ANALYSIS OF VIBRATION ON AERIAL TRAMWAY

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Abstract : An aerial tramway is a type of aerial lift. An aerial tramway consists of one or more passenger or cargo cabins that are supported entirely by stationary cables and a propulsion cable for cabin movement. The cables are anchored at either end by towers or piers. In further section the vibration analysis of aerial tramway with two cabins are done. For analysis we consider above system as two masses fixed on a tightly stretched string. Also we develop the response waves at various modes and simulation of the system using matlab software.

***IndexTerms* – aerial tramway, natural frequency, mode shape, fast fourier transformation.**

I. INTRODUCTION

Vibration is a mechanical phenomenon whereby oscillations occur about an equilibrium point. The oscillations may be periodic, such as the motion of a pendulum or random, such as the movement of a tire on a gravel road. Free vibration occurs when a mechanical system is set in motion with an initial input and allowed to vibrate freely. Forced vibration is when a time-varying disturbance (load, displacement or velocity) is applied to a mechanical system. The disturbance can be a periodic and steady-state input, a transient input, or a random input.

II. VIBRATION ANALYSIS OF AERIAL TRAMWAY

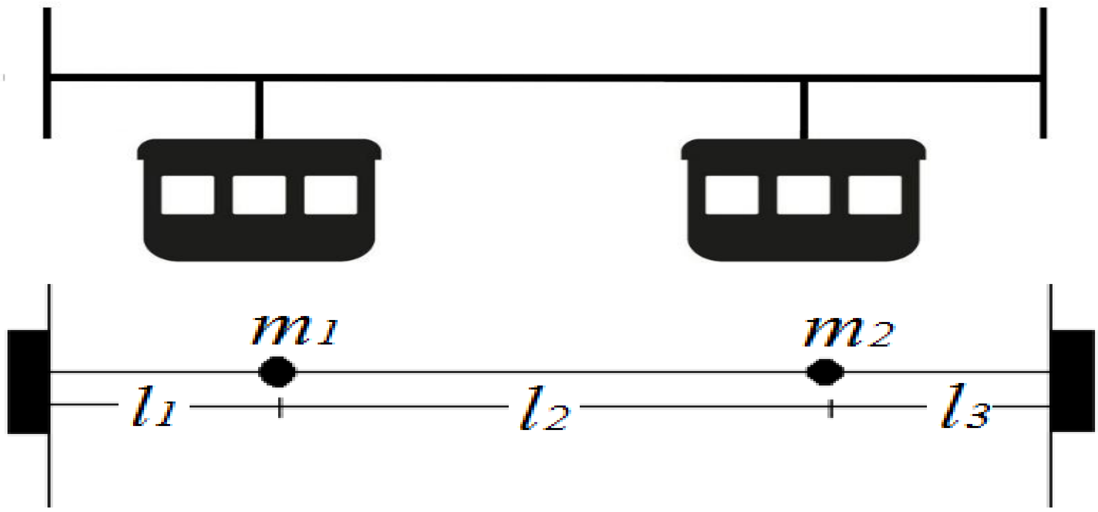

Fig1.Comparing aerial tramway with a simple system

In the further section we do the vibration analysis of aerial tramway with two cabin using matlab software. For analysis we only consider the towers and cabins of the aerial tramway as they are the main parts which transfer the load acting. In order to conduct the analysis in matlab software we have to simplify the aerial tramway to some other simple system. The fig.1 shows that comparison of aerial tramway can be done with another system of two masses suspended over tightly stretched string. Here we consider the aerial tramway to be viewed from a large distance we can see the cabins as point mass objects and cables as strings connecting them. Hence in further section we consider the analysis of aerial tramway using two masses suspended over a tightly stretched string system.

III. TWO MASSES FIXED ON TIGHTLY STRETCHED STRING

Consider two masses m_1 and m_2 fixed on a tight string stretched between two supports as shown in figure below and having a tension T. Let the amplitude of vibration of the two masses be small and tension T large so that it remains appreciably constant during the vibration of the two masses. At any instance let x_1 and x_2 be the displacements of the two masses respectively as shown in figure below. The components of tension T along the original direction of the string are $T\cos(\varphi_1)$, $\cos(\varphi_2)$ and $T\cos(\varphi_3)$, and each one of these is approximately equal to T for small amplitudes since the angles are small (considering Taylor series).

