of Work completed} / \text{Percentage of effort expended})}$	Estimated Effort (in person days) = 10 Actual Effort (in person days) = 5 Effort Required to complete remaining work (in person days) = 7 %Work Completed = $(5 / (5 + 7)) * 100 = 41.66\%$

		$\% \text{Effort Expended} = (5/10) * 100 = 50\%$ $\text{Work Efficiency Index} = (41.66 / 50) = 0.83$
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Risk Identification Efficiency %

This metric determines the efficiency of identifying risk in a project. This helps in planning for the mitigation and contingency to be carried out in a project. This metric is computed as Number of Risk identified to Number of Risk occurred, expressed as percentage figure.

When it should be measured: It should be measured at overall project level.

Input/Measure	Formula	Example
Total no of Risks Identified Total no of Unanticipated Risk Occurred	$(\text{Total no of Risks Identified} / (\text{Total no of risks identified} + \text{Total no of Unanticipated Risk Occurred})) * 100$	Total no of Risks Identified = 6 Total no of Unanticipated Risk Occurred = 2 Risk Identification Efficiency % = $(6/8) * 100 = 75\%$

Benefits:

- This metric determines the efficiency of identifying risk in a project
- This helps in planning for the mitigation and contingency to be carried out in a project

Risk Mitigation Efficiency %

When it should be measured: It should be measured at overall project level.

Input/Measure	Formula	Example
Total no of Risks Identified for mitigation No. of Risk mitigated but occurred	$((\text{No. of Risk identified for mitigation} - \text{No. of Risk mitigated but occurred}) / (\text{No. of Risk identified for mitigation})) * 100$	Total no of Risks Identified for mitigation = 10 No. of Risk mitigated but occurred = 1 Risk Mitigation Efficiency % = $(9/10) * 100 = 90\%$

Benefits:

- Gives insight into efficiency to mitigate risk and take further actions.

Review Efficiency

It's the ratio of number of review defects to total defects in software (review and testing)

Objective: This metric shows the efficiency of the review process. A higher ratio of Review Defects to Total Defects indicates a very efficient Review Process. However, a lower ratio need not necessarily mean that the review is inadequate. This may be due to frequent changes in scope, requirements etc. Typically, the review defects include all reviews, starting from the Requirements Stage. All review comments that arise from the review of Code/unit / integration / system test plans / procedures / cases shall be counted under Review defects. The total number of Defects includes all the defects found in reviews and testing.

When it should be measured: It should be measured at overall project level and stage level.

Input/Measure	Formula	Example
Number of Review defects Total number of Testing Defects including customer reported test defects	$(\text{Number of Review defects}) / (\text{Total number of Review} + \text{Testing Defects [including customer reported test defects]}) * 100$	Number of Review defects = 10 Total number of Testing Defects including customer reported test defects = 100 Review Efficiency% = $(10/110) * 100 = 9.09$

Benefits:

- This helps to know effectiveness of the reviews performed and to improve the review process.
- Increase in review efficiency decrease the defect leakage to next stage.

16. Case Study 1:

Power Equipment Limited (PEL) is an Indian Subsidiary of an MNC that has its headquarters in Europe. The Company customizes designs of its parent company and manufactures machines for industrial use. Its annual turnover is Rs 1500 crore. The Company Ships about 600 machines every month. Approximately 30% of this volume is exported to various countries such as UK, US and other European countries. Its turnover from the domestic market in India is approximately 60%. PEL was a market leader and enjoyed a monopolistic position from 1970 to 1990. However Competition entered the scenario in 1990 resulting in pressures on quality, cost and delivery. As a result in the past few years profit did not increase in proportion to its turnover. The company is a certified to ISO 9000 since 1993 and has a fairly strong quality management system. PEL embraced various quality improvement programs and obtained fair gains from these. The engineer's workers and staff were given training in technology and management practice such as JIT, Kaizen, and Six Sigma etc. The Company evolved its own excellence model and developed a check list of excellence norms. IN spite of that PEL has been losing some of its key personnel to its competitors. Retention of people has become a concern and a priority for the HR head. PEL spends about 3% of its sales turnover on warranty.

PEL purchases about 60% of parts by value from suppliers. These parts include pistons, Crankshafts, heavy castings like cylinder block, Cylinder head, electrical parts, starters, radiators, cables, belts, hardware etc. The Company has developed a good supplier base and has enjoyed good relationship with most of them. In the past few years the company has been forced to negotiate with the suppliers to reduce costs. With the new initiative of senior management the supply base has been rationalized significantly. PEL has a modern machine Shop that has the capability to manufacture machine parts with High precision. This includes heavy medium and small parts.

PEL Ships its machines to several original equipment manufacturers (OEMs) who assemble them on their equipment. Most of the OEMs are not happy about the quality performance of PEL now, since the inferior quality machine parts have resulted in dissatisfied end customers. Apart from the defects reported OEMs have been constantly complaining about the quality of the paint. They reported that the end customers have given a feedback that the machines do not look like new. Moreover the replacement of defective parts typically takes 2 weeks to reach OEMs. Under pressure of the delivery of the equipment, OEMs sometimes have to remove parts from other machines and use them in the place of defective ones. The top management of the company has assured the OEMs that it will make efforts to improve its performance within a year. They have assigned the responsibility of improving the performance to the existing Six Sigma Steering Committee. They have asked the committee to assess the current situation with inputs from the quality manager and recommend an action plan.

