RELIABILITY ANALYSIS OF STOCHASTIC MODEL OF MINI DAIRY PLANT WITH TWO OUTPUT MACHINES

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ABSTRACT

This research paper deals with reliability analysis of a mini dairy plant with two output machines. This model consists of different sub-system of various natures. The analysis of this system has been done with the help of Regenerative point technique by taken repair time distributions as general and failure time distribution as exponential. In particular, we have taken the repair time distribution as exponential distribution. Some different measures such as Reliability, Mean time to system failure, Availability analysis, Busy period analysis and Profit analysis have been obtained for the model. To highlight the important results, a graphical study has also been done.

Keywords: Renewal Process, MTSF, Availability, Profit function

INTRODUCTION

Many authors have analysed the real existing industrial systems. Pandey *et al*. [8] worked with reliability analysis of star type local area network. Gupta and Shivakar [3] working with the analysis of stochastic model of cloth weaving system. Gupta and Taneja [4] have analysed a reliability model on a cement grinding system with failure in its nine components. Sharma *et al.* [10] has worked with reliability analysis of a stochastic model of cheese making plant. But very few authors have studied the mini industrial real existing system. Sharma and Kumar [9] have worked with reliability analysis of stochastic model of mini dairy plant. In this model only one output machine has been taken. But there are several mini dairy plants in Noida and its area in which two output machines are being used.

Keeping this idea in our mind we in this paper have analysed a real existing system model of mini dairy plant with two output machines. This mini dairy plant is of complex type reparable engineering system, involving high risk which consists of four units namely Bulk milk cooler (R), Pasteurization unit (P), Pouch making machine (H_1) and Bottle filling machine (H_2). These type plants have been manufactured by different engineering companies and installed in Noida and its area as small scale industries.

Various sub-systems of mini Dairy Plant and their functioning

The sub-system and their working is described as given below

(a) **Bulk Milk Cooler (R)** : Bulk milk cooler (B.M.C.) is used for cooling the raw milk from 30^0C to 4^0C and milk can be stored in it at 4^0C for upto 96 hours.

(b) **Pasteurization Unit (P)** : Pasteurization is a process of heating milk to at least 72^0C (160^0F) for 17 seconds and cooled immediately to 4^0C. This process makes milk safe for human consumption by destruction of cent percent pathogenic.

(c) **Pouch Making Machine (H_1)** : After pasteurization the milk is sent to the storage tank. This tank is connected to the pouch making machine. This machine automatically makes the different size pouches.

(d) **Bottle Filling Machine (H_2)** : After pasteurization the milk is send to the storage tank. This storage tank is also connected to the bottle filling machine. This machine automatically fills the milk to the different size bottles.

The network presentation of the system configuration is shown by the following figure 1.