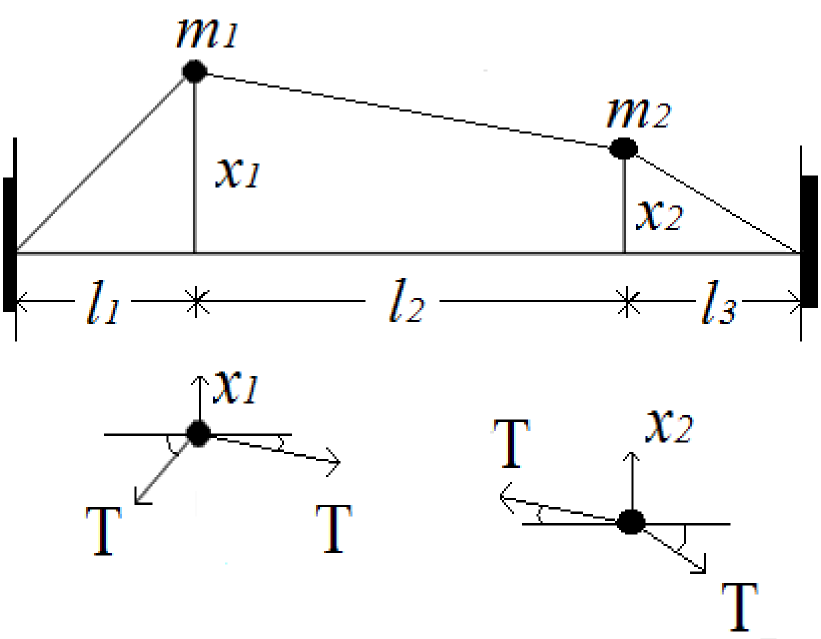

Fig2.Free body diagram

IV. GENERAL EQUATION

$$m_1 * \ddot{x}_1 = -(T\sin\theta_1) - (T\sin\theta_2),$$

$$\sin\theta_1 = \frac{x_1}{l_1}, \quad \sin\theta_2 = \frac{x_1 - x_2}{l_2},$$

$$m_1 * \ddot{x}_1 = -(T * \frac{x_1}{l_1}) - (T * \frac{(x_1 - x_2)}{l_2}),$$

$$m_1 * \ddot{x}_1 = -((\frac{T}{l_1} + \frac{T}{l_2}) * x_1) + (\frac{T}{l_2} * x_2) \rightarrow \ (1)$$

$$\mathrm{m}_2 * \ddot{x}_2 = (T * \sin\theta_2) - (T * \sin\theta_3),$$

$$\sin\theta_3 = \frac{x_2}{l_3},$$

$$\mathrm{m}_2 * \ddot{x}_2 = (T * \frac{x_1 - x_2}{l_2}) - (T * \frac{x_2}{l_3}),$$

$$\mathrm{m}_2 * \ddot{x}_2 = (\frac{T}{l_2} * x_1) - (\frac{T}{l_2} + \frac{T}{l_3}) * x_2 \rightarrow \ (2)$$

General Equations (1) & (2) can be written in matrix form as:-

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} (\frac{T}{l_1} + \frac{T}{l_2}) & -(\frac{T}{l_2}) \\ -(\frac{T}{l_2}) & (\frac{T}{l_2} + \frac{T}{l_3}) \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \ (3)$$

V. SOLUTION FOR GENERAL EQUATION

Let:-

$$x_1 = A_1 \sin(\omega_1 t),\ \ddot{x}_1 = -A_1\omega_1^2 \sin(\omega_1 t),$$

$$x_2 = A_2 \sin(\omega_2 t),\ \ddot{x}_2 = -A_2\omega_2^2 \sin(\omega_2 t),$$

Eqn(1):-

$$-m_1 A_1 \omega_1^2 \sin(\omega_1 t) + (\frac{T}{l_1} + \frac{T}{l_2}) A_1 \sin(\omega_1 t) - (\frac{T}{l_2}) A_2 \sin(\omega_1 t) = 0,$$

$$(-m_1\omega_1^2 + (\frac{T}{l_1} + \frac{T}{l_2})) * A_1 = (\frac{T}{l_2}) * A_2,$$

$$r_1 = \frac{A_2}{A_1} = \frac{(\frac{T}{l_1} + \frac{T}{l_2}) - m_1\omega_1^2}{(\frac{T}{l_2})} \rightarrow (1)^1$$

