Sigma coloring of Bull graph and related graphs

Preethi K Pillai[1] and J. Suresh Kumar[2]
PG and Research Department of Mathematics,
N.S.S. Hindu College, Changanacherry, Kerala, India 686102.

ABSTRACT: The Bull graph is a graph with 5 vertices and 5 edges consisting of a triangle with two disjoint pendant edges. In this paper, we obtain the σ-coloring of the Bull graph and some related graphs such as Middle graph, Total graph, Splitting graph, Degree splitting graph, Shadow graph and Litact graph of the Bull graph.

Key Words: Graph, Sigma coloring, Bull graph.

1. INTRODUCTION

Graph coloring take a major stage in Graph Theory since the advent of the famous four color conjecture. Several variations of graph coloring were investigated [5] and still new types of coloring are available such as σ −coloring (Sigma coloring) [4] and Roman coloring [6, 7]. In this Paper, we consider the proper coloring for the Bull graph and related graphs. By a graph, we mean a finite undirected graph without loops and parallel edges.

The Bull graph is a planar undirected graph with five vertices and five edges in the form of a triangle with two disjoint pendant edges. There are three variants of Bull graph, the name being derived from the fact that the structure of the graph is meant to represent the face of a "Bull".

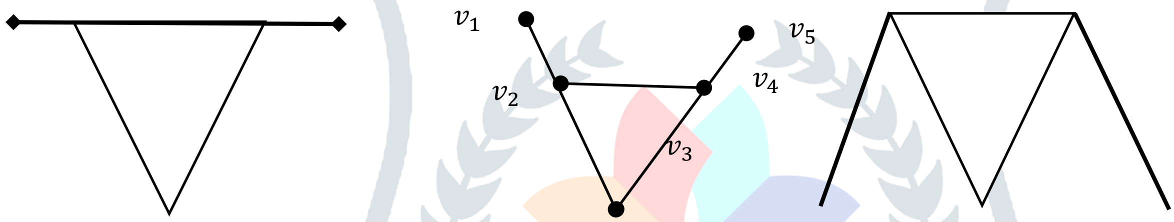

Figure 1. The Bull Graph

The Bull graph was introduced by Weisstein [2]. The concept of Bull-free graphs was studied, but there are no works related to the coloring of Bull graph and its related graphs. In this Paper, we investigate the Sigma coloring of Bull graph and its related graphs. We recall some basic definitions.

Definition. 1.1. Let G be a simple connected graph and $f: V(G) \to \mathbb{N}$, where $\mathbb{N}$ is the set of positive integers, be a coloring of the vertices in G. For any v∈V(G), let σ (v) denotes the sum of colors of the vertices adjacent to v then f is called a Sigma coloring ($\sigma -$ coloring) of G if for any two adjacent vertices $u, v \in V(G)$, $\sigma(v) \neq \sigma$(u). The minimum Number of colors used in a sigma coloring of G is called the sigma chromatic number of G and is denoted by $\sigma(G)$.

Definition. 1.2. The Middle graph M(G) of a graph G is the graph whose vertex set is V(G)∪E(G) and in which two vertices of M(G) are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge of G incident to it.

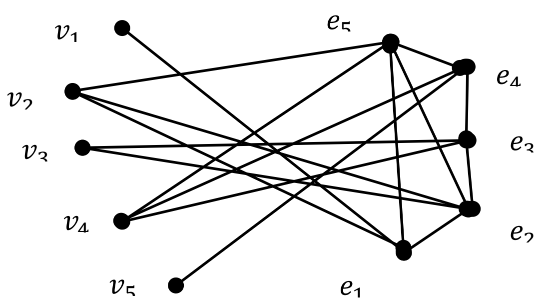

Figure.2. Middle graph of the Bull Graph

Definition. 1.3. The Total graph T(G) of a graph G is the graph whose vertex set is V(G)∪E(G) and in which two vertices of T(G) are adjacent whenever they are adjacent or incident in G..

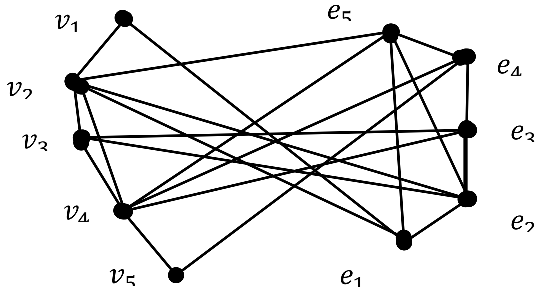

Figure. 3. Total graph of the Bull graph

Definition.1.4. The Splitting graph S(G) of a graph G is obtained by adding a new vertex v′ corresponding to each vertex v of G such that N(v)=N(v′) where N(v) and N(v′) are the neighbourhood set of v and v′ respectively.

