Graph Energies of Anthraquinone: Energy, Seidel Energy, Distance Energy, Harary Energy, Maximum Degree Energy

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Abstract:

In this paper I compute Energy, Seidel energy, Distance energy, Harary energy, Maximum degree energy of Anthraquinone.

Key words:

Anthraquinone, Eigen values, Characteristic equation, Energy, Seidel energy, Distance energy, Harary energy, Maximum degree energy.

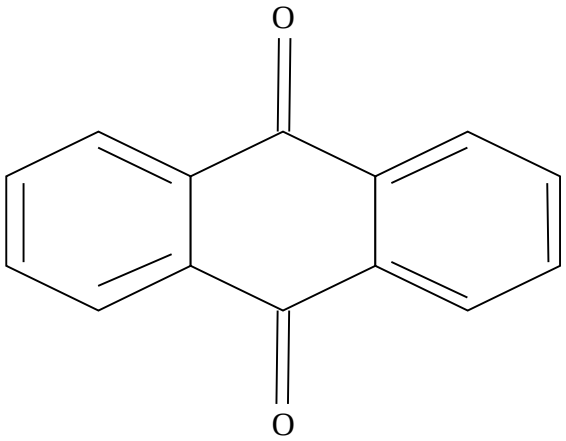
1. Introduction

Anthraquinones are active components of many plant blends which are used as medicines and exhibit laxative, diuretic, estrogenic and immunomodulatory effects. Anthraquinones are structurally related to anthracene and possess the 9,10-dioxoanthracene. Anthraquinones typically occur in their glycosidic forms. These compounds impart colour to plants and have been widely utilized as natural dyes. Anthraquinone is an important and widely used raw material for the manufacture of vat dyes, which are a class of water-insoluble dyes that can easily be reduced to a water soluble and usually colourless leuco form that readily impregnates fibres and textiles. In addition, they are also used as laxatives and possess antifungal and antiviral activities. It is also used as a seed dressing or in seed treatments. Other major uses are as a pesticide, as a bird repellent (especially for geese), and as an additive in chemical alkaline pulp processes in the paper and pulp industry.

So far, 79 naturally occurring anthraquinones have been identified which include emodin, physcion, cascarin, catenarin and Rhein. A large body of literature has demonstrated that the naturally occurring anthraquinones possess a broad spectrum of bioactivities, such as cathartic, anticancer, anti-inflammatory, antimicrobial, diuretic, Vaso relaxing and phytoestrogen activities, suggesting their possible clinical application in many diseases.

In this paper I compute Energy, Seidel energy, Distance energy, Harary energy, Maximum degree energy of Anthraquinone.

2. Structural and Molecular formulae

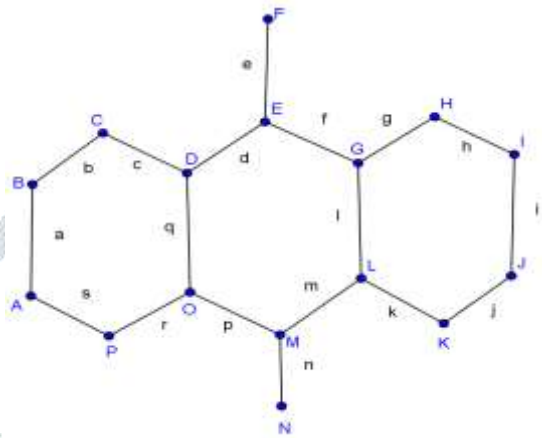


Molecular formulae: $C_{14}H_8O_2$

3. Energy of a Graph

The energy of a graph is one of the emerging concepts within graph theory. This concept serves as a frontier between chemistry and mathematics and is defined in1978 by I. Gutman [1] and originating from theoretical chemistry. In this paper we consider all graphs are simple without loops and multiple edges, finite and undirected. For standard terminology and notations related to graph theory, we follow Balakrishnan and Ranganathan [2]. The energy of a graph is zero if and only if it is trivial. The energy of a graph is one of the emerging concepts within graph theory. This concept serves as a frontier between chemistry and mathematics [3].

Let us consider the graph of Anthraquinone (i.e., Graph G) as shown in the following fig.



Graph G: Graph of Anthraquinone

Here the vertices A, B, C, D, ...M, N, O, P are treated as the vertices $v_1, v_2, ...v_{13}, v_{14}, v_{15}, v_{16}$.