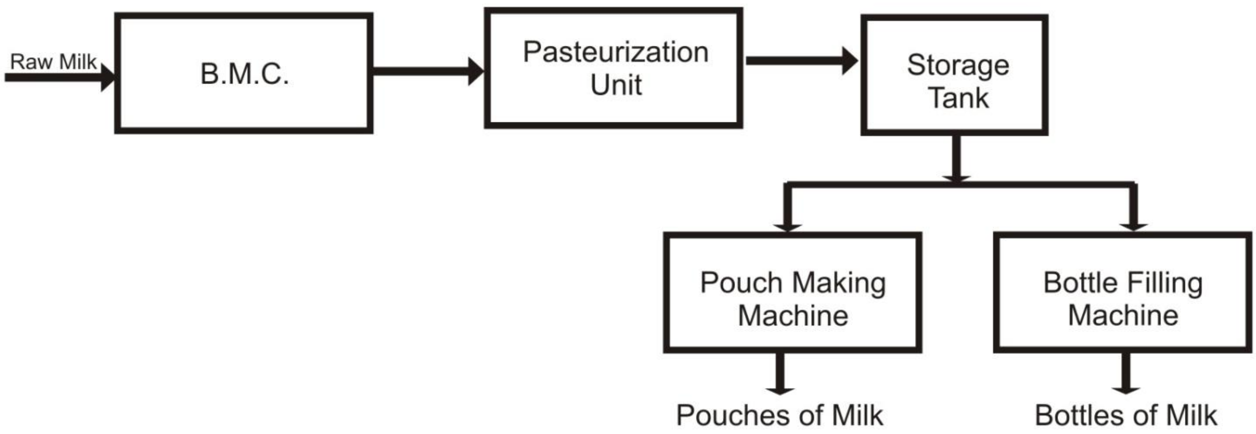


Figure 1

ASSUMPTIONS

(i) Failure and repair are stochastically independent.
(ii) A single repair facility is used to repair a failed sub-system/unit.
(iii) Service discipline is FCFS.
(iv) A repaired unit is good as new and immediately reconnected to the system.
(v) Failure rates of all the units are constant.
(vi) Repair time distributions of all the units are general.

By using regenerative point technique in Markov Renewal Process the following measures of the system effectiveness are obtained as

(1) Steady state transition probabilities.
(2) Mean sojourn time.
(3) Reliability and mean time to system failure of the system.
(4) Availability analysis.
(5) Expected up time of the system and expected busy period of the repairman in time interval (0, t].
(6) Net expected Profit by the system in (0, t) and in steady state.

NOTATIONS AND STATES OF THE SYSYEM

α_i constant failure rate of the units R/P/H_1/H_2 respectively for i=1, 2, 3, 4

$g_i(\cdot), G_i(\cdot)$ P.d.f. and c.d.f. of the repair time units R/P/H_1/H_2 respectively for i=1, 2, 3, 4

q_{ij} P.d.f. of transition time from state S_i to S_j

P_{ij} Steady state transition probability from S_i to S_j

$Z_i(t)$ Probability that the sojourn in state S_i upto time t

ψ_i Mean sojourn time in state S_i

$*$ Symbol for Laplace Transformation

Symbols which are used for states of the system

$R_g/R_o/R_r/R_{wr}$ Unit R is good/operative/under repair/waiting for repair

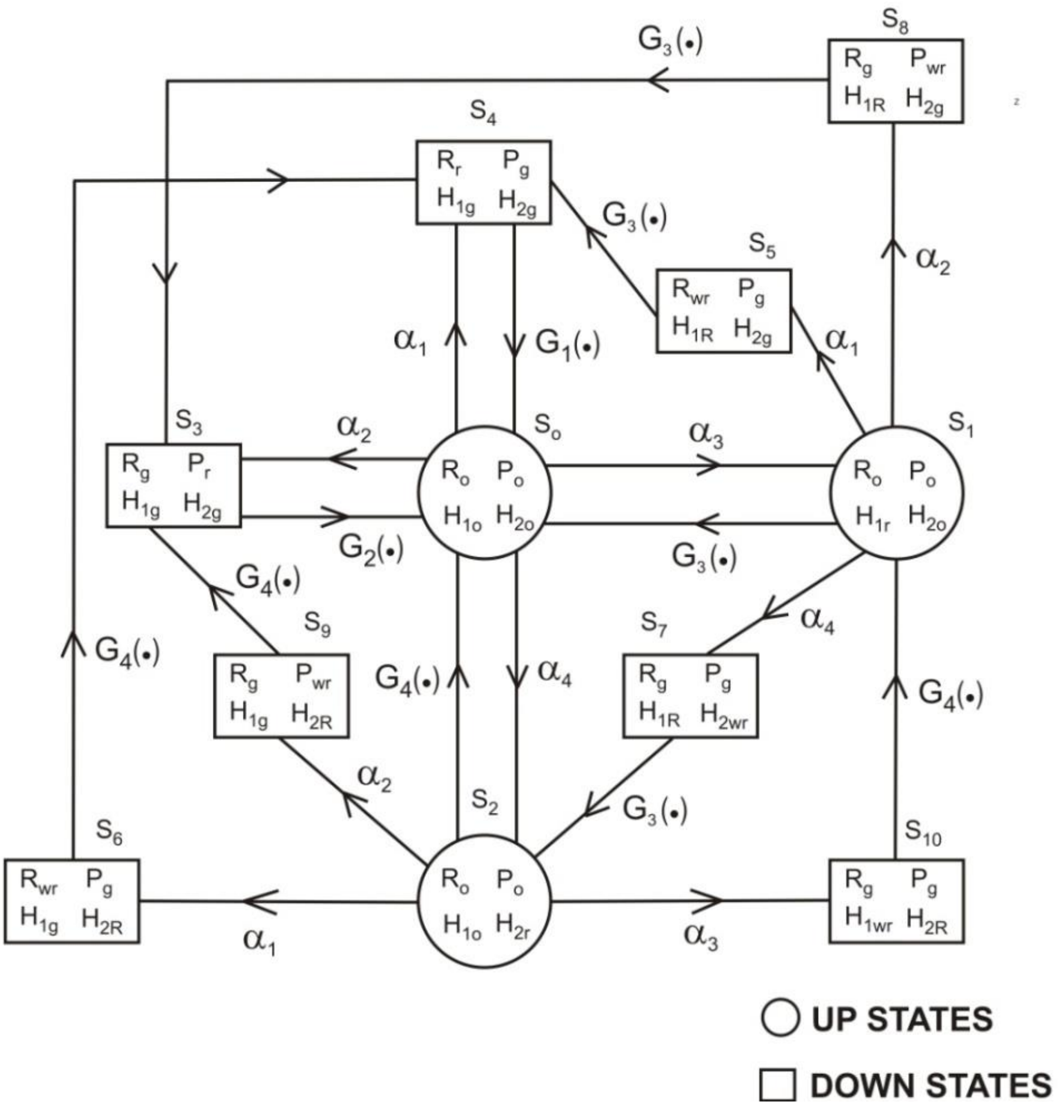
$P_g/P_o/P_r/P_{wr}$ Unit P is good/operative/under repair/waiting for repair

