The Primary Theoretical Framework in Quantum Physics - A Review

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Abstract

This paper explores quantum mechanics and the basic theoretical framework that underpins it all, first developed in the 1920s by Niels Bohr, Werner Heisenberg, Erwin Schrödinger and others.Quantum theory provides a framework for modern theoretical physics that enjoys enormous predictive and explanatory success. Yet, in view of the so-called "measurement problem", there is no consensus on how physical reality can possibly be such that this framework has this success. The theory is thus an extremely well-functioning algorithm to predict and explain the results of observations, but no consensus on which kind of objective reality might plausibly underlie these observations.

Amongst the many attempts to provide an "interpretation" of quantum theory to account for this predictive and explanatory success, one class of interpretations hypothesizes backward-in-time causal influences—retrocausality—as the basis for constructing a convincing foundational account of quantum theory. This entry presents an overview of retrocausal approaches to the interpretation of quantum theory, the main motivations for adopting this approach, a selection of concrete suggested retrocausal models, and a review of the objections brought forward against such approaches. From the birth of the theory of quantum mechanics in 1925/6 to the outbreak of war in Europe, a clear orthodoxy emerged in the conceptual and ontological framework for understanding quantum theory. Now known as the Copenhagen interpretation, this framework embodied the positivistic tendencies of Heisenberg and Bohr, and was set opposed to the more realist tendencies of de Broglie, Einstein, and Schrödinger. It was not until Bell's theorem in the 1960s, and its experimental tests in the 1970s and 1980s, that new energy was breathed into this interpretational debate. However, beginning in the mid-1940s, the first suggestions of retrocausality as part of the conceptual and ontological framework in quantum theory had already materialized.There are two key ideas that punctuate the historical development of the notion of retrocausality in quantum mechanics. The first proposal of retroactive influence in quantum mechanics comes from a suggestion made by Wheeler and Feynman (1945, 1949). They were led to this idea while considering the potentially classical origins of some of the difficulties of quantum theory. Consider the following problem from classical electrodynamics: an accelerating electron emits electromagnetic radiation and, through this process, the acceleration of the electron is damped. Various attempts to account for this phenomenon in terms of the classical theory of electrodynamics lacked either empirical adequacy or a coherent physical interpretation

Key words: ontological framework, Einstein, Schrödinger, retroactive influence, quantum mechanics.

Introduction

Wheeler and Feynman attempted to remedy this situation by reinterpreting Dirac's (1938) theory of radiating electrons. The core of Wheeler and Feynman's proposed "absorber theory of radiation" is a suggestion that the process of electromagnetic radiation emission and absorption should be thought of as an interaction between a source and an absorber rather than as an independent elementary process. (This idea has its roots as far back as Tetrode 1922 and G. Lewis 1926.) Wheeler and Feynman imagine an accelerated point charge located within an absorbing system and consider the nature of the electromagnetic field associated with the acceleration. An electromagnetic disturbance can be imagined "initially" to travel outwards from the source to perturb each particle of the absorber. The particles of the absorber then generate together a subsequent field. According to the Wheeler-Feynman view, this new field is comprised of half the sum of the retarded (forward-in-time) and advanced (backward-in-time) solutions to Maxwell's equations. The sum of the advanced effects of all the particles of the absorber then yields an advanced incoming field that is present at the source simultaneous with the moment of emission (although see §5 for more on how one should understand this "process"). The claim is that this advanced field exerts a finite force on the source which has exactly the required magnitude and direction to account for the observed energy transferred from source to absorber; this is Dirac's radiative damping field. In addition, when this advanced field is combined with the equivalent half-retarded, half-advanced field of the source, the total observed disturbance is the full retarded field known empirically to be emitted by accelerated point charges.

The crucial point to note about the Wheeler-Feynman schema is that due to the advanced field of the absorber, the radiative damping field is present at the source at exactly the time of the initial acceleration. This schema of advanced and retarded waves now forms the basis for the most fully-formed retrocausal model of quantum mechanics, the transactional interpretation (see §5).