In general, G be a graph possessing n vertices and m edges. Let $v_1, v_2, ...v_n$ be the vertices of G. Then the adjacency matrix $A(G)$ of the graph G is the square matrix of order n whose (i, j) entry is defined as

$$a_{ij} = \begin{cases} 1 & if \quad i \neq j \quad and\ v_i\ and\ v_j\ are\ adjacent \\ 0 & if \quad i \neq j \quad and\ v_i\ and v_j\ are\ not\ adjacent \\ 0 & if \quad i = j \end{cases}$$

The eigenvalues $\lambda_1, \lambda_2, \lambda_3, ...\lambda_n$ of the graph G are the eigen values of its adjacency matrix. Since $A(G)$ is real symmetric, the eigen values of G are real numbers whose sum equal to zero.

The energy of a graph G is the sum of absolute values of the eigen values of a graph G and denoted it by $E(G)$. Hence

$$E(G) = \sum_{i=1}^{n} |\lambda_i|$$

$E(G)$ will be referred as the ordinary energy of the graph G.

Energy of the Anthraquinone:

Adjacency matrix of Anthraquinone is shown in matrix 1.

The Characteristic equation is

$$x^{16} - 18x^{14} + 127x^{12} - 456x^{10} + 903x^8 - 998x^6 + 593x^4 - 168x^2 + 16 = 0$$

The Eigen values of above characteristic equation are

$\lambda_1 = -2.49889, \lambda_2 = 2.49889, \lambda_3 = -2, \lambda_4 = 2, \lambda_5 = -1.6624, \lambda_6 = 1.6624, \lambda_7 = -1.49592, \lambda_8 = 1.49592\ \lambda_9 = -1,\ \lambda_{10} = 1, \lambda_{11} = -1, \lambda_{12} = 1,$
$\lambda_{13} = -0.757366, \lambda_{14} = 0.757366, \lambda_{15} = -0.424945,\ \lambda_{16} = 0.424945.$

The Energy of Anthraquinone is

$$\varepsilon(C_{14}H_8O_2) = |-2.49889| + |2.49889| + |-2| + |2| + |-1.6624| + |1.6624| + |-1.49592| + |1.49592| + |-1| + |1| + |-1| + |1| + |-0.757366| + |0.757366| + |-0.424945| + |0.424945|$$

$$= 21.679042$$

4. Seidel Energy of a Graph:

LIU Jian-ping and LIU Bo-lian defined the Seidel energy of the graph G.

The Seidel matrix of a graph is a real square symmetric matrix $S(G) = \lfloor s_{ij} \rfloor$ and whose (i, j) entry is defined as

$$s_{ij} = \begin{cases} -1 & if \quad i \neq j \quad and\ v_i\ and\ v_j\ are\ adjacent \\ 1 & if \quad i \neq j \quad and\ v_i\ and v_j\ are\ not\ adjacent \\ 0 & if \quad i = j \end{cases}$$

Obviously $S(G) = J - I - 2A$ where J denotes a square matrix all of whose entries are 1 and A is the adjacency matrix of the graph G. Let $\lambda_1, \lambda_2, \lambda_3, ..., \lambda_n$ be the eigenvalues of the Seidel matrix $S(G)$. The Seidel energy [4] of the graph G is defined as $SE(G) = \sum_{i=1}^{n} |\lambda_i|$. The Seidel energy definition is in analogy with the graph energy $E(G) = \sum_{i=1}^{n} |\lambda_i|$ where $\lambda_1, \lambda_2, ..., \lambda_n$ are the eigenvalues of the adjacency matrix of G. The Seidel energy is always greater than the energy of the graph. The graphs G_1 and G_2 are said to be Seidel equi energetic if $SE(G_1) = SE(G_2)$.

Seidel Energy of the Anthraquinone:

Seidel matrix of Anthraquinone is as shown in matrix 2.