$H_{1g}/H_{1o}/H_{1r}/H_{1R}/H_{1wr}$ Unit H_1 is good/operative/under repair/repair continued/waiting for repair

$H_{2g}/H_{2o}/H_{2r}/H_{2R}/H_{2wr}$ Unit H_2 is good/operative/under repair/repair continued/waiting for repair

The possible states of the system are S_0 to S_{10} in which S_0, S_1 and S_2 are operative and the states S_3 to S_{10} are failed states.

Transition Diagram



TRANSITION PROBABILITIES AND SOJOURN TIMES

Using simple probabilistic laws the expressions for transition probabilities in steady state are given as

$$P_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt$$

$P_{01} = \frac{\alpha_3}{\Sigma\alpha_i}$, $P_{02} = \frac{\alpha_4}{\Sigma\alpha_i}$, $P_{03} = \frac{\alpha_2}{\Sigma\alpha_i}$, $P_{04} = \frac{\alpha_1}{\Sigma\alpha_i}$, where $\Sigma\alpha_i = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$

$P_{10} = g_3^*(\alpha_1 + \alpha_2 + \alpha_4)$, $P_{12}^{(7)} = \frac{\alpha_4}{\alpha_1+\alpha_2+\alpha_4}[1 - g_3^*(\alpha_1 + \alpha_2 + \alpha_4)]$

$P_{13}^{(8)} = \frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_4}[1 - g_3^*(\alpha_1 + \alpha_2 + \alpha_4)]$, $P_{14}^{(5)} = \frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_4}[1 - g_3^*(\alpha_1 + \alpha_2 + \alpha_4)]$

$P_{20} = g_4^*(\alpha_1 + \alpha_2 + \alpha_3)$, $P_{21}^{(10)} = \frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3}[1 - g_4^*(\alpha_1 + \alpha_2 + \alpha_3)]$

$P_{23}^{(9)} = \frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3}[1 - g_4^*(\alpha_1 + \alpha_2 + \alpha_3)]$, $P_{24}^{(6)} = \frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3}[1 - g_4^*(\alpha_1 + \alpha_2 + \alpha_3)]$