The second key idea in the historical development of retrocausality in quantum mechanics occurs around the same time as Wheeler and Feynman's absorber theory. French physicist Costa de Beauregard, a student of de Broglie, noticed a potential objection to the reasoning found in Einstein, Podolsky, and Rosen's famous paper (1935) on the completeness of quantum mechanics (see the entry on the Einstein-Podolsky-Rosen argument in quantum theory). Now widely known as the EPR argument, They argue that quantum mechanics must be incomplete on the basis of the following assumption: no reasonable definition of reality could be expected to permit the reality of some system being dependent upon the process of measurement carried out on some other distant system which does not in any way disturb the first system.

There is a tradition that stretches back at least as far as Russell (1913) that denies that there is any place for causal notions in the fundamental sciences, including physics: the notion serves no purpose, and simply does not appear, in the fundamental sciences. The argument goes that, since at least the nineteenth century, the laws that govern physical behavior in fundamental sciences such as physics are almost always differential equations. Such equations are notable for specifying, given some initial conditions, exact properties of systems for all time. And thus if everything is specified for all time, there is no place left for causality. Thus Russell advocates that "causality" should be eliminated from the philosophers lexicon, because it is certainly not a part of the scientific lexicon.

In contrast to Russell's position, Cartwright (1979: 420) claims that we do have a need and use for a causal vocabulary in science: "causal laws cannot be done away with, for they are needed to ground the distinction between effective strategies and ineffective ones". One of the main contemporary accounts of causation, the interventionist account of causation (Woodward 2003; see also the entry on causation and manipulability), is an embodiment of Cartwright's dictum. In a nutshell, the interventionist account claims that A is a cause of B if and only if manipulating A is an effective means of (indirectly) manipulating B. Causality in the present entry, unless specified otherwise, should be understood along broadly interventionist lines. According to accounts of quantum theory that hypothesize retrocausality, manipulating the setting of a measurement apparatus can be an effective means of manipulating aspects of the past. A broadly interventionist view of causality indeed underlies most contemporary attempts to harness the tool kit of causal modeling (see the entry on causal models; Spirtes, Glymour, & Scheines 2000; Pearl 2009) in the foundations of quantum theory (Leifer & Spekkens 2013; Cavalcanti & Lal 2014; Costa & Shrapnel 2014; Allen et al. 2014).

Using the notion of causality along broadly interventionist lines in the foundations of quantum theory does not commit one to realism (or anti-realism) about the causal relations at issue. Woodward combines interventionism with realism about causality while acknowledging

important differences between, on the one hand, the way in which causal notions figure in common sense and the special sciences and the empirical assumptions that underlie their application and, on the other hand, the ways in which these notions figure in physics. (Woodward 2007: 67; although see Frisch 2014: chs. 4 and 5 for a response)

Objective:

This paper intends to explore Quantum physics frameworks, which are is described here in two guises: indeterminacy with its concomitant indeterminism of measurement outcomes, and fuzziness, or unsharpness. Also features were long seen as obstructions of experimental possibilities that were available in the realm of classical physics.

Model reasonable definition of reality

The "reasonable definition of reality" to which Einstein et al. refer is implicitly an assumption of relativistic "locality" (made explicit in Einstein 1948), which combines causal asymmetry with the Lorentz invariance of special relativity (more on this in a moment). Costa de Beauregard, however, was alert to a particular kind of unorthodox interpretation of this assumption which undermined its role in the EPR argument. His proposal was that two distant systems could “remain correlated by means of a successively advanced and retarded wave” (Costa de Beauregard 1953: 1634); that is, one system could influence, via an advanced wave, the state of the combined systems in their common past, which then, via a retarded wave, could influence the state of the distant system in a kind of “zigzag” through spacetime. This way, there could be a dependence between the two distant systems without any violation of Lorentz invariance. Thus, as Costa de Beauregard (1987b: 252) puts it,

Einstein of course is right in seeing an incompatibility between his special relativity theory and the distant quantal correlations, but only under the assumption that advanced actions are excluded.