The Characteristic equation is

$$x^{16} - 120x^{14} - 320x^{13} + 3516x^{12} + 12096x^{11} - 43080x^{10} - 165888x^9 + 268806x^8 + 1048704x^7$$
$$-1005512x^6 - 3102528x^5 + 2554300x^4 + 3401792x^3 - 3496440x^2 + 379008x + 145665 = 0$$

The Eigen values are

$\lambda_1 = 3, \lambda_2 = 1, \lambda_3 = 1, \lambda_4 = -3, \lambda_5 = -3, \lambda_6 = -5, \lambda_7 = 3.99779, \lambda_8 = 1.99185, \lambda_9 = -0.15011,\ \lambda_{10} = -2.51473, \lambda_{11} = -4.32479, \lambda_{12} = 10.6458,$
$\lambda_{13} = 0.377308, \lambda_{14} = -2.1297, \lambda_{15} = -4.20226, \lambda_{16} = 2.3089.$

The Seidel Energy of Anthraquinone is

$$S\varepsilon(C_{14}H_8O_2) = |3| + |1| + |1| + |-3| + |-3| + |-5| + |3.99779| + |1.99185| + |-0.15011| + |-2.51473| + |-4.32479| + |10.6458| + |0.377308| + |-2.1297| + |-4.20226| + |2.3089|$$

$$= 48.63901$$

5. Distance Energy of a Graph:

On addressing problem for loop switching, R. L. Graham, H. O. Pollak defined distance matrix of a graph. The concept of distance energy was defined by G. Indulal etal [5] in the year 2008.

The distance matrix of a graph has several applications in chemistry, music theory, ornithology, molecular biology, psychology, archaeology etc. The distance matrix, contains information on various walks and self-avoiding walks of chemical graphs, is extremely helpful in the calculation of topological indices such as the Wiener index.

Wiener index is useful in the computation of thermodynamic properties such as pressure and temperature coefficients and distance matrix contain more structural information compared to a simple adjacency matrix.

The Distance matrix of G is the square matrix $D(G)$ of order n whose (i, j) th entry is the distance (length of shortest path) between the vertices v_i and v_j . the Distance energy $DE(G)$,is defined as, $DE(G)=\sum_{i=1}^{n}|\lambda_i|$, where the eigen values $\lambda_1,\lambda_2,.....\lambda_n$ are the D-eigen values of the distance matrix of G. Since D(G) is a real symmetric matrix with zero trace, its eigenvalues are real numbers. The sum of these eigen values is zero. The diameter of G, denoted by diam(G), is the maximum distance between any two vertices of G. The girth of G is the length of a shortest cycle in G. Two graphs with the same D-energy are called D - equi energetic.

Distance Energy of Anthraquinone:

Distance matrix of Anthraquinone is as shown in matrix 3.

The Characteristic equation is

$$x^{16}-1500x^{14}-36320x^{13}-369356x^{12}-1953936x^{11}-5613328x^{10}-7949696x^{9}-2045120x^{8}$$
$$+8439808x^{7}+9352960x^{6}+1817600x^{5}+1588224x^{4}-589824x^{3}=0$$

The Eigen values are

$$\lambda_1=0,\lambda_2=0,\lambda_3=-19.607,\lambda_4=-11.4669,\lambda_5=-6.27375,\lambda_6=-3.7968,\lambda_7=-2.59614,\lambda_8=-2.37451\,\lambda_9=-2.07734,\ \lambda_{10}=-1.06372,$$
$$\lambda_{11}=-0.731705,\lambda_{12}=-0.52857,\lambda_{13}=0.430395,\lambda_{14}=0,\lambda_{15}=0.989282,\lambda_{16}=49.0968.$$

The Distance Energy of Anthraquinone is

$$DE(C_{14}H_8O_2)=\ |-19.607|+|-11.4669|+|-6.27375|+|-3.7968|+|-2.59614|+|-2.37451|+|-2.07734|$$
$$+|-1.06372|+|-0.731705|+|-0.52857|+|0.430395|+|0.989282|+|49.0968|$$
$$=101.032912$$

6. Harary energy of a graph:

The concept of Harary energy was introduced by A. Dilek Gungor and A. Sinan Cevik [6]. The Harary matrix is also called as reciprocal distance matrix of a graph g is $n\times n$ matrix $RD(G)=\lfloor r_{ij}\rfloor$, in which

$$r_{ij}=\begin{cases}\dfrac{1}{d(v_i,v_j)} & if \quad i\neq j\\ 0 \quad otherwise\end{cases}$$

where $d(v_i,v_j)$ is the distance between v_i and v_j vertices.

The characteristic polynomial of $RD(G)$ is defined as $\psi(G:\mu)=\det(\mu I-RD(G))$, where I is the identity matrix of order n. The eigen values of the Harary matrix, $RD(G)$, denoted by $\lambda_1,\lambda_2,\lambda_3,.....,\lambda_n$ are said to be the Harary eigen values or H-Eigen values of G and their collection is called the Harary spectrum or H-spectrum of G. Since the Harary matrix of G is symmetric, its eigen values are real. Two non-isomorphic graphs are said to be Harary co spectral or H-co spectral if they have same H-eigen values.