$P_{30} = P_{40} = 1$

These probabilities satisfy the following relations

$$P_{01} + P_{02} + P_{03} + P_{04} = 1$$

$$P_{10} + P_{12}^{(7)} + P_{13}^{(8)} + P_{14}^{(5)} = 1$$

$$P_{20} + P_{21}^{(10)} + P_{23}^{(9)} + P_{24}^{(6)} = 1$$

MEAN SOJOURN TIME

Mean sojourn time ψ_i in state S_i is defined as the expected time for which the system stays in state S_i before transiting to any other state. Let X_i denotes the sojourn time in state S_i, is given as

$$\psi_i = \int P[X_i > t]dt$$

so that

$\psi_0 = \frac{1}{\Sigma\alpha_i}$, $\psi_1 = 1 - \frac{\tilde{G}_3(\alpha_1+\alpha_2+\alpha_4)}{\alpha_1+\alpha_2+\alpha_4}$, $\psi_2 = 1 - \frac{\tilde{G}_4(\alpha_1+\alpha_2+\alpha_3)}{\alpha_1+\alpha_2+\alpha_3}$

$\psi_3 = \int \bar{G}_2(t)dt$, $\psi_4 = \int \bar{G}_1(t)dt$

RELIABILITY AND MEAN TIME TO SYSTEM FAILURE

Let the random variable T_i be the time to system failure when the system initially starts from states $S_i \in E$, then the reliability of the system is given by

$$R_i(t) = P[T_i > t]$$

To determine $R_i(t)$, we assume that the failed states (S_3 to S_{10}) of the system as observing. By using the simple probabilistic arguments, we observe that $R_0(t)$ is the sum of the following mutually exclusive contingencies.

(a) The system remains up in state S_0 upto time t then the probability of this contingency is $Z_0(t) = e^{-\Sigma\alpha_i t}$.

(b) System first enters into the state S_i (i=1, 2) from the state S_0 during (u, u+du) u<t, and then starting from S_i (i=1, 2) it remains up continuously during remaining time (t-u).

The probability of this contingency is

$$\int_0^t q_{01}(t)R_1(t-u)du + \int_0^t q_{02}(t)R_2(t-u)du = q_{01}(t)©\mathrm{R}_1(\mathrm{t}) + \mathrm{q}_{02}(\mathrm{t})©\mathrm{R}_2(\mathrm{t})$$

Therefore,

$$R_0(t) = Z_0(t) + q_{01}(t)©R_1(t) + q_{02}(t)©R_2(t)$$

Similarly

$$R_1(t) = Z_1(t) + q_{10}(t)©R_0(t)$$

$$R_2(t) = Z_2(t) + q_{20}(t)©R_0(t)$$

(1-3)

where

$$Z_1(t) = e^{-(\alpha_1+\alpha_2+\alpha_4)t}\bar{G}_3(t)$$

$$Z_2(t) = e^{-(\alpha_1+\alpha_2+\alpha_3)t}\bar{G}_4(t)$$

Taking Laplace Transform of the relations (1-3), we have

$$R_0^*(s) = Z_0^*(s) + q_{01}^*(s)R_1^*(s) + q_{02}^*(s)R_2^*(s)$$

$$R_1^*(s) = Z_1^*(s) + q_{10}^*(s)R_0^*(s)$$

$$R_2^*(s) = Z_2^*(s) + q_{20}^*(s)R_0^*(s)$$

(4-6)

After solving the relation (4-6), we have

$$R_0^*(s) = \frac{Z_0^*(s) + q_{01}^*(s)Z_1^*(s) + q_{02}^*(s)Z_2^*(s)}{1 - q_{01}^*(s)q_{10}^*(s) - q_{02}^*(s)q_{20}^*(s)} \quad (7)$$