Eqn (2):-

$$-m_2 A_2 \omega_2^2 \sin(\omega_2 t) + (\frac{T}{l_2} + \frac{T}{l_3}) A_2 \sin(\omega_2 t) - (\frac{T}{l_2}) A_1 \sin(\omega_2 t) = 0,$$

$$r_2 = \frac{A_2}{A_1} = \frac{(\frac{T}{l_2})}{(\frac{T}{l_2} + \frac{T}{l_3}) - m_2\omega_2^2} \rightarrow (2)^1$$

Equating $(1)^1$ & $(2)^2$:-

$$let : \omega_1 = \omega_2 = \omega,$$

$$((\frac{T}{l_1} + \frac{T}{l_2}) - m_1\omega^2) * ((\frac{T}{l_2} + \frac{T}{l_3}) - m_2\omega^2) = (\frac{T}{l_2})^2,$$

$$(\omega^2)^2 - ((\frac{T}{m_1 * m_2}) * (\frac{m_2}{l_1} + \frac{m_1 + m_2}{l_2} + \frac{m_1}{l_3})) * (\omega)^2 + ((\frac{1}{l_1 * l_2} + \frac{1}{l_1 * l_3} + \frac{1}{l_2 * l_3}) * (\frac{T^2}{m_1 * m_2})),$$

Let:-

a=1,

$$\text{b} = -((\frac{T}{m_1 * m_2}) * (\frac{m_2}{l_1} + \frac{m_1 + m_2}{l_2} + \frac{m_1}{l_3})),$$

$$\text{c} = ((\frac{1}{l_1 * l_2} + \frac{1}{l_1 * l_3} + \frac{1}{l_2 * l_3}) * (\frac{T^2}{m_1 * m_2})),$$

Natural frequencies:-

$$\omega^2{}_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

$$\omega^2{}_{1,2} = \frac{((\frac{T}{m_1 * m_2}) * (\frac{m_2}{l_1} + \frac{m_1 + m_2}{l_2} + \frac{m_1}{l_3})) \pm \sqrt{(-(\frac{T}{m_1 * m_2}) * (\frac{m_2}{l_1} + \frac{m_1 + m_2}{l_2} + \frac{m_1}{l_3}))^2 - 4*1*((\frac{1}{l_1 * l_2} + \frac{1}{l_1 * l_3} + \frac{1}{l_2 * l_3}) * (\frac{T^2}{m_1 * m_2}))}}{2*1}$$

The normal modes of vibration corresponding to $\omega_1{}^2$ and $\omega_2{}^2$ can be expressed respectively as:-

$$\vec{A}^{(1)} = \begin{Bmatrix} A_1{}^{(1)} \\ A_2{}^{(1)} \end{Bmatrix} = \begin{Bmatrix} A_1{}^{(1)} \\ r_1 A_1{}^{(1)} \end{Bmatrix},$$

$$\vec{A}^{(2)} = \begin{Bmatrix} A_1{}^{(2)} \\ A_2{}^{(2)} \end{Bmatrix} = \begin{Bmatrix} A_1{}^{(2)} \\ r_2 A_1{}^{(2)} \end{Bmatrix},$$

The vectors $\vec{A}^{(1)}$ and $\vec{A}^{(2)}$ which denote normal modes of vibration are known as the modal vectors of the system.

$$\vec{x}^{(1)}(t) = \begin{Bmatrix} x_1{}^{(1)}(t) \\ x_2{}^{(1)}(t) \end{Bmatrix} = \begin{Bmatrix} A_1{}^{(1)} \cos(\omega_1 t + \varphi_1) \\ r_1 A_1{}^{(1)} \cos(\omega_1 t + \varphi_1) \end{Bmatrix} = \text{first mode},$$

$$\vec{x}^{(2)}(t) = \begin{Bmatrix} x_1{}^{(2)}(t) \\ x_2{}^{(2)}(t) \end{Bmatrix} = \begin{Bmatrix} A_1{}^{(2)} \cos(\omega_2 t + \varphi_2) \\ r_2 A_1{}^{(2)} \cos(\omega_2 t + \varphi_2) \end{Bmatrix} = \text{second mode},$$

Where the constants $A_1{}^{(1)}, A_1{}^{(2)}, \varphi_1$ and φ_2 are determined by the initial conditions.