The Harary energy or H-energy of a graph G, denote $HE(G)$, is defined as $HE(G)=\sum_{i=1}^{n}|\lambda_i|$

Harary Energy of Anthraquinone:

Harary matrix of Anthraquinone is as shown in matrix 4.

The Characteristic equation is

$$x^{16}-\frac{1746359}{58800}x^{14}-\frac{4348417}{50400}x^{13}+\frac{136792875601}{31116960000}x^{12}+\frac{3664060153}{10080000}x^{11}+\frac{9812867722626607}{18670176000000}x^{10}-\frac{15371825558145179}{672126336000000}x^{9}$$
$$-\frac{69745292738072496553}{100818950400000000}x^{8}-\frac{702978257060136727399}{1209827404800000000}x^{7}-\frac{5274277974553359873937}{483930961920000000000}x^{6}+\frac{1955453165895558578701547}{871075731456000000000}x^{5}$$
$$+\frac{31408076725881816130765937}{2613227194368000000000000}x^{4}+\frac{68266540410184306852176 4909}{3135872633241600000000000}x^{3}+\frac{88111868171672299946529809}{1254349053296640000000000}x^{2}$$
$$-\frac{32415613512375216548717 11}{56445707398348800000000000}x+\frac{17486178526405741221629 3}{48382034912870400000000000}=0$$

The Eigen values are

$\lambda_1=-1.54739, \lambda_2=-1.39082, \lambda_3=-1.35457, \lambda_4=-1.2643, \lambda_5=-1.20339, \lambda_6=-1.17632, \lambda_7=-1.03347, \lambda_8=-0.863715\,\lambda_9=-0.311932$, $\lambda_{10}=-0.112658, \lambda_{11}=0.00703627, \lambda_{12}=0.0306486, \lambda_{13}=0.725306, \lambda_{14}=1.14082, \lambda_{15}=1.92447, \lambda_{16}=6.43037$.

The Harary Energy of Anthraquinone is

$$\begin{aligned}H\varepsilon(C_{14}H_8O_2)= &|-1.54739|+|-1.39082|+|-1.35457|+|-1.2643|+|-1.20339|+|-1.17632|+|-1.03347|\\ &+|-0.863715|+|-0.311932|+|-0.112658|+|0.00703627|+|0.0306486|+|0.725306|\\ &+|1.14082|+|1.92447|+|6.43037|\\ &=20.5172\end{aligned}$$

7. Maximum Degree Energy of a graph:

In the year 2009 Professor Adiga and Smitha [7] defined maximum degree energy of a graph. Let G be a simple graph with n vertices $v_1, v_2, \ldots v_n$ and let d_i be the degree of v_i, $i=1,2,\ldots\ldots, n$.
Define

$$d_{ij}=\begin{cases}\max\{d_i,d_j\} & \text{if } v_i \text{ and } v_j \text{ are adjacent,}\\ 0 & \text{otherwise}\end{cases}$$

then the $n\times n$ matrix $M(G)=\lfloor d_{ij}\rfloor$ is called the maximum degree matrix of G. The characteristic polynomial of the maximum degree matrix $M(G)$ is defined by $\phi(G:\lambda)=\det(\lambda I-M(G))$ where I is the unit matrix of order n. The roots $\lambda_1,\lambda_2,\lambda_3,\ldots\ldots\ldots,\lambda_n$ of the equation $\phi(G:\lambda)=0$ are the maximum degree eigen values of G. Since $M(G)$ is a real symmetric matrix with zero trace, these maximum degree eigen values are real with sum equal to zero and the sum of these eigen values is equal to zero. The maximum degree energy of a graph is defined as $E_M(G)=\sum_{i=1}^{n}|\lambda_i|$

Maximum degree energy of Anthraquinone:

Maximum degree matrix of Anthraquinone is as shown in the matrix 5.