By taking the inverse Laplace Transform of the equation (7), we get the reliability of the system starting from S_0 for known values of parameters, mean time to system failure is given by

$$E(T_0) = \int R_0(t)dt = \lim_{s\to 0} R_0^*(s) = \frac{\Psi_0 + P_{01}\Psi_1 + P_{02}\Psi_2}{1 - P_{01}P_{10} - P_{02}P_{20}} \quad (8)$$

as $Z_i^*(0) = \Psi_i$ and $q_{ij}^* = P_{ij}$

AVAILABILITY ANALYSIS

Let $A_i(t)$ be the probability that the system is in up state at instant t, given that the system entered in regenerative state S_i at t=0. By using the probabilistic argument, the recursive relations for $A_i(t)$ are given as

$$A_0(t) = Z_0(t) + q_{01}(t)©A_1(t) + q_{02}(t)©A_2(t) + q_{03}(t)©A_3(t) + q_{04}(t)©A_4(t)$$

$$A_1(t) = Z_0(t) + q_{10}(t)©A_0(t) + q_{12}^{(7)}(t)©A_2(t) + q_{13}^{(8)}(t)©A_3(t) + q_{14}^{(5)}(t)©A_4(t)$$

$$A_2(t) = Z_2(t) + q_{20}(t)©A_0(t) + q_{21}^{(10)}(t)©A_1(t) + q_{23}^{(9)}(t)©A_3(t) + q_{24}^{(6)}(t)©A_4(t)$$

$$A_3(t) = q_{30}(t)©A_0(t)$$

$$A_4(t) = q_{40}(t)©A_0(t)$$

(9-13)

where

$$Z_0(t) = e^{-\Sigma\alpha_i t}, \quad Z_1(t) = e^{-(\alpha_1+\alpha_2+\alpha_4)t}\bar{G}_3(t), \quad Z_2(t) = e^{-(\alpha_1+\alpha_2+\alpha_4)t}\bar{G}_4(t)$$

Taking the Laplace Transform of the relation (9-13) and solving them for $A_0^*(s)$ and then steady state availability of the system is given by

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \frac{N_1}{D_1'} \tag{14}$$

where

$$N_1 = \left(1 - P_{12}^{(7)}P_{21}^{(10)}\right)\psi_0 + \left(\mathrm{P}_{01} + \mathrm{P}_{02}\mathrm{P}_{21}^{(10)}\right)\psi_1 + \left(\mathrm{P}_{02} + \mathrm{P}_{01}\mathrm{P}_{12}^{(7)}\right)\psi_2$$

By using $q'^{*}_{ij}(0) = -\int tq_{ij}^*(t) = -m_{ij}$ and $\psi_\mathrm{i} = \sum_j m_{ij}$, we get

$$D_1' = 4\left(1 - P_{12}^{(7)}P_{21}^{(10)}\right)\psi_0 + 4\left(\mathrm{P}_{01} + \mathrm{P}_{02}\mathrm{P}_{21}^{(10)}\right)\psi_1 + 4\left(\mathrm{P}_{02} + \mathrm{P}_{01}\mathrm{P}_{12}^{(9)}\right)\psi_2 + \left[\mathrm{P}_{03}\left(1 - \mathrm{P}_{12}^{(7)}\mathrm{P}_{21}^{(10)}\right) + \mathrm{P}_{01}\left(\mathrm{P}_{13}^{(8)} + \mathrm{P}_{12}^{(7)}\mathrm{P}_{23}^{(9)}\right) + \mathrm{P}_{02}\left(\mathrm{P}_{23}^{(9)} + \mathrm{P}_{21}^{(10)}\mathrm{P}_{13}^{(8)}\right)\right]\psi_3 + \left[\mathrm{P}_{01}\left(\mathrm{P}_{14}^{(5)} + \mathrm{P}_{12}^{(7)}\mathrm{P}_{24}^{(6)}\right) + \mathrm{P}_{02}\left(\mathrm{P}_{24}^{(6)} + \mathrm{P}_{21}^{(10)}\mathrm{P}_{14}^{(5)}\right) + \mathrm{P}_{04}\left(1 - \mathrm{P}_{12}^{(7)}\mathrm{P}_{21}^{(10)}\right)\right]\psi_4$$