Initial Conditions:-

$$x_1(t=0) = A_1 = \text{constant}, \qquad x_2(t=0) = r_1 A_1,$$

$$\dot{x}_1(t=0) = 0, \qquad \dot{x}_2(t=0) = 0,$$

The resulting motion, which is given by the general solutions of Eqns (1) & (2) can be obtained by a linear superposition of the two normal modes:-

$$\vec{x}(t) = c_1 \vec{x}^{(1)}(t) + c_2 \vec{x}^{(2)}(t),$$

$$\text{Let:- } c_1 = c_2 = 1,$$

$$x_1(t) = x_1{}^{(1)}(t) + x_1{}^{(2)}(t),$$

$$x_1(t) = A_1{}^{(1)} \cos(\omega_1 t + \varphi_1) + A_1{}^{(2)} \cos(\omega_2 t + \varphi_2),$$

$$x_2(t) = x_2{}^{(1)}(t) + x_2{}^{(2)}(t),$$

$$x_2(t) = r_1 A_1{}^{(1)} \cos(\omega_1 t + \varphi_1) + r_2 A_1{}^{(2)} \cos(\omega_2 t + \varphi_2),$$

Where the unknown constants $A_1{}^{(1)}, A_1{}^{(2)}, \varphi_1$ and φ_2 can be determined from the initial conditions:-

When t=0,

$$x_1(0) = A_1{}^{(1)} \cos(\varphi_1) + A_1{}^{(2)} \cos(\varphi_2),$$

$$\dot{x}_1(0) = -A_1{}^{(1)} \omega_1 \sin\varphi_1 - A_1{}^{(2)} \omega_2 \sin\varphi_2,$$

$$x_2(0) = r_1 A_1{}^{(1)} \cos(\varphi_1) + r_2 A_1{}^{(2)} \cos(\varphi_2),$$

$$\dot{x}_2(0) = -r_1 A_1{}^{(1)} \omega_1 \sin\varphi_1 - r_2 A_1{}^{(2)} \omega_2 \sin\varphi_2,$$

$$A_1{}^{(1)} \cos(\varphi_1) = \left\{ \frac{r_2 x_1(0) - x_2(0)}{r_2 - r_1} \right\},$$

$$A_1^{(2)}\cos(\varphi_2)=\left\{\frac{-r_1x_1(0)+x_2(0)}{r_2-r_1}\right\},$$

$$A_1^{(1)}\sin(\varphi_1)=\left\{\frac{-r_2\dot{x}_1(0)+\dot{x}_2(0)}{\omega_1*(r_2-r_1)}\right\},$$

$$A_1^{(2)}\sin(\varphi_2)=\left\{\frac{r_1\dot{x}_1(0)-\dot{x}_2(0)}{\omega_2*(r_2-r_1)}\right\},$$

From above equations we obtain the desired solution:-

$$A_1^{(1)}=\sqrt{(A_1^{(1)}\cos(\varphi_1))^2+(A_1^{(1)}\sin(\varphi_1))^2},$$

$$A_1^{(1)}=\frac{1}{(r_2-r_1)}*\sqrt{(r_2x_1(0)-x_2(0))^2+\frac{(-r_2\dot{x}_1(0)+\dot{x}_2(0))^2}{\omega_1^2}},$$

$$A_1^{(2)}=\sqrt{(A_1^{(2)}\cos(\varphi_2))^2+(A_1^{(2)}\sin(\varphi_2))^2},$$

$$A_1^{(2)}=\frac{1}{(r_2-r_1)}*\sqrt{(-r_1x_1(0)+x_2(0))^2+\frac{(r_1\dot{x}_1(0)-\dot{x}_2(0))^2}{\omega_2^2}},$$

$$\varphi_1=\tan^{-1}\left\{\frac{A_1^{(1)}\sin(\varphi_1)}{A_1^{(1)}\cos(\varphi_1)}\right\}=\tan^{-1}\left\{\frac{-r_2\dot{x}_1(0)+\dot{x}_2(0)}{\omega_1(r_2x_1(0)-x_2(0))}\right\},$$

$$\varphi_2=\tan^{-1}\left\{\frac{A_1^{(2)}\sin(\varphi_2)}{A_1^{(2)}\cos(\varphi_2)}\right\}=\tan^{-1}\left\{\frac{r_1\dot{x}_1(0)-\dot{x}_2(0)}{\omega_2(-r_1x_1(0)+x_2(0))}\right\}$$

From above derivations the equations denoted in blue color is used in matlab for the analysis. Matlab is a software which uses the above equations to obtain a proper response. Proper coding of above equations along with some inputs like masses, initial displacement of masses helps us to provide the response. The response is obtained in the form of amplitude vs time graph. The graphs obtained always will be a combination of two masses from which finding frequency is much difficult. Hence we use fft (fast fourier transformation) which converts amplitude vs time domain graphs to amplitude vs frequency domain graphs. From fft we can determine the actual natural frequency of vibration and their amplitude.