When Costa de Beauregard in 1947 suggested this response to the EPR argument to his then supervisor de Broglie, de Broglie was “far from willing to accept” the proposal (1987b: 252) and forbade Costa de Beauregard to publish his unorthodox idea (Price & Wharton 2014). However, in 1948 Feynman had developed his eponymous diagrams in which antiparticles were to be interpreted as particles moving backward-in-time along the particle trajectories, and so by 1953 de Broglie had endorsed the publication of Costa de Beauregard’s response. On the seeming craziness of the proposal, Costa de Beauregard claims, “[t]oday, as the phenomenon of the EPR correlations is very well validated experimentally, and is in itself a ‘crazy phenomenon’, any explanation of it must be ‘crazy’”

Another suggested strategy to take into account Russell’s worry while continuing to apply causal notions in physics in a consistent manner is to understand interventionism in “perspectival” terms (Price 2007; Price & Corry 2007; Price & Weslake 2010; Ismael 2014). Perspectivalism is usually staged, as seems natural in the setting of modern physics (although more will be said on this below), in the framework of a block universe view where the past, present, and future are equally real. In this framework, causality cannot have anything to do with *changing* the future or the past because both are—from an “external” perspective—completely “fixed”. But one can understand causation in the block universe from an “internal” perspective, according to which causal correlations are precisely those that are stable under interventions on those variables that we refer to as the “causes”.

The important difference between the two viewpoints—internal and external to the block—is that there is a discrepancy between the parts of the spacetime block that are epistemically accessible from each perspective. The spatiotemporally constrained perspective by which we are bound permits us only limited epistemic accessibility to other spatiotemporal regions. This is the perspective in which, according to causal perspectivalism, causal notions are perfectly serviceable. Once, on the other hand, we imagine ourselves to be omniscient beings that have epistemic access to the whole spatiotemporal block, it should not come as a surprise that our causal intuitions get confused when we attempt to consider how a spatiotemporally bound agent can deliberate about whether or not to affect a particular event that is already determined from our imagined omniscient perspective. It is because we do not know which events are determined to occur and are ignorant about many others that we can be deliberative agents at all. Again, these considerations are relevant just as much to ordinary forward-in-time causation as they are to backward-in-time causation.

Retrocausal approaches Modelling

Many of the retrocausal approaches to quantum theory considered in §6 are best understood with some type of perspectival interventionist account of causality in mind. A notable exception is the transactional interpretation (§5), in which causality might be best understood in terms of processes underscored by conserved quantities. The possibilist extension of the transactional interpretation, defended by Kastner (2006, 2013), moreover eschews the block universe picture.

According to Bell's theorem (Bell 1964; Clauser et al. 1969; see also the entry on Bell's theorem) and its descendants (e.g., Greenberger, Horne, & Zeilinger 1989; see also Goldstein et al. 2011; Brunner et al. 2014 for an overview), any theory that reproduces all the correlations of measurement outcomes predicted by quantum theory must violate a principle that Bell calls local causality (Bell 1976, 1990; see also Norsen 2011; Wiseman & Cavalcanti 2014). In a locally causal theory, probabilities of spatiotemporally localized events occurring in some region 1 are independent of what occurs in a region 2 that is spacelike separated from region 1, given a complete specification of what occurs in a spacetime region 3 in region 1's backward light cone that completely shields off region 1 from the backward light cone of region 2. (See, for instance, Figs. 4 and 6 in Bell 1990 or Fig. 2 in Goldstein et al. 2011.)

In a relativistic setting, then, the notion of locality involves prohibiting conditional dependences between spacelike separated events, provided that the region upon which these spacelike separated events are conditioned constitutes their common causal (Minkowski) past. This characterization of locality implicitly assumes causal asymmetry. Thus locality is the idea that there are no causal relations between spacelike separated events.

There is another sense of "local" that is sometimes used that will be worth avoiding for the purposes of clarity. This is the idea that causal influences are constrained along timelike trajectories. Thus, given Costa de Beauregard's suggestion of "zigzag" causal influences, it is perfectly possible for a retrocausal model of quantum phenomena to be nonlocal in the sense that causal relations exist between spacelike separated events, but "local" in the sense that these causal influences are mediated by timelike trajectories. To avoid ambiguity, it will be useful to refer to this latter sense as "action-by-contact" (set apart from action-at-a-distance).