After the review of the inputs the steering committee identified and finalized two six sigma projects:

1. Reducing Shipment defects
2. Improvement in the quality of the painting.

Project Execution Phase 2: Improvement in the Quality of Painting

PEL paints machine before dispatch. These machines are dispatched to the original equipment manufacturers (OEMs) for installation on their equipment. Many OEM’s had expressed dissatisfaction regarding the quality of paint. They had informed that the end users were unhappy. Some customers even doubted whether the machines were new.

The Define Phase

As customers were highly dissatisfied with the current quality of painting the management decided to take up a six sigma project to achieve break through improvement in that area. A partial charter of the project is shown below.

PEL LTD						
Six Sigma Project Charter						
Product or Process	Painting of Machines'					
Category(BB/GB)	BB					
Name of Belt	XYZ					
Sponsor	ABC					
Start Date	5 th Septem 2016					
Opportunity Improvement	Paint Gloss improvement and Cost reduction due to excess coating					
Short Statement of project delivera (Problem to be Solved	Improve average paint gloss from 75 to 85 and capability better than 1.5					
		Current	Target	Benchmark	Units	
Objective and measu (as applicable)	External Custor Defects	5 Per 100	0 Per 100		Number	
	Reduction in CO	250000	25000		US \$/ Year	
	DPMO/DPU/% Defective	27	1			
	Cycle time		Same or better			
	Productivity		Same or Better			
	Sigma Level	2.1	4.5			
	Payback					
Project Scope	Painting Machine of Mo V Series					

The Measure Phase

Each machine was inspected for gloss and was reworked for conformance. However approximately 27% of machines did not meet the specification with the estimated Sigma level that was 2.1. The target sigma level was 4.5. The existing process was mapped to identify KPOVs and KPIVs as shown in the table below

KPIVs (Xs)		C/U	Process	KPOVs	
X1	Type of Paint Gun	C	Painting Machines	Gloss	Y1
X2	Type of Paint	C		Adhesion	Y2
X3	Surface Cleanliness	C		Dry Film Thickness	Y3
X4	Paint Viscosity	C			
X5	Distance of Gun	C			
X6	Painting time	C			
X7	Painter Skill	C			
X8	Drying oven Temperature	C			
X9	Drying time	C			
X10	Ambient Temperature	U			

Measurement system analysis showed that the system was adequate with R&R value of 9%.

A cause and effect (C&E) matrix was evolved by the team members this is as shown in the table below.

C&E matrix for the Painting Process

Output Variable (Y's)	Coating Thickness T	Gloss G	Adhesion	
Importance Score (1-10)	5	7	8	
Input/Process Variable (X's)				Weighted Score
Type of Paint Gun	3	9	1	86 ←
Type of Paint	3	9	3	102 ←
Surface Cleanliness	1	1	9	84
Paint Viscosity	9	9	1	116 ←
Distance of the Gun	9	3	3	90 ←
Painting time	9	3	1	74
Painter Skill	3	9	3	102 ←
Drying oven Temperature	0	9	9	135 ←
Drying time	0	9	0	63
Ambient Temperature	0	3	3	45
Ambient Humidity	0	3	3	45

The Improve Phase

The team then decided to conduct an experiment with 5 factors refer table given below.

Factors	Name	Type	Low	High
A	Paint Source	Numeric	1	2
B	Gun type	Numeric	1	2
C	Distance	Numeric	50	80
D	Viscosity	Numeric	20	24
E	Temp	Numeric	120	140

17. Case Study 2:

Mumbai Dabbawallas achieved a level better than 6 sigma. The Tiffinwallas deliver tiffin boxes from their respective houses to their customers. Approximately 200,000 tiffin boxes are delivered to different offices in Mumbai by about 5000 tiffinwallas. The error rate is less than 1 in 60,00,000 deliveries this corresponds to the six sigma level of 6.6; the performance is superb considering the average education of Tiffinwallas is class 8. They created a simple coding system that every tiffin walla would understand with a short training; this code is painted on the tiffin boxes with specific colors. BBC made a TV News story on this achievement, Prince Charles met the tiffinwallas in November 2003 to appreciate their effort. Later in 2005, the prince invited the office bearers of 'dabbawallas' for his wedding; they stayed at Taj mahal hotel in London where they found that superstar Amitabh Bachchan was also staying next to their room but was not invited for the royal wedding.

References: The Certified Master Black belt by Kubiak; lean Six Sigma by Lawton ; Six Sigma for Business Excellence by Hemant Urdhwareshe; Six Sigma Handbook by Thomas Pyzdek and Paul Keller, Six Sigma Black Belt Handbook by Thomas Mc Carry-Michael Bremer & Praveen Gupta ; Kaner & Walter P. Bond-Software Engineering Metrics-What do they Measure and How do we Know; Google Search Engine.

Reference Software Free Trial Used: 30 Days-Minitab18; SigmaXL; Companion by Minitab Version 5 Software.

Statistical Tables used as Reference to do critical Statistic Calculation: T-Table; Mann Whitney Table; Chi-Square Table; F- ANOVA table which are available in Google search.

18. About the Author

The Author © Suresh .V. Menon is a lean Six Sigma Master Black Belt Certified Accredited by IASSC and has an experience of 27 years in the IT Industry, has delivered more than 4000 hours of six sigma trainings across the globe; Currently pursuing (PhD) in Computer Science is also a Six Sigma Principal Trainer, Strategic Management Consultant and Principal Consultant, he can be contacted at sureshmenonr1009@gmail.com for comments and queries.