VI. MODE SHAPES

Mode shape is a characteristic manner in which vibration occurs. In a freely vibrating system, oscillation is restricted to certain characteristic frequencies these motions are called normal modes of vibration.

In the case of masses suspended over a tightly stretched string the mode of vibration are shown below.

To make the analysis simple, let us take a special case when,

$m_1=m_2=m$,

$l_1=l_2=l_3=l$,

We get two mode shapes and corresponding wave form is provided below:-

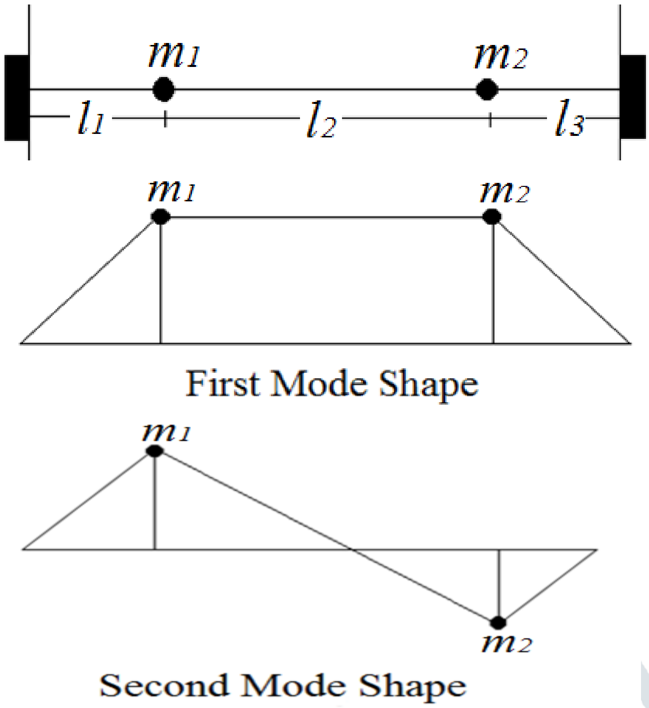

Fig3.Mode shapes

VII. GRAPHICAL REPRESENTATION

For the first mode shape we get response plot as follows :-

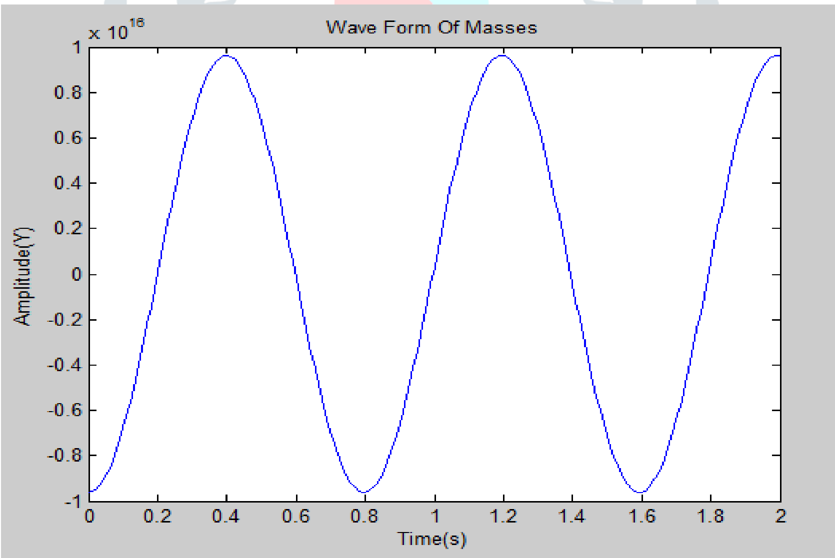