The Characteristic equation is

$$x^{16}-132x^{14}+6512x^{12}-156462x^{10}+1990768x^{8}-13849884x^{6}+51903585x^{4}$$
$$-95790600x^{2}+65610000=0$$

The Eigen values are

$\lambda_1=-2.51691, \lambda_2=2.51691, \lambda_3=-2.27299, \lambda_4=2.27299, \lambda_5=-5.24392, \lambda_6=5.24392, \lambda_7=-1.26496, \lambda_8=1.26496\,\lambda_9=-1.86092,$
$\lambda_{10}=1.86092, \lambda_{11}=-3.18713, \lambda_{12}=3.18713, \lambda_{13}=-4.91432, \lambda_{14}=4.91432, \lambda_{15}=-7.32315, \lambda_{16}=7.32315.$

The maximum degree Energy of Anthraquinone is

$$\begin{aligned}E_M(C_{14}H_8O_2)= &|-2.51691|+|2.51691|+|-2.27299|+|2.27299|+|-5.24392|+|5.24392|+|-1.26496|+|1.26496|\\ &+|-1.86092|+|1.86092|+|-3.18713|+|-4.91432|+|4.91432|+|-7.32315|+|7.32315|\\ &=57.1686\end{aligned}$$

$$A\left(C_{14}H_8O_2\right)=$$

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}	v_{15}	v_{16}
v_1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
v_2	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
v_3	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
v_4	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0
v_5	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0
v_6	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
v_7	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0
v_8	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0
v_9	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0
v_{10}	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0
v_{11}	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0
v_{12}	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	0
v_{13}	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0
v_{14}	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
v_{15}	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1
v_{16}	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0

Matrix 1: Adjacency Matrix of Anthraquinone

$$S(C_{14}H_8O_2)=\begin{array}{c|cccccccccccccccc} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \\ \hline v_1 & 0 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 \\ v_2 & -1 & 0 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ v_3 & 1 & -1 & 0 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ v_4 & 1 & 1 & -1 & 0 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 \\ v_5 & 1 & 1 & 1 & -1 & 0 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ v_6 & 1 & 1 & 1 & 1 & -1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ v_7 & 1 & 1 & 1 & 1 & -1 & 1 & 0 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 \\ v_8 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 0 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ v_9 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 0 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\ v_{10} & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 0 & -1 & 1 & 1 & 1 & 1 & 1 \\ v_{11} & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 0 & -1 & 1 & 1 & 1 & 1 \\ v_{12} & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 0 & -1 & 1 & 1 & 1 \\ v_{13} & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 0 & -1 & -1 & 1 \\ v_{14} & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 0 & 1 & 1 \\ v_{15} & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 0 & -1 \\ v_{16} & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 0 \end{array}$$

Matrix 2: Seidel matrix of Anthraquinone

$$D(C_{14}H_8O_2) = \begin{bmatrix} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \\ v_1 & 0 & 1 & 2 & 3 & 4 & 5 & 5 & 6 & 7 & 6 & 5 & 4 & 3 & 4 & 2 & 1 \\ v_2 & 1 & 0 & 1 & 2 & 3 & 4 & 4 & 5 & 6 & 7 & 6 & 5 & 4 & 5 & 3 & 2 \\ v_3 & 2 & 1 & 0 & 1 & 2 & 3 & 3 & 4 & 5 & 6 & 5 & 4 & 3 & 4 & 2 & 3 \\ v_4 & 3 & 2 & 1 & 0 & 1 & 2 & 2 & 3 & 4 & 5 & 4 & 3 & 2 & 3 & 1 & 2 \\ v_5 & 4 & 3 & 2 & 1 & 0 & 1 & 1 & 2 & 3 & 4 & 5 & 4 & 3 & 4 & 2 & 3 \\ v_6 & 5 & 4 & 3 & 2 & 1 & 0 & 2 & 3 & 4 & 5 & 4 & 5 & 4 & 5 & 3 & 4 \\ v_7 & 5 & 4 & 3 & 2 & 1 & 2 & 0 & 1 & 2 & 3 & 2 & 1 & 2 & 3 & 3 & 4 \\ v_8 & 6 & 5 & 4 & 3 & 2 & 3 & 1 & 0 & 1 & 2 & 3 & 2 & 3 & 4 & 4 & 5 \\ v_9 & 7 & 6 & 5 & 4 & 3 & 4 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 5 & 6 \\ v_{10} & 6 & 7 & 6 & 5 & 4 & 5 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 4 & 5 \\ v_{11} & 5 & 6 & 5 & 4 & 5 & 4 & 2 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 3 & 4 \\ v_{12} & 4 & 5 & 4 & 3 & 4 & 5 & 1 & 2 & 3 & 2 & 1 & 0 & 1 & 2 & 2 & 3 \\ v_{13} & 3 & 4 & 3 & 2 & 3 & 4 & 2 & 3 & 4 & 3 & 2 & 1 & 0 & 1 & 1 & 2 \\ v_{14} & 4 & 5 & 4 & 3 & 4 & 5 & 3 & 4 & 5 & 4 & 3 & 2 & 1 & 0 & 2 & 3 \\ v_{15} & 2 & 3 & 2 & 1 & 2 & 3 & 3 & 4 & 5 & 4 & 3 & 2 & 1 & 2 & 0 & 1 \\ v_{16} & 1 & 2 & 3 & 2 & 3 & 4 & 4 & 5 & 6 & 5 & 4 & 3 & 2 & 3 & 1 & 0 \end{bmatrix}$$