The expected up time of the system during time interval (0, t) is given by

$$\mu_{up}(t) = \int_0^t A_0(u)du \tag{15}$$

so that

$$\mu_{up}^*(s) = \frac{A_0^*(s)}{s} \tag{16}$$

BUSY PERIOD ANALYSIS

Let $B_i(t)$ is defined as the probability that the repairman is busy at epoch t starting from $S_i \in E$. By using the elementary probabilistic arguments, we have the following relations

$$B_0(t) = q_{01}(t)©\mathrm{B}_1(\mathrm{t}) + \mathrm{q}_{02}(\mathrm{t})©\mathrm{B}_2(\mathrm{t}) + \mathrm{q}_{03}(\mathrm{t})©\mathrm{B}_3(\mathrm{t}) + \mathrm{q}_{04}(\mathrm{t})©\mathrm{B}_3(\mathrm{t})$$

$$B_1(t) = \delta_3 Z_1(t) + q_{10}(t)©\mathrm{B}_0(\mathrm{t}) + \mathrm{q}_{12}^{(9)}(\mathrm{t})©\mathrm{B}_2(\mathrm{t}) + \mathrm{q}_{13}^{(8)}(\mathrm{t})©\mathrm{B}_3(\mathrm{t}) + \mathrm{q}_{14}^{(5)}(\mathrm{t})©\mathrm{B}_4(\mathrm{t})$$

$$B_2(t) = \delta_4 Z_2(t) + q_{20}(t)©\mathrm{B}_0(\mathrm{t}) + \mathrm{q}_{21}^{(10)}(\mathrm{t})©\mathrm{B}_1(\mathrm{t}) + \mathrm{q}_{23}^{(9)}(\mathrm{t})©\mathrm{B}_3(\mathrm{t}) + \mathrm{q}_{24}^{(6)}(\mathrm{t})©\mathrm{B}_4(\mathrm{t})$$

$$B_3(t) = \delta_2 Z_3(t) + q_{30}(t)©B_0(t)$$

$$B_4(t) = \delta_1 Z_4(t) + q_{40}(t)©B_0(t)$$

(17-21)

where

$Z_3(t) = \bar{G}_2(t)$, $Z_4(t) = \bar{G}_1(t)$

Taking the Laplace Transform of equations (17-21) and then solving them for $B_0^*(s)$. Omitting the arguments 's' for brevity, we get

$$B_0^*(s) = \frac{N_2(s)}{D_1(s)} \tag{22}$$

Now, if $B_0^R(t)$, $B_0^P(t)$, $B_0^{H_1}(t)$ and $B_0^{H_2}(t)$ respectively be the probability that the system is under repair at epoch t, when system initially starts from state S_0, due to the failure of R, P, H_1 and H_2. Then the separate values of the probabilities in terms of their Laplace Transform can be obtained from (22) by putting

$$(\delta_1 = 1\ ,\ \delta_2 = \delta_3 = \delta_4 = 0) \text{ for } B_0^R$$
$$(\delta_2 = 1\ ,\ \delta_1 = \delta_3 = \delta_4 = 0) \text{ for } B_0^P$$
$$(\delta_3 = 1\ ,\ \delta_1 = \delta_2 = \delta_4 = 0) \text{ for } B_0^{H_1}$$
$$(\delta_4 = 1\ ,\ \delta_1 = \delta_2 = \delta_3 = 0) \text{ for } B_0^{H_2}$$

then

$$B_0^{R*}(s) = \frac{N_2^R(s)}{D_1(s)},\ B_0^{P*}(s) = \frac{N_2^P(s)}{D_1(s)},\ B_0^{H_1*}(s) = \frac{N_2^{H_1}(s)}{D_1(s)},\ B_0^{H_2*}(s) = \frac{N_2^{H_2}(s)}{D_1(s)} \tag{23-26}$$

In a long run, the probability that the repair facility will be busy in the repairing of failed R unit, is given as follows

$$B_0^R = \lim_{t\to\infty} B_0^R(t) = \lim_{s\to 0} sB_0^{R*}(s) = \lim_{s\to 0} \frac{N_2^R(s)}{D_1(s)} = \frac{N_2^R}{D_1'} \tag{27}$$

where

$$N_2^R = \left(P_{04} + P_{01}P_{14}^{(5)} + P_{02}P_{24}^{(6)} + P_{01}P_{12}^{(7)}P_{24}^{(6)} + P_{02}P_{21}^{(10)}P_{14}^{(5)} - P_{04}P_{12}^{(7)}P_{21}^{(10)}\right)\Psi_4$$