The first of two main motivating considerations for invoking retrocausality in the foundations of quantum mechanics derives from the exploitation of what is essentially the same loophole in a range of theorems collectively known as "no-go theorems". According to these theorems, any theory or model that is able to account for the empirically confirmed consequences of quantum theory must be unavoidably *nonlocal*, *contextual*, and ψψ-*ontic* (i.e., ascribe reality to the quantum states ψψ).

Retrocausality in circumventing quantum mechanics

One way to understand the role that retrocausality plays in circumventing the results of the no-go theorems is to consider each theorem to be underpinned by what is known as the ontological models framework (Harrigan & Spekkens 2010; Leifer 2014; Ringbauer 2014). The ontological models framework formalizes and captures the central notion of realism in quantum theory (and so subsumes local hidden variable approaches to quantum mechanics). The framework consists of an operational description of a general quantum process, which describes observed statistics for outcomes of measurements given both preparations and transformations, along with an ontological model (or "ontic extension") accounting for the observed statistics. For every preparation procedure, which is usually said to result in a quantum state ψψ, the quantum system is in fact prepared in an "ontic" state λλ, chosen from a set of states ΛΛ, which completely specifies the system's properties. The framework leaves open (and is ultimately used to define) whether the quantum state ψψ is itself an ontic or epistemic state (if ψψ is ontic, λλ either includes additional ontic degrees of freedom, or is in one-to-one correspondence with ψψ;). Each preparation is assumed to result in some λλ via a classical probability density over ΛΛ, and a set of measurement procedures that determine conditional probabilities for outcomes dependent upon λλ (which thus screens off the preparation procedure ψψ); explicitly, λλ does not causally depend on any future measurement setting αα. Finally, the operational statistics must reproduce the quantum statistics.

Important for our purposes here is the qualification that λλ does not causally depend on the measurement setting αα. This assumption is referred to as "measurement independence" (λλ is "conditionally independent" of αα). This assumption is explicitly violated by allowing retrocausal influences to be at play in the system. Thus, in so far as the no-go theorems are underpinned by the ontological models framework, the no-go theorems are no longer applicable to models that allow retrocausality. And in so far as there is motivation to avoid the consequences of the no-go theorems for quantum theory, retrocausality is well placed to provide such a model (or so the argument goes). Notably, it has been argued that admitting retrocausality (i) makes it possible to account for the correlations entailed by quantum theory using action-by-contact causal influences (and so ensures Lorentz invariance); (ii) undermines as implausible the assumption of (certain types of) noncontextuality from the outset; and (iii) may enable an independently attractive ψψ-epistemic interpretation of the wavefunction underpinned by local hidden variables.

Principle of local causality

The principle of local causality, according to Bell, is meant to spell out the idea that

[t]he direct causes (and effects) of events are near by, and even the indirect causes (and effects) are no further away than permitted by the velocity of light. (1990: 105)

Violation of this principle, according to some researchers in the foundations of quantum theory, indicates a fundamental incompatibility between quantum theory and the spirit, perhaps even the letter, of relativity theory

(Maudlin 2011). That the correlations entailed by quantum theory which violate local causality actually occur in nature has been experimentally documented many times (for example, by Freedman & Clauser 1972; Aspect, Dalibard, & Roger 1982; and Aspect, Grangier, & Roger 1982).

Bell's result crucially depends not only on the assumption of local causality, but also on the assumption that whatever variables λλ describes in some spacetime region, which Bell calls "local beables", do not depend probabilistically on which measurement setting αα some experimenters choose in the future of that region:

P(λ|α)=P(λ).(1)(1)P(λ|α)=P(λ).

This is the aforementioned assumption of measurement independence. It is also sometimes referred to as "no superdeterminism" because it is incompatible with a particularly strong form of determinism ("superdeterminism") according to which the joint past of the measurement setting αα and the measured system state λλ determines them both completely and induces a correlation between them. But, as pointed out in the first instance by Costa de Beauregard (1977a) and then by Price (1994, 1996), superdeterminism is not the only can be violated: if there is retrocausality (understood along interventionist lines), the choice of measurement setting αα may causally influence the physical state λλ at an earlier time and thereby also render. Bell's theorem can no longer be derived. Thus, admitting the possibility of retrocausality in principle reopens the possibility of giving a causal account of the nonlocal correlations entailed by quantum theory as mediated by purely action-by-contact, spatiotemporally contiguous, Lorentz invariant causal influences (of the type envisaged by Costa de Beauregard) acting between systems described by local beables.