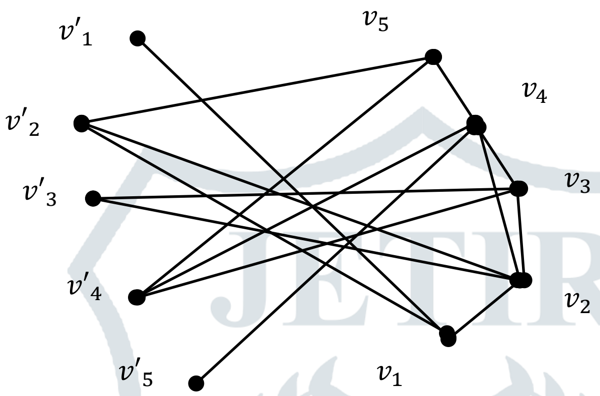

Figure.4. Splitting graph of the Bull graph

Definition 1.5. Let G = (V, E) be a graph with V= $S_1 \cup S_2 \cup \ldots\ldots \cup S_t \cup T$, where each S_i is a set of vertices having at least two vertices and having the same degree and T=V-∪ S_i. The Degree Splitting graph DS(G) of a graph G is obtained from G by adding vertices $w_1, w_2, \ldots\ldots w_t$ and joining w_i, to each vertex of S_i (1≤ i≤ t)

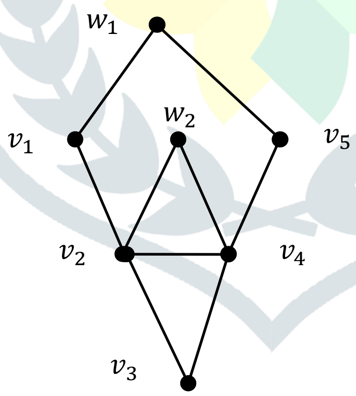

Figure.5. Degree Splitting graph of Bull graph

Definition.1.6. The Shadow graph, D_2(G) of a connected graph G is constructed by taking two copies of G, say G' and G" and joining each vertex u′ in G' to all the adjacent vertices of the corresponding vertex, u" in G".

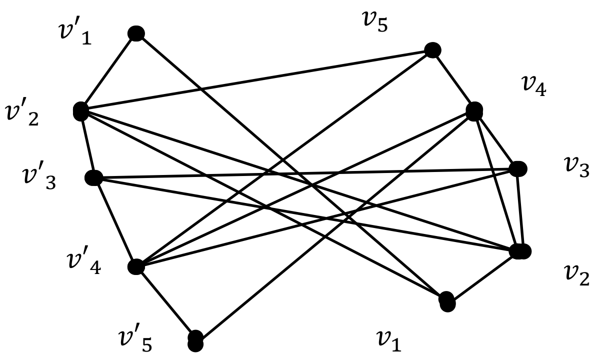

Figure.6. Shadow graph of Bull graph

Definition.1.7. Let c(G) denotes the cut vertex set of Bull graph G. The Litact graph m(G) of a graph G is the graph whose vertex set is V(G)∪c(G) in which two vertices are adjacent if and only if they correspond to adjacent edges of G or to adjacent cut-vertices of G or one corresponds to an edge e_i of G and the other corresponds to a cut-vertex c_j of G and e_i is incident with c_j.

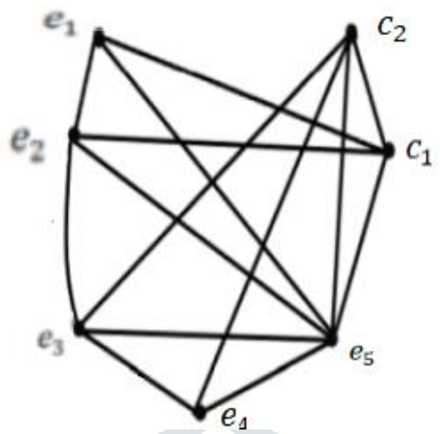

Figure.7. The Litact graph of Bull graph

2. MAIN REESULTS:

In this section, we discuss the Sigma Coloring of Bull Graphs and related Graphs. For the terms and definitions not explicitly defined here, reader may refer Harary [10].

Theorem. 2.1. The Middle graph of Bull graph is σ -colorable.