Similarly, other steady state probabilities can be obtained as follows

$$B_0^P = \frac{N_2^P}{D_1'},\quad B_0^{H_1} = \frac{N_2^{H_1}}{D_1'},\quad B_0^{H_2} = \frac{N_2^{H_2}}{D_1'} \tag{28-30}$$

where

$$N_2^P = \left(P_{03} + P_{01}P_{13}^{(8)} + P_{02}P_{23}^{(9)} + P_{01}P_{12}^{(7)}P_{23}^{(9)} + P_{02}P_{21}^{(10)}P_{13}^{(8)} - P_{03}P_{12}^{(7)}P_{21}^{(10)}\right)\Psi_3$$

$$N_2^{H_1} = \left(P_{01} + P_{02}P_{21}^{(10)}\right)\Psi_1$$

$$N_2^{H_2} = \left(P_{02} + P_{01}P_{12}^{(7)}\right)\Psi_2$$

Now, the expected busy period of the repair facility in repairing R unit, during the time interval (0, t) is given as

$$\mu_b^R(t) = \int_0^t B_0^R(u)du \tag{31}$$

so that

$$\mu_b^{R*} = \frac{B_0^{R*}(s)}{s} \tag{32}$$

Similarly, the other expected busy period can be obtain as

$$\mu_b^{P*} = \frac{B_0^{P*}(s)}{s},\quad \mu_b^{H_1*} = \frac{B_0^{H_1*}(s)}{s},\quad \mu_b^{H_2*} = \frac{B_0^{H_2*}(s)}{s} \tag{33-35}$$

PROFIT ANALYSIS

The expected profit incurred by the system during (0, t] is given by

P(t)= Expected total revenue in (0, t]-Expected total repair cost in (0, t]

$$= C_0\mu_{up}(t) - C_1\mu_b^R(t) - C_2\mu_b^P(t) - C_3\mu_b^{H_1}(t) - C_4\mu_b^{H_2}(t) \tag{36}$$

Where C_0 be the per unit up time revenue by the system and C_1, C_2, C_3 and C_4 be the cost per unit down time when the system is under repair due to the failure of R, P, H_1 and H_2 units respectively. The expected profit per unit in steady state is given by

$$P = C_0A_0 - C_1B_0^R - C_2B_0^P - C_3B_0^{H_1} - C_4B_0^{H_2} \quad (37)$$

GRAPHICAL INTERPRETATION

In particular, let all the repair time distributions are also follow the exponential distribution.

$G_1(t) = 1 - e^{-\lambda_1 t}$, $G_2(t) = 1 - e^{-\lambda_2 t}$, $G_3(t) = 1 - e^{-\lambda_3 t}$, $G_4(t) = 1 - e^{-\lambda_4 t}$

Fig. 3 shows that the MTSF with respect to the failure rate of bottle filling machine (α_4) for different value of α_3 =0.02, 0.03, 0.04 when the other parameters are kept fixed as α_1=0.001, α_2=0.002, λ_3=0.02 and λ_4=0.03, decreases when α_3 and α_4 increase.