Fig4.Response curve for first mass

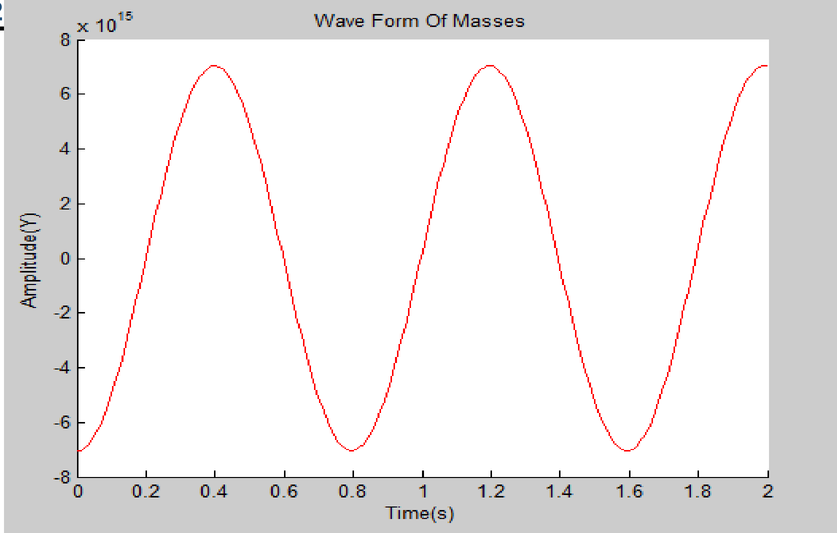

Fig5.Response curve for second mass

The above response is obtained for first mode. Here we can see that both the mass having amplitude at same phase which shows that both the mass have displaced in same direction.

For the second mode shape we get response plot as follows:-

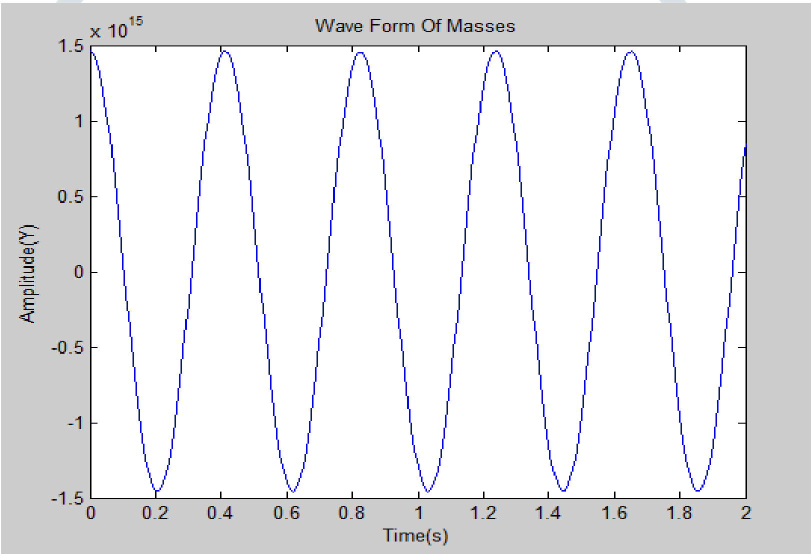

Fig6.Response curve for first mass

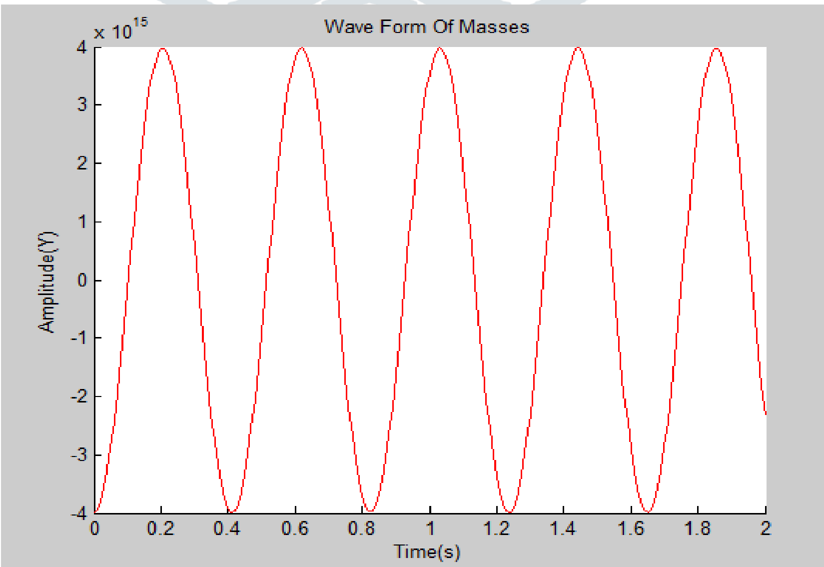

Fig7.Response curve for second mass

The above response is obtained for second mode. Here we can see that the amplitude of both the masses is out of phase which shows that the masses have displacement in opposite direction.