Matrix 3: Distance matrix of Anthraquinone

$$H(C_{14}H_8O_2)=\begin{array}{c|cccccccccccccccc}
 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \\
\hline
v_1 & 0 & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{6} & \frac{1}{5} & \frac{1}{4} & \frac{1}{3} & \frac{1}{4} & \frac{1}{2} & \frac{1}{1} \\
v_2 & 1 & 0 & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{6} & \frac{1}{5} & \frac{1}{4} & \frac{1}{5} & \frac{1}{3} & \frac{1}{2} \\
v_3 & \frac{1}{2} & 1 & 0 & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{5} & \frac{1}{4} & \frac{1}{3} & \frac{1}{4} & \frac{1}{2} & \frac{1}{3} \\
v_4 & \frac{1}{3} & \frac{1}{2} & 1 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & \frac{1}{3} & 1 & \frac{1}{2} \\
v_5 & \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & 1 & 0 & 1 & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{4} & \frac{1}{3} & \frac{1}{4} & \frac{1}{2} & \frac{1}{3} \\
v_6 & \frac{1}{5} & \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & 1 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{4} & \frac{1}{5} & \frac{1}{4} & \frac{1}{5} & \frac{1}{3} & \frac{1}{4} \\
v_7 & \frac{1}{5} & \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & 1 & \frac{1}{2} & 0 & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{3} & \frac{1}{4} \\
v_8 & \frac{1}{6} & \frac{1}{5} & \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & \frac{1}{3} & 1 & 0 & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{4} & \frac{1}{5} \\
v_9 & \frac{1}{7} & \frac{1}{6} & \frac{1}{5} & \frac{1}{4} & \frac{1}{3} & \frac{1}{4} & \frac{1}{2} & 1 & 0 & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{5} & \frac{1}{6} \\
v_{10} & \frac{1}{6} & \frac{1}{7} & \frac{1}{6} & \frac{1}{5} & \frac{1}{4} & \frac{1}{5} & \frac{1}{3} & \frac{1}{2} & 1 & 0 & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{4} & \frac{1}{5} \\
v_{11} & \frac{1}{5} & \frac{1}{6} & \frac{1}{5} & \frac{1}{4} & \frac{1}{5} & \frac{1}{4} & \frac{1}{2} & \frac{1}{3} & \frac{1}{2} & 1 & 0 & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{3} & \frac{1}{4} \\
v_{12} & \frac{1}{4} & \frac{1}{5} & \frac{1}{4} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{2} & 1 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{3} \\
v_{13} & \frac{1}{3} & \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & 1 & 0 & 1 & 1 & \frac{1}{2} \\
v_{14} & \frac{1}{4} & \frac{1}{5} & \frac{1}{4} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & 1 & 0 & \frac{1}{2} & \frac{1}{3} \\
v_{15} & \frac{1}{2} & \frac{1}{3} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & 1 & \frac{1}{2} & 0 & 1 \\
v_{16} & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{5} & \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & \frac{1}{3} & 1 & 0
\end{array}$$

Matrix 4: Harary matrix of Anthraquinone

$$M(C_{14}H_8O_2)=\begin{array}{c|cccccccccccccccc} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \\ \hline v_1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ v_2 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ v_3 & 0 & 2 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ v_4 & 0 & 0 & 3 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ v_5 & 0 & 0 & 0 & 3 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ v_6 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ v_7 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 3 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ v_8 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ v_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ v_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ v_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 3 & 0 & 0 & 0 & 0 \\ v_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 3 & 0 & 3 & 0 & 0 & 0 \\ v_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 3 & 0 \\ v_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ v_{15} & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 3 \\ v_{16} & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \end{array}$$

Matrix 5: Maximum degree matrix of Anthraquinone

8. Conclusion:

In this article, I compute Seidel energy, Distance energy, Harary energy, Maximum degree energy of Anthraquinone.

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