Standard Model of elementary particle physics

The laws of nature at the most fundamental level at which they are currently known are combined in the Standard Model of elementary particle physics. These laws are CPT-invariant, i.e., they remain the same under the combined operations of charge-reversal *C* (replacing all particles by their anti-particles), parity *P* (flipping the signs of all spatial coordinates), and time-reversal *T*. The asymmetries in time which are pervasive in our everyday lives are a consequence not of any temporal asymmetry in these laws but, instead, of the boundary conditions of the universe, notably in its very early stages. It seems natural to assume that the time-symmetry of the laws (modulo the combined operation of *C* and *P*) extends to causal dependences at the fundamental "ontic" level that underlies the empirical success of quantum theory. If so, there may be backward-in-time no less than forward-in-time causal influences at that ontic level.

Price (2012) turns these sketchy considerations into a rigorous argument. He shows that, when combined with two assumptions concerning quantum ontology, time-symmetry implies retrocausality (understood along broadly interventionist lines). The ontological assumptions are (i) that at least some aspects of the quantum state ψψ are real (notably, in Price's example, there is a "beable" encoding photon polarization angle), and (ii) that inputs and outputs of quantum experiments are *discrete* emission and detection events. Moreover, it is important to Price's argument that dynamical time-symmetry (that the dynamical laws of the theory are time-symmetric) be understood as implying that operational time-symmetry (that the set of all possible combinations of preparation and measurement procedures in a theory, with associated probabilities for outputs given inputs, is closed under interchange of preparation and measurement) translates into ontic time-symmetry (operational time-symmetry plus a suitable map between the ontic state spaces of the symmetric combinations). Given these conditions, any foundational account that reproduces the empirical verdicts of quantum theory must be retrocausal.

Leifer and Pusey (2014) (Other Internet Resources) strengthen Price's argument by showing that his assumption about the reality of (aspects of) the quantum state ψψ can be relaxed. As they demonstrate, if measurement outcomes depend only on a system's ontic state λλ, i.e., if that state completely mediates any correlations between preparation procedures and measurement outcomes ("λλ-mediation"), this suffices for operational time-symmetry to entail the existence of retrocausality. Foundational accounts which like Bohmian mechanics (Bohm 1952a,b) or GRW-theory (Ghirardi, Rimini, & Weber 1986) avoid postulating retrocausality do so by violating time-symmetry in some way. The GRW-theory does so by introducing explicitly time-asymmetric dynamics. In Bohmian mechanics the dynamics is time-symmetric, but the theory is applied in a time-asymmetric manner when assessing which quantum states are actually realized. Notably, one assumes that the quantum states of a measured system and a measurement device connected to it are uncorrelated prior to measurement, whereas they are in general correlated after measurement

Conclusion

Spekkens' (2005) claim that no noncontextual ontological model can reproduce the observed statistics of quantum theory based on his principle of parsimony (that there can be no ontological difference without operational difference) was sidestepped by retrocausal approaches due to the explicit assumption of the ontological models framework that the ontic state is independent of the measurement procedure (i.e., that there is no retrocausality). It was noted there the possibility that Spekkens' principle of parsimony might be recast to apply more generally to retrocausal models. Shrapnel and Costa (2014) achieve just this in a no-go theorem that applies to any exotic causal structure used to sidestep the ontological models framework, including retrocausal accounts, rendering such models contextual after all.

Shrapnel and Costa's result is based on a generalization of the ontological models framework which replaces the operational preparation, transformation, and measurement procedures with the temporally and causally neutral notions of local controllables and environmental processes that mediate correlations between different local systems, and generate the joint statistics for a set of events. "These include any global properties, initial states, connecting mechanisms, causal influence, or global dynamics"

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