Response plot of amplitude vs time for natural frequencies w_1 & w_2 :-

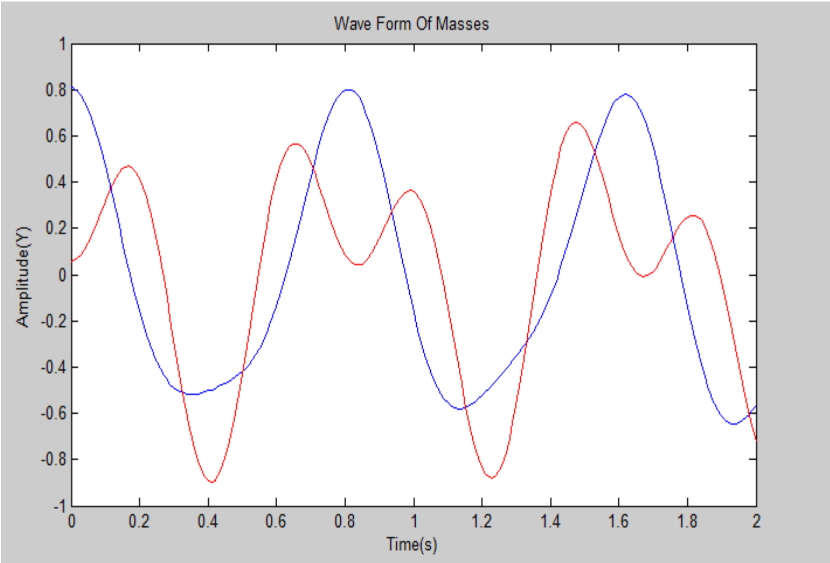

Fig8.Response curve of both modes

The response is a combination of first and second modes. The blue curve shows the first mode and the red curve shows the second mode.