Proof. In the Middle graph of Bull graph, let the vertices be v_i for i = 1 to 5 and e_j for j= 1 to 5. Suppose v_1, v_3, e_1, e_2, e_3 are colored with color 1 and v_2, v_4, v_5, e_4, e_5 are colored with color 2 (Figure.8). Then sums of colors of adjacent vertices are different. So it is σ–colorable and $\sigma(G)$ =2.

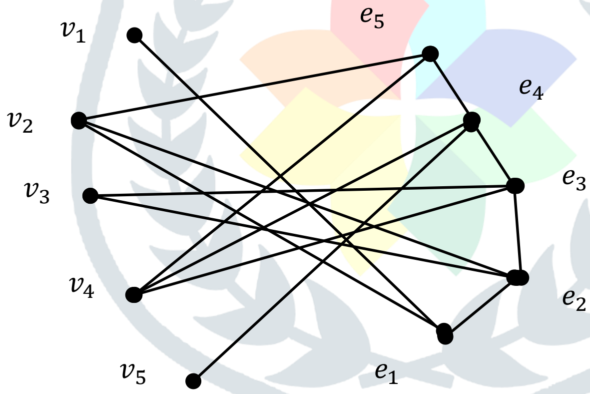

Figure 8

Theorem. 2.2. The Total graph of Bull graph is σ –colorable.

Proof. In the Total graph of Bull graph, let the vertices be v_i for i = 1 to 5 and e_j for j= 1 to 5. Figure.9 shows a Sigma coloring of TG with 2 colors (colors are shown in brackets). Then the sum of colors of adjacent vertices is different. So the coloring **is** σ –colorable and $\sigma(G)$ =2.

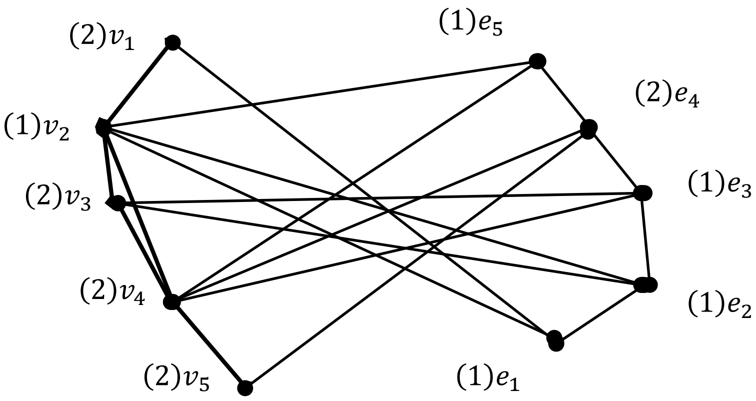

Figure 9

Theorem. 2.3. The Splitting graph of Bull graph is σ –colorable.

Proof. In the Splitting graph of Bull graph, let the vertices be v_i for I = 1 to 5 and v'_j for j= 1 to 5. Suppose that v_1, v_2 , $v_{3,}$ v_4 , v_5, v'_1, v'_2 , v'_4 , v'_3 are colored with the color 1 and v'_5 is colored with the color 2 so that the sum of colors (shown in square brackets) of adjacent vertices are different (Figure.10). Hence, the Splitting graph of Bull graph is σ–colorable and $\sigma(G)$ =2.

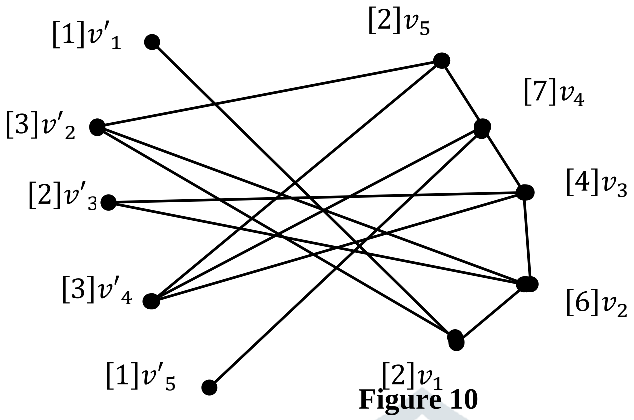

Figure 10

Theorem. 2.4. The Degree Splitting graph of Bull graph is σ –colorable.

Proof. Let the vertices of the Degree Splitting graph of Bull graph be v_i for i = 1 to 5 and w_j for j= 1 to 2. Suppose that v_1 is colored with the color 1, v_2 , $v_3 v_4$ v_5 w_1,, w_2 are colored with the color 2 (Figure.11). Then the sums of colors of adjacent vertices are different. So the Degree Splitting graph of Bull graph **is** σ –colorable and $\sigma(G)$ =2.