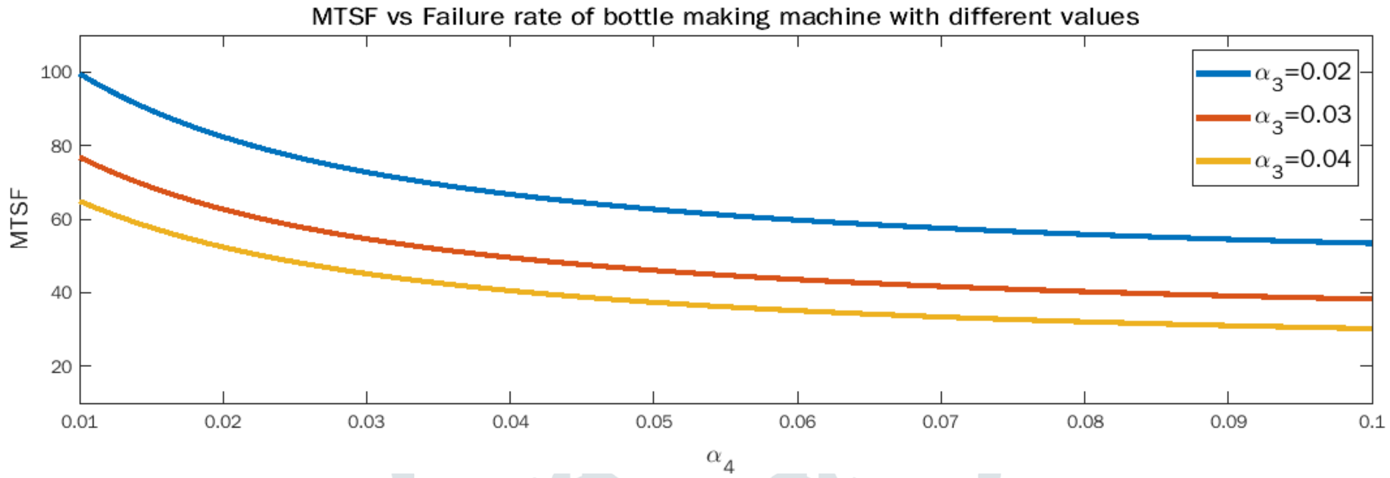


Figure 3

Fig. 4 shows the profit with respect to time for different values of repair rate of bottle filling machine λ_4=0.05, 0.10, 0.20 when the other parameters are kept fixed α_1=0.001, α_2=0.002, α_3=0.01, α_4=0.02, λ_1=0.01, λ_2=0.02, λ_3=0.03, C_0=6000, C_1=250, C_2=350, C_3=400 and C_4=450, decreases when t increases. It is also observed from the graph that if λ_4=0.05, then the profit decreases rapidly. But if λ_4 increases then the profit decreases slowly.

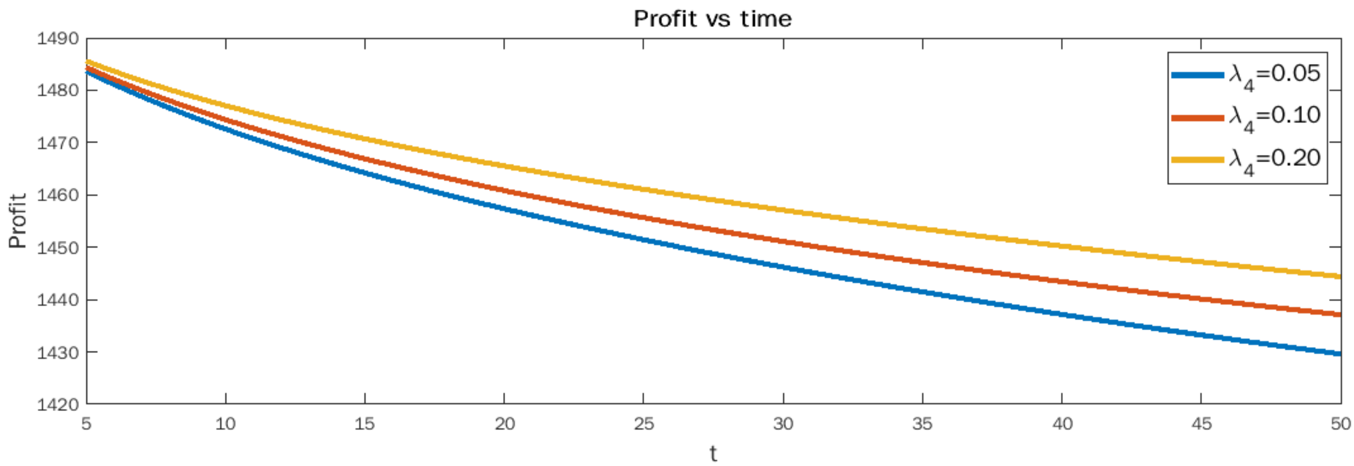


Figure 4

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