VIII. SIMULATIONS

Simulations corresponding to the mode shapes are shown below :-

For first mode shape:-

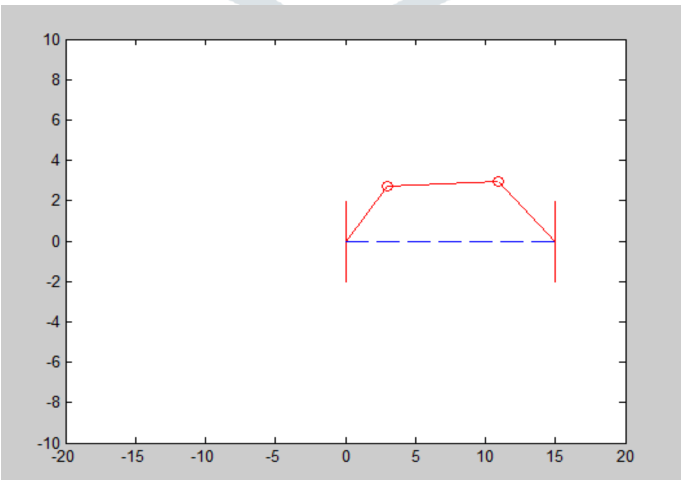

Fig9.Simulation of first mode

For second mode shape:-

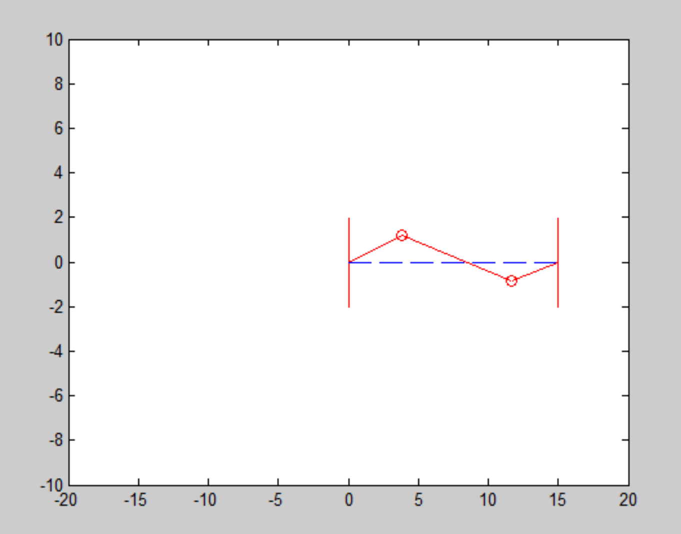

Fig10.Simulation of second mode

IX. FAST FOURIER TRANSFORM

The fast fourier transform for the natural frequencies are as follows:-

For first natural frequency:-

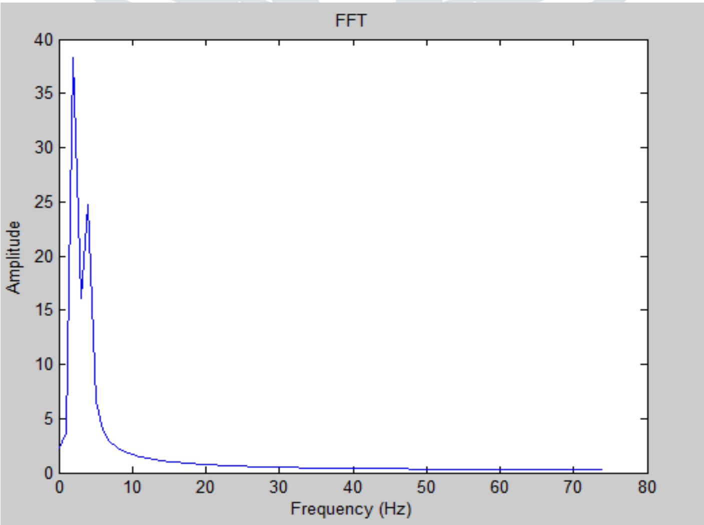

Fig11.Natural frequency for first mode

For second natural frequency:-

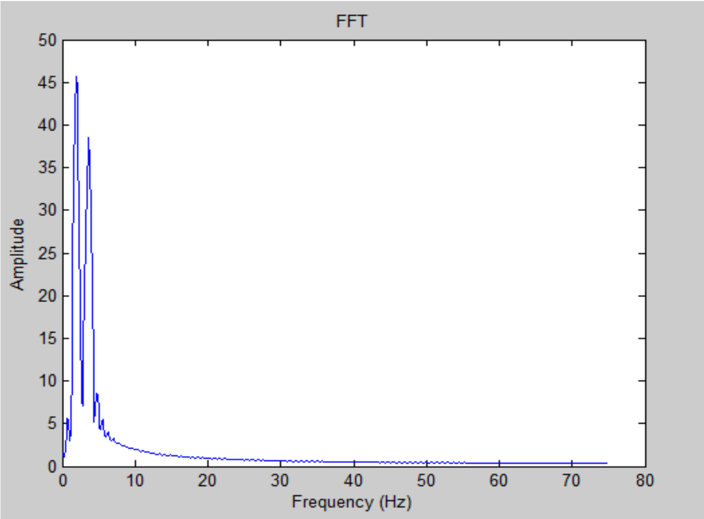

Fig12.Natural frequency for second mode

The fast fourier transformation of both the modes helps to provide us the natural frequencies of both mass as shown in above figures. We get particular values for frequency and amplitude and thus we obtain our required result.

X. MATHLAB BASIC FUNCTIONS USED

Matlab functions are the basic function that are defined within a single Matlab Statement. It consist of a single Matlab expression and any number of input and output arguments. It can be define an anonymous function right at the Matlab command line or within a function or script.
Some Matlab functions used are:

1) plot([0 0],[-2 2],'r-');// is a function which draws a line connecting the co- ordinates (0,0)and(-2,2) And 'r-' provides line red color.

2) a=viscircles([10 10],r);// is a function which draws a circle whose center is at the co-ordinate(10,10) with radius r. And the function name is viscircles.

3) axis([-20 20 -10 10]);//is a function used to get the coordinate system of given values with equal intervals.

4) pause(0.001);//is a function to pause 0.001s.

5) delete(ali1);//is a function to delete ali1(Which is the name of a line).

6) plot(x,t);//is a function to plot the necessary graph with respect to the values of x and t.

7) title('Wave Form Of Masses');// is a function to provide title to the graph.

8) xlabel('Time(s)');// is a function to provide x label similarly for y label.

9) i=1:3;// is a function for iterating the motion of system.

XI. CONCLUSION

In this analysis we have dealt with the study of vibrations. The above report provides vibrational analysis of aerial tramway with two passenger or cargo cabins using matlab software. The equations of motions, general solutions and simulations were done using Matlab, Math type and Edraw. The final result obtain was the natural frequency of the aerial tramway for various input parameters such as masses, tension on strings, initial displacement when force is applied and also the lengths of cables connecting the towers. By providing the required input parameters for the aerial tramway we obtain the corresponding natural frequency.

XII. REFERENCES

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