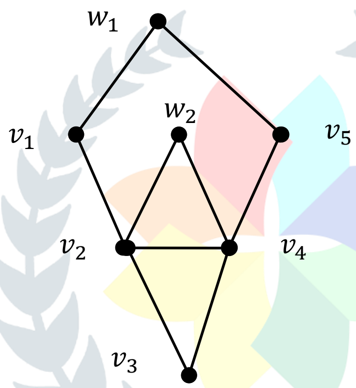

Figure 11

Theorem. 2.5. The shadow graph of Bull graph is σ –colorable.

Proof. Let the vertices of the Shadow graph of Bull graph be v_i for i = 1 to 5 and v'_j for j= 1 to 5. If v_2, v_4, v_5, v'_1 , v'_2 v'_4 , v'_5 are colored with the color 1 and v_1,v_3,v'_3 are colored with the color, 2 (Figure.12), then the sums of colors of adjacent vertices are different so that the shadow graph of Bull graph is σ –colorable**.** So $\sigma(G)$ =2.

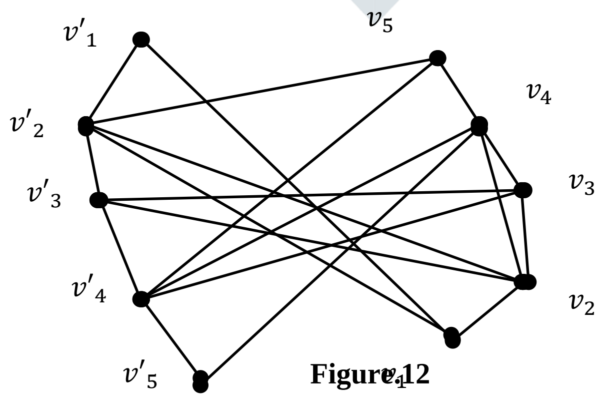

Figure 12

Theorem. 2.6. The Litact graph of Bull graph is σ –colorable.

Proof. In Litact graph of Bull graph, let the vertices be e_i for i = 1 to 5 and c_j for j= 1 to 2. Suppose that c_1, c_2 , e_5 , e_3 *are* colored with color 1, , e_4 e_1,, e_2 are colored with color 2 (figure.13).. We see that sum of colors of adjacent vertices are different. Here no two adjacent vertices have the same sum. We use only two colors. So the coloring **is** σ –colorable**.** So $\sigma(G)$ =2.

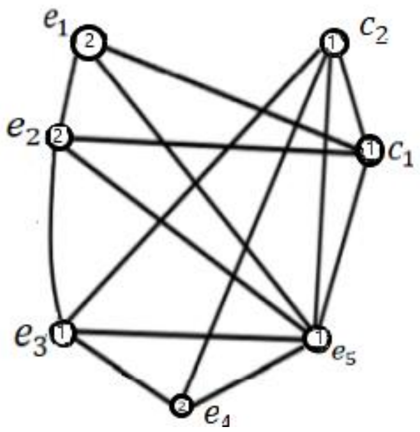

Figure.13

References:

[1] Gallian JA Dynamic Survey of Graph labelling, The Electronic Journal of Combinatorics, 2012.

[2] Weisstein, Eric W , Bull Graph, Math World

[3] Poulomy Ghosh, Sachchida N, Mishra, Anita pal, Various labelling on Bull graph and some related graphs, International ,Journal of Application of Fuzzy Sets and artificial intelligence (ISSN 2241-1240) vol:5 2015 23-35

[4] Gary Chartrand, Futaba Okamoto, Ping Zhang, The Sigma Chromatic Number of a Graph, Graphs and Combinatorics (2010) 26:755–773

[5] Suresh Kumar J, Graph Coloring Parameters – A survey, International Journal for Research in applied Science and Engineering Technology, Vol:7, Issue IV, April 2019.

[6] Suresh Kumar J, Roman colouring of Graphs and Application to Military Strategy, International Journal for Research in Applied Science & Engineering Technology, Volume 8, Issue III, March 2020.

[7] Suresh Kumar J, and Preethi K Pillai, Proper Roman coloring of graphs, International Journal of Creative Research Thoughts, March, 2020

[8] Suresh Kumar J, and Preethi K Pillai, Proper coloring of Bull graphs and related graphs, International Journal of Creative Research Thoughts, March, 2020 (Pre Print)

[9] Suresh Kumar J, and Preethi K Pillai, Roman coloring of Bull graph and related graphs, International Journal of Research in Applied Science and Engineering technology, April, 2020 (Pre Print)

[10] Harary Frank, Graph Theory, Addison Wesley, Reading Mass